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Preliminary Remarks

The present D 658 76a is a translation of the original Russian text edited by Ja. E. Binowitsch, N. J. Grusdjew, P. J. Iwanow and A. A. Prokofiew. Its contents include theoretical considerations of tractive effort, stability, turning ability and amphibiousness of tanks in addition to their various spring types.

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Section 1.

CALCULATION OF TRACTIVE EFFORT

1. Purpose and Method of Calculating Tractive Effort

Tractive effort must be calculated in order to determine the main characteristics of a tank which enable it to cross over ground of a given type at the prescribed speed and to overcome assumed differences in height of the terrain.

When designing a tank or a tracked towing vehicle[1)] (tractor with tracks) for transport purposes, the following values must be known in order to calculate the tractive effort:

a) Braking power of the engine N_e in HP.

b) Minimum traveling speed v_{min} in km/h.

c) Speed gradations (between maximum and minimum speed) v_2, $v_3 \ldots v_{r-1}$ in km/h.

d) Average speeds in the various speed levels v_d in km/h.

e) Angle of inclination and gradient of slope (in relation to the pertinent preconditions for the trip) α (in degrees) or $i = tg\ \alpha$ (in percentage).

f) Accelerations b in m/sec^2, which come into consideration with the given preconditions for the trip.

g) The weight of the load C_p in kg, which can be towed with the pertinent preconditions for the trip.

h) The distance s in m and the time span T in sec. for the start-up and braking of the tank.

As preliminary data for solving the given problems we use the so-called preconditions for the construction plan of the tank.

Customarily the following data are given in the construction plan:

a) Combat weight G (of the tank) or the weight of the towed load G_a (or the tracked towing vehicle);

Note: Henceforth the tracked towing vehicle will be mentioned only when some special feature is involved in the calculation.

b) Armor plating and armaments;

c) Maximum speed v_{max} at the best preconditions for the trip (height differences of the terrain and type of ground);

d) Maximum angle of gradient to be surmounted α_{max}.

Sometimes, in addition to those enumerated so-called main requirements, there are additional ones such as:

a) Minimum allowable number of gear ratios i_1;

b) Desirable speeds v_{i1};

c) Allowable specific pressure on the ground q (km/cm^2);

d) Maximum allowable dimensions -- length L, width B and height H;

e) Maximum angle of longitudinal and transverse stability α_{max} and β_{max};

f) Ground clearance (height of the lowest point of the body above the road).

The following method was introduced for calculating tractive effort:

First, all exterior forces and movement resistances were determined which effect the tank in its forward progress.

In the case of a towing vehicle we determine in addition its weight at the prescribed most difficult travel conditions.

Then, the forces when towing and operating the tank are determined.

When we have determined the required power output of the engine and when we know the most difficult preconditions for the trip according to the technical requirements, there are no further difficulties in determining the required minimum travel speed v_{min} and solving all other problems in regard to calculating tractive effort.

It is natural then, that we should be able to check the above-cited values for the tank in question according to the method for calculating tractive effort.

2. The Exterior, Inertia and Moment Resistances which generally Effect a Tank Under Motion.

We now wish to give closer consideration to the mechanical movement of a tank. The drive sprocket α of the tank is driven by the transmission of the tank engine (Fig. 1).

In the process, the drive sprocket tightens the track and moves it under the tank. Since the track is pressed against the earth's surface by the weight G of the tank, and the counterpressure P of the earth's surface is exerted upward between the track and the earth's surface, the track is resisted from slipping along the ground.

The one-piece track pulled by the drive sprocket under the road wheels causes the tank to move forward onto the path formed by the track.

The part of the track links thus freed are moved away from the earth's surface, over the drive sprocket and back onto the earth's surface.

The exterior propulsive force for any movement of the tank is designated as tractive effort and is exerted in the direction opposite track motion.

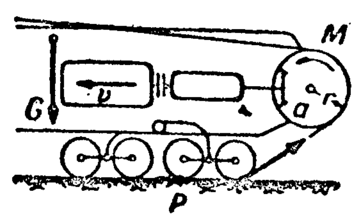


Figure 1. Mechanics of Motion on a Tank (movement phenomena)

The amount of this force is significantly influenced either by the extent of the torque exerted on the drive sprocket by the engine or by available ground traction.

The tractive effort P_d is determined according to the amount of torque exerted on the track.

If M is the torque exerted on the drive sprocket, and r is its radius, then the tractive effort of the engine is expressed by the following formula:

$$P_d = \frac{M}{r} \tag{1}$$

It appears that the larger the torque M or the smaller the radius of the drive sprocket r, the greater the initial thrust P transmitted to the earth's surface.

In reality there is a maximum value of this force which, when exceeded, causes the track to slip on the earth's surface.

The maximum value of the tractive effort which is governed by the tractive capacity of the earth's surface is designated as P_φ.

P_φ is proportional to the weight of the tank, which is equal to the counteraction of the earth's surface Q on the resistance surface of the tank.

The coefficient of the direct ratio is designated as traction coefficient.

Thus,

$$P_\varphi = \varphi Q \tag{2}$$

The dependence and size of the coefficient φ is treated in more detail later on in the text.

On the basis of this information we have the inequality

$$P_d < P_\varphi \tag{3}$$

a precondition that the tracks do not "grind"or slip.

We wish to determine the point of application of the force P.

For this purpose we wish to consider some of the track links (Fig. 2).

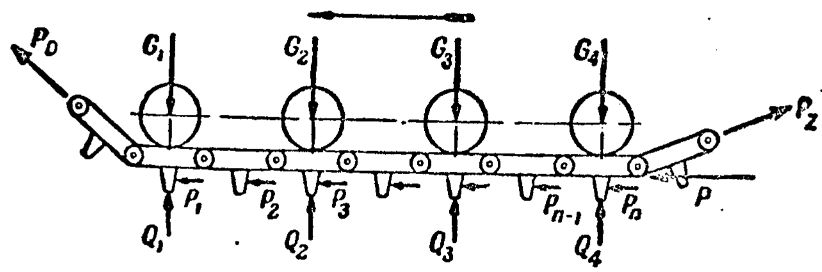


Figure 2. Effect of Ground Reaction on the Track Links

In Fig. 2, P_z and P_D represent the points of application of the track ends, while G_1, G_2 and G_3 .....are the loads transmitted by the road wheels.

As a result of friction and traction of the grips on each track link, countereffects are caused by the earth's surface, each of which can be analyzed in the following individual components: the vertical components Q, and the horizontal components P_i.

Of course the tangential countereffects (reactions) P of the earth's surface, which are exerted in the direction of travel, are distributed on the under surfaces of the track links, insofar as these are subject to traction on the earth's surface, as well as between the ends of the grips.

Without committing greater errors, this force can be considered as lying in the plane of the track tread.

The next group of exterior forces are the weights of the individual tank parts, which can be replaced by the force G, which is equal to the weight of the tank as a whole and to the force of gravity exerted at its center point.

This force can be analyzed in the following components: the components $G \sin \alpha$ running (parallel) to the plane of motion and the components $G \cos \alpha$ situated vertically to this plane (see Fig. 3).

The force $G \sin \alpha$ is designated as slope resistance.

When traveling uphill, the force $G \sin \alpha$ is exerted opposite the direction of travel and exerts resistance against forward motion while, when traveling downhill, it is exerted in the direction of travel and becomes a moving force.

Since the tank cannot surmount gradients of more than 45°, the force $G \sin \alpha$ fluctuates between 0 and 0.7 G.

The force $G \cos \alpha$ is the adhesion weight of the tank when traveling without a trailer; it changes in relation to the amount of the angle of gradient from G to 0.7 G.

When the angle of gradient α increases, the components of the weight $G \sin \alpha$ increase relatively more rapidly than the other components. $G \cos \alpha$ becomes smaller.

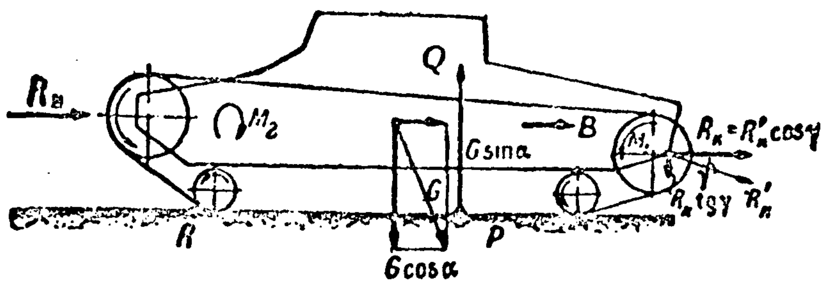


Figure 3. Exterior Forces and Moments of Force effecting the Tank

We will assume that the tank is loaded on the towing shackle with the force R'_K, the direction of which forms the angle γ with the tank's plane of travel.

Consequently, the force R'_K can be analyzed in components:

those running with the plane of travel

$$R_K = R'_K \cos \gamma,$$

which exerts travel resistance and those

$$R'_K \sin \gamma = R_K \operatorname{tg} \gamma,$$

which run vertically to this plane and increase or decrease the adhesion weight or the countereffect of the earth's surface Q.

We assume that the force of air resistance R_w is exerted at the center point of the bow.

This force originates during travel of the tank for the following reasons:

a) by the pressure of air particles on the frontal surface of the tank;
b) by the resultant thinning out of the air behind the tank, so that the pressure on the front of the tank is not completely equalized by the air pressure on the rear;
c) by the friction of air particles on the upper surface of the tank.

The force of air resistance is calculated according to the following empirical formula:

$$R_w = KFv^2 \tag{4}$$

where v is the assumed tank speed, K is the coefficient of air resistance equal to 0.065 $kg/s^2/m^4$, in case v is expressed in m/s or 0.0048 $kg/h^2/m^2km^2$, and in case v is expressed in km/h.

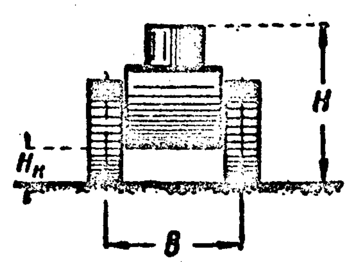


Figure 4.

F we assume is approximately equal to the product of the full tank height H, subtracting ground clearance H_K, with the tank track width B (Fig. 4):

$$F = (H - H_K)\, B \tag{5}$$

At speeds below 50 km/h the force of air resistance R_w can be disregarded because it is so small.

We do not wish to deal with individual cases in which a strong headwind is encountered at low speed.

EXAMPLE

It is assumed that a tank is traveling along a road at 30 km/h.

The numerical data for the tank is as follows:

$$B = 2.03 \text{ m}$$
$$H = 2.19 \text{ m}$$
$$H_k = 0.29 \text{ m}$$

Thus,

$$R_w = kFv^2 = K(H - H_k)Bv^2,$$

that means

$$R_v = 0{,}0048 \cdot 1{,}9 \cdot 2{,}03 \cdot 30^2 = \text{ approx. } 17 \text{ kg}$$

(we do not consider the force of the wind).

If the output of the tank engine is N_e = 90 HP, then the tractive force P_d exerted by the engine can be expressed as follows:

$$P_d = \frac{75\, N_e \eta}{v} \tag{6}$$

where η is the tank efficiency, while v is the tank speed in m/s.

If the speed of the tank v is given in km/h, then the tractive force emanating from the engine takes the following form:

$$P_d = \frac{75 \cdot 3{,}6 \cdot N_e \eta}{v} = \frac{270\, N_e \eta}{v} \tag{7}$$

In our case, P_d is approximately 610 kg, the efficiency of the tank η is assumed to be 0.75.

In other words:

$$R_w = \text{ approx. } 0.03\, P_d,$$

that is to say a value which can be disregarded because when making a numerical comparison of the travel resistance $f(G_1-H_2)$ of two tanks of one and the same manufacturing series, the difference can attain a still higher value.

When we assume another relation with the same tank and the same preconditions, that the speed will amount to 50 km/h, we obtain for air resistance:

$$R_w = \text{approx. } 0.09\ P_d$$

that is to say a value which is so small that it can be neglected.

When traveling over soft ground, the tank compresses the ground with its tracks which causes vertical and horizontal counterpressures.

If the load of all road wheels is equal, or is greater on the first road wheel than on the succeeding ones, then in the first-mentioned case the track pressure has a rectangular form (Fig. 5a) and in the second case has the form of a trapezoid (Fig. 5b).

In this case the first road wheel deforms the ground when it comes into contact with non-compacted soil, while the other road wheels move in the track which has already been made.

The opposite occurs when, for example, loads in the trapezoidal form (Fig. 5c) occur when the tank is rear-heavy. Now the other road wheels will contribute to the deformation of the earth.

The resultant force of all horizontal countereffects of the earth on the track R (Fig. 6) which is exerted opposite the direction of travel, is designated as "Resistive force against the direction of travel". From this force it is assumed that it stands in direct ratio to the adhesion weight of the tank, where the coefficient of the direct ratio is designated as coefficient of travel resistance f

$$R = f\,Q \tag{8}$$

The significance of this coefficient f, the values with which it is associated as well as the method of ascertaining it, will be studied later.

The resultant force R is to be assumed between the lower contact surfaces of the track links with the ground and between the ends of the grips at a certain height A above the track level.

If we are dealing with a smooth tank track and if the tank is traveling over solid ground, it can be calculated that this force is exerted on the contact surface of the track links by the earth's surface.

Finally, it is necessary to calculate the resultant force Q of all normal counteractions of the earth's surface to the total of exterior forces (see Fig. 3). This force appears at the pressure center point of the track on the earth's surface.

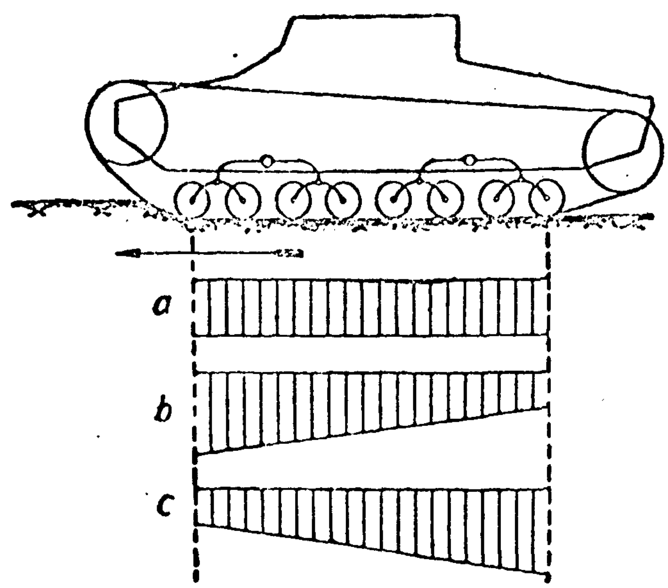


Figure 5. Load Plan

The position of the pressure center point is in no sense uniform, but changes within the limits of the tank supporting surface in relation to its type of movement and the total number of forces exerted on it (see Section II Stability).

Correspondingly, in the general case of tank movement, the following forces are exerted on the tank (see Fig. 3):

= Tractive effort P,
= Weight components $G \cos \alpha$ and $G \sin \alpha$,
= Traction components on the hook R_k and $R_k \operatorname{tg} \gamma$,
= Force of the travel resistance R,
= Resultant force of all normal reactions of the earth's surface Q.

Since later on the principle of d'Alembert is applied when equilibrium is established, we now wish to consider the inert forces and the moments of inertia more closely.

When considering the non-uniform forward movement of the tank, we introduce the concept of the inertia forces of forward moving masses.

The force B (Fig. 3) resulting from these forces is exerted on the center of gravity of the tank and is the product of the mass of the tank and its acceleration.

It is parallel to the road and in the opposite direction to the forward acceleration of the tank b

$$B = \frac{G}{g} b \tag{9}$$

The force of inertia can reach a considerable value when starting up suddenly from a standing position or when braking suddenly.

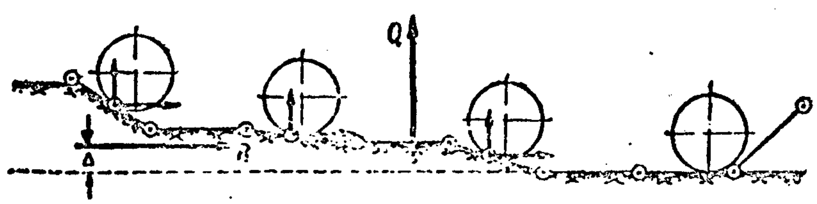


Figure 6. Deformation of the Earth's Surface by the Road Wheels

All individual tank parts participate in its forward movement. Some of them undergo a rotational movement opposite the tank body.

The tangential forces of inertia, which result from a non-uniform rotational movement, form corresponding moments.

We wish to designate the sum of the moments of tangential forces of inertia of the rotating components from the engine to the drive sprocket with reference to this gear as M_1 (Fig. 3) and the sum of the moments of the tangential inert forces of the rotating components behind the drive sprocket with reference to this gear as M_2.

To the first group belong the main clutch, the steering clutches, the final drive, the shafts and the wheels of the gear shift.

The moment of inertia of some individual parts of this group is expressed in the following formula:

$$M_i = J_i \varepsilon_i \tag{10}$$

Herein:

J_i is the moment of inertia of this individual part opposite its rotational axis and

ε_i is the angular acceleration of the part.

If the angular acceleration ε_i is converted into a linear acceleration b of the tank (without taking slip of the tracks into account), we obtain the following equation:

$$\varepsilon_i = \frac{d\omega_i}{dt} = \frac{i_i}{r}\frac{dv}{dt} \tag{11}$$

In this case, i is the ratio of the respective individual parts to the drive sprocket and

r is the radius of the drive sprocket,
ω = angular velocity.

After recalculating the moments of inertia of the parts of this group to the drive sprocket circumference, we obtain the following formula:

$$M_1 = b\sum \frac{J_i\, i_i^2}{r} \tag{12}$$

To the second group belong: The drive sprocket and idler wheel, the track supporting wheel and the road wheels as well as the track. The moment of inertia of an individual part of this group is expressed in the following formula:

$$M_i = J_i \varepsilon_i \tag{13}$$

where J_i is the moment of inertia of this individual part opposite the rotational axis and ε_i is the angular acceleration of this part.

When we transform this angular acceleration ε_i to linear acceleration b_i of the tank (without considering the slip of the wheels), we obtain the following formula

$$M_i = \frac{J_i}{r_i} b, \tag{14}$$

because

$$\varepsilon_i = \frac{d\omega_i}{dt} = \frac{dv}{dt}\,\frac{1}{r_i} = \frac{1}{r_i}\,b \tag{15}$$

Here r_i is the rotational radius of the appropriate individual part.

After recalculating these moments of inertia to the circumference of the drive sprocket, we obtain the total moment in the following formula:

$$M_\Sigma = \left(\sum \frac{J_i}{r_i^2}\,r + \frac{J}{r}\right) b \tag{16}$$

Here J and r are the moments of inertia relative to the radius of the drive sprocket and its radius.

In addition, the tracks move around the outside of the tank at a relative velocity v_r which is equal to the forward moving speed of the tank v (if the track does not "grind" on the spot).

The corresponding force of inertia is

$$J_r = \frac{G_r}{g}\,b, \tag{17}$$

where G_r is the weight of the track.

Consequently, the total force of inertia of the non-uniformly rotating individual parts of the tank, if this force is recalculated to the drive sprocket, is expressed in the following formula:

$$J_\delta = \left(\sum \frac{J_i}{r_i^2} + \frac{J}{r^2} + \sum \frac{J_i^2\, i}{r^2} + \frac{G_r}{g}\right) b \tag{18}$$

The full force of inertia of the tank is expressed by:

$$B = \frac{G}{g}\left(1 + \frac{g}{G}\sum \frac{J_i}{r_i^2} + \frac{g}{G}\,\frac{J}{r^2} + \frac{g}{G}\sum \frac{J_i^2\, i}{r^2} + \frac{G_r}{G}\right) b \tag{19}$$

or we obtain, if we replace the expression in parentheses by δ, finally:

$$B = \delta\,\frac{G}{g}\,b \tag{20}$$

The coefficient δ is designated as the coefficient of the apparent increase of the mass due to the presence of relative movements of the individual tank parts.

3. Equilibrium of Forces of the Tank

Applying the principle of d'Alembert, we write the equation of the projections on the longitudinal axis of the tank of all the forces effecting it, and of the mass acceleration.

We are assuming that the tank surmounts a slope having an angle α under acceleration. In this case

$$P = fQ + G \sin \alpha + R_k + R_w + \frac{G}{g} b \qquad (21)$$

From this it can be seen that all exterior forces which exert resistance during travel as well as the force of inertia under forward movement are equalized by the tractive effort of the tracks. This exterior tractive effort is the tangential reaction of the earth's surface itself on the tracks, exerted in the direction of travel.

It has already been mentioned that the maximum value of P_{φ} of this exterior tractive effort is stipulated by the adhesive properties of the ground and is designated as adhesion.

Thus, the exterior tractive effort corresponds to the circumferential force of the drive sprocket. This force must be dissipated by the engine (we will temporarily neglect the force losses in the track and the slip loss in the track).

Equation (21) can be reconverted into the following form:

$$P = \sum R_i + \frac{G}{g} b,$$

in which $\sum R_i$ is the sum of all resistances with the exception of the force of inertia of drive acceleration.

If, then,

$$\sum R_i = P \leqq P_{\varphi} \qquad (22)$$

$$\sum R_i > P, < P_{\varphi} \qquad (23)$$

the movement will proceed uniformly.

Other conditions correspond to starting up and acceleration.

Finally, if the inequality

$$\sum R_i > P \leq P_\varphi \tag{24}$$

applies, where

$$P_\varphi > \sum R,$$

a deceleration occurs, which, as will be proven, can lead to the engine stalling.

If the road wheels do not rotate uniformly, formula (21) is written in the following form:

$$P = fQ + G \sin \alpha + R_k + R_w + \delta_1 \frac{G}{g} b \tag{25}$$

If we knew the traveling speed v, then if we knew the tractive effort of the track, we could calculate the power efficiency required for traveling on the track, i.e.

$$N_r = \frac{P v}{75}$$

(where we assume for the time being that the coefficient (efficiency) of the track is equal to 1).

Even if we assume that the efficiency of the transfer of force onto the drive sprockets is equal to 1, this power efficiency N_r cannot equal that power efficiency required from the engine; it must be either larger or smaller according to whether we are dealing with an acceleration or a deceleration.

This may be explained by the fact that on the path which the flux of force takes from the engine to the drive sprockets or from the drive sprockets to the ground, a series of individual phenomena occur which store up energy at acceleration, or which give off stored up energy at deceleration.

Consequently, the efficiency of the engine under travel at non-uniform speed must be determined according to the following formula

$$N_d = \frac{P' v}{75},$$

where

$$P' = P + \delta \frac{G}{g} b \qquad (26)$$

When the tank travels without a trailer ($P'_k = 0$) (R_w = approx. 0) Eq. (21) takes on the following appearance:

$$P = fG \cos \alpha + G \sin \alpha + R_w + \frac{G}{g} b \qquad (21a)$$

At uniform speed (b =0) under rapid travel (but below 50 km/h), the tank can surmount a maximum angle of inclination from 5-6^o; consequently $\cos \alpha \sim 1$ and $\sin \alpha \sim tg \alpha = i$.

Then,

$$P = (f + i) G \qquad (26a)$$

When surmounting an inclination greater than $\alpha = 6^o$ and with uniform travel, such an approximate value is too rough; consequently, P must be determined according to the following formula:

$$P = (f \cos \alpha + \sin \alpha) G \qquad (26b)$$

With regard to the coefficient δ of the apparent increase of the tank mass, the following observations can be made.

As is evident from formula (19), the value of the gear ratio i from the engine to the drive sprockets exerts the greatest influence on the size of the coefficient δ, because the value of this ratio is contained in the formula squared.

Of the resultant empirical formulas for the determination of the coefficient δ, the following formula gives corresponding results with reasonable accuracy in a series of cases:

$$\delta = 1{,}2 + 0{,}002\, i_0^2 \qquad (27)$$

Here i_o is the general numerical value of the transfer from the engine to the track.

When accelerating the tank with the steering clutch disengaged, the coefficient δ is approximately 1.3-1.4.

The combined multipliers $f + i$ and $f \cos \alpha + \sin \alpha$ in formulas (26a) and (26b) are designated as the coefficient of total travel resistance of the tank; it is designated with the sign f_o.

4. Operating Equilibrium of the Tank

The operating equilibrium of the tank is designated as the distribution of engine power efficiency on the individual types of resistance.

Consequently, in case the traction equilibrium of the tank is expressed in the formula:

$$P' = fQ + G\sin\alpha + R_K + R_w + \delta\frac{G}{g}b \tag{28}$$

its operating equilibrium is expressed by the following formula:

$$N_e = N_f \pm N_\alpha + N_K + N_w \pm N_b \tag{29}$$

If, however, the power loss of the power plant is considered in addition, then

$$N_e = N_\eta + N_f \pm N_\alpha + N_K + N_w \pm N_b \tag{30}$$

N_η is the power loss brought about by the power plant.

As is well known, the actual power efficiency of an engine is expressed by the following formula:

$$N_e = N_i - N_r \tag{31}$$

in which N_i is the nominal power and N_r the friction loss.

In connection with what types of power losses are included in N_r and what is understood by N_i, the value is determined by N_e.

Usually we include in the value N_r only the loss of power due to inner friction and power loss of the engine by the following individual parts: Oil and water pump, the usual electric equipment and the fuel pump.

The power loss caused by the blower is considered only in the case of the engine with air cooling.

Consequently, when using high power engines patterned after aircraft engines with a power of N_e, the loss caused by the ventilator is not taken into account; it would have to be taken into account, however, and formula (30) would have to be written in the following form:

$$N_e - N_1 = N_d = N_\eta + N_f \pm N_\alpha + N_k + N_w \pm N_b. \qquad (32)$$

The power N_1 required from the blower is expressed by the following ratio:

$$N_1 \sim C\gamma n,$$

in which C is a constant coefficient which characterizes the blower, γ is the air density in kg/m^3, n is the rotational velocity of the engine per minute.

N_e of Figure 7is the fuel consumption of the blower at various engine rotational velocities recorded for an engine output of 500 HP at n = 1445 rpm.

The consideration of the loss by the blower at various engine rotational velocities can be summarized in the following formula:

$$N_i = N_1 \frac{n_i^3}{n^3} \qquad (33)$$

The power N_i, required from the blower at various velocities (rotational) of the engine is

Table 1.

n_d	N_i
1200	46
1300	58
1400	73
1500	89
1600	109

The top part of Fig. 7 depicts the characteristic line of an airplane engine, below the characteristic line with the engine choked and on the very bottom the same characteristic line having subtracted blower output.

All calculations must be conducted taking into consideration this changed characteristic line.

To make a comparison between the various tanks in regard to their maximum velocity, the following formula must be used:

$$N_G = \frac{N_e}{G} \qquad (34)$$

here N_G is the output which the tank develops per ton of weight, G is the weight of the tank in tons.

Sometimes the output is related to the product of weight and velocity

$$N_{Gv} = \frac{N_e}{G v_{max}} \qquad (35)$$

where N_e represents the actual power output of the tank engine without consideration of the power consumption by the blower.

With the assumed maximum velocity of the tank, the power equilibrium of the tank can be written down in the following form:

$$N_d = N_f + N_\alpha \qquad (36)$$

The power equation of the tank with the same type of travel is as follows:

$$P = (f + i) G$$

The value i of the inclination which is to be surmounted at maximum speed without shifting gears is usually assumed to be from 0.04 - 0.06.

If (according to the technical preconditions for the construction of the tank) the maximum speed of the tank v_{max} is known, and if at this velocity the tank travels in a uniform manner, then the required output of the tank can be calculated according to the following formula

$$N_d = \frac{P v_{max}}{270 \eta}, \qquad (37)$$

in which η is the output of the tank.

This formula is also used for determining the required power of the tank.

The required power of a tracked towing vehicle (tractor with tracks) is determined in a different manner.

In reality, in the case of technical preconditions for the construction of a towing vehicle with tracks, the weight G of the towing vehicle is not prescribed, but only the weight of the load G_n to be towed.

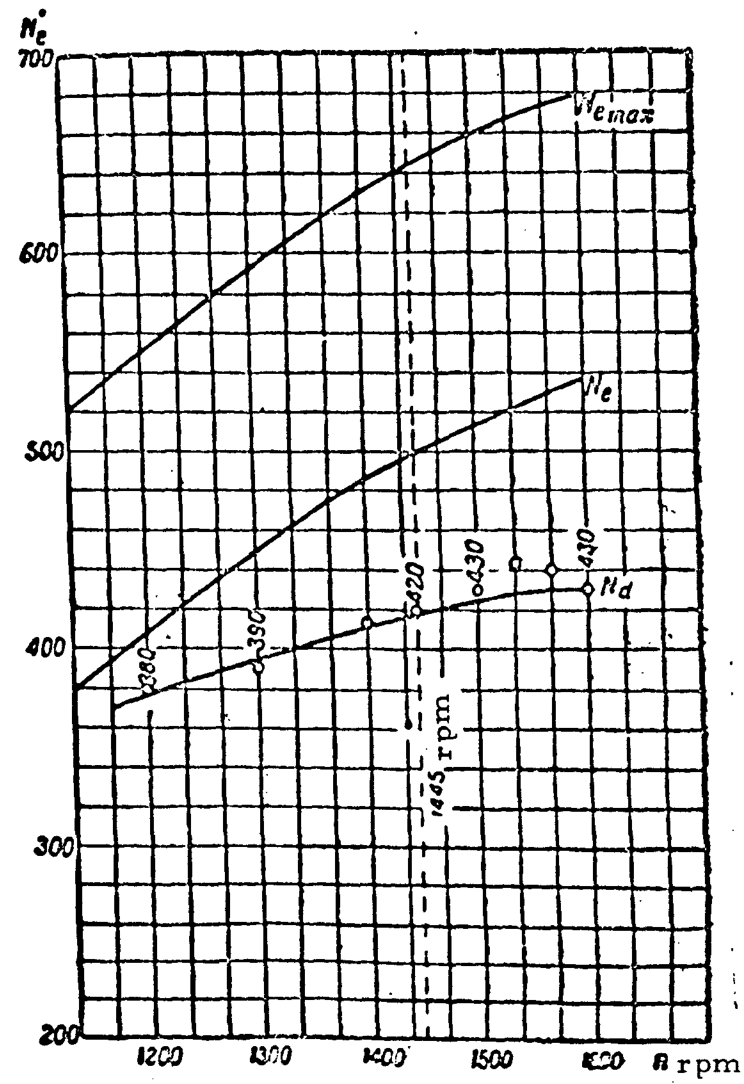


Figure 7. Engine Characteristic Lines with Consideration of Power Loss

In order now to apply formula (37), the weight of the towing vehicle must be determined beforehand.

The weight of the towing vehicle is determined for trips under the most difficult conditions, that is to say for the maximum possible tractive effort on the tracks which, as already mentioned, cannot be greater than P_φ because of ground adhesion.

In this manner we obtain the equation

$$P_\varphi = (f\cos\alpha + \sin\alpha)\,G + (f_n\cos\alpha + \sin\alpha)\,G_n = f_0 G + f_{0n} G_n.$$

for travel under the most difficult preconditions.

Since now $P_\varphi = \varphi G \cos\alpha$, the weight of the towing vehicle to be determined, assuming that the resistance coefficients against the movement of the towing vehicle and of the trailer are equal, appears in the following formula:

$$G = G_n \frac{(f \cos\alpha + \sin\alpha)}{(\varphi - f)\cos\alpha - \sin\alpha} = G_n \frac{f_0}{\varphi \cos\alpha - f_0} = G_n \frac{f + \operatorname{tg}\alpha}{\varphi - f - \operatorname{tg}\alpha} \qquad (38)$$

As soon as the weight of the towing vehicle has been determined, there are no further difficulties in calculating the required power output of the engine; it is

$$N_d = \frac{P\, v_{\max}}{270\,\eta},$$

where:

$$P = (f + i)\,(G + G_n)$$

5. The Mechanical Efficiency of the Tank

The mechanical efficiency of the tank is expressed by the product of the efficiencies of all series-connected transmission mechanisms from the engine up to and including the track.

When the power output of the tank engine is N_e and the power losses in the power plant are N_η, then the efficiency of the tank appears in the following formula:

$$\eta = \frac{N_e - N_\eta}{N_e} \qquad (39)$$

On the other hand the efficiency is

$$\eta = \eta_1\, \eta_2 \ldots\ldots \eta_n, \qquad (40)$$

in which $\eta_1, \eta_2 \ldots\ldots \eta_n$ are the mechanical efficiencies of the series-connected power plant parts.

Such mechanical power plant parts are spur gear pairs, bevel gear pairs, sometimes worm gears and finally, the tracks.

The mechanical efficiency of the toothed wheel gear (according to Kammerer and Kranz) is stipulated by the lubricated condition of the gear wheels and the appropriate oiled locations, by the load on the teeth and the level of rotational speed.

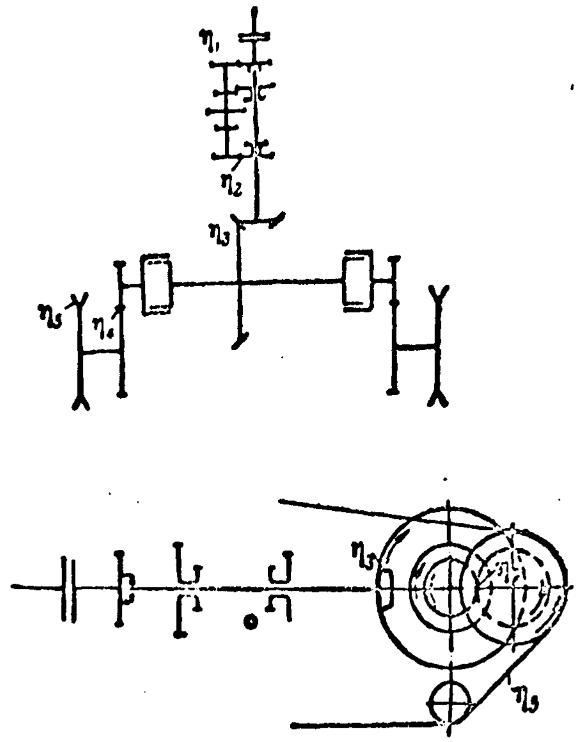


Figure 8. Schematic Representation of the Tank Power Plant
Drive Schematic of the Tank

Table 2 contains a compilation of the basic values in regard to the mechanical efficiency of individual tanks.

Table 2.

Efficiencies of Tank-Power Plant Parts

Name of the Components	Efficiency
Spur Gears	0.97 - 0.95
Bevel Gears	0.95 - 0.93
Worm Gears	0.91 - 0.85
Track	0.90 - 0.75

Table 3 gives the basic values of coefficients pertinent to the track dependent on tank traveling speed.

Table 3.

Efficiency of the Track (according to Karatschian)

Traveling Speed in km/h	Efficiency
0 - 5	0.95 - 0.90
5 - 10	0.90 - 0.85
10 - 20	0.80 - 0.75

The results of the tests on the training grounds in Aberdeen (United States) have proven that the efficiency of the track is influenced by the following factors:

a) By the traveling speed of the tank: the higher the traveling speed, the greater the power loss in the track. In fact, centrifugal force and the stress of the track, thus also the power loss increases with an increase of tank traveling speed due to the interactions of the track with the drive sprocket, which is unavoidable;

b) By the transmitted power efficiency: the efficiency becomes smaller with an increase of transmitted power;

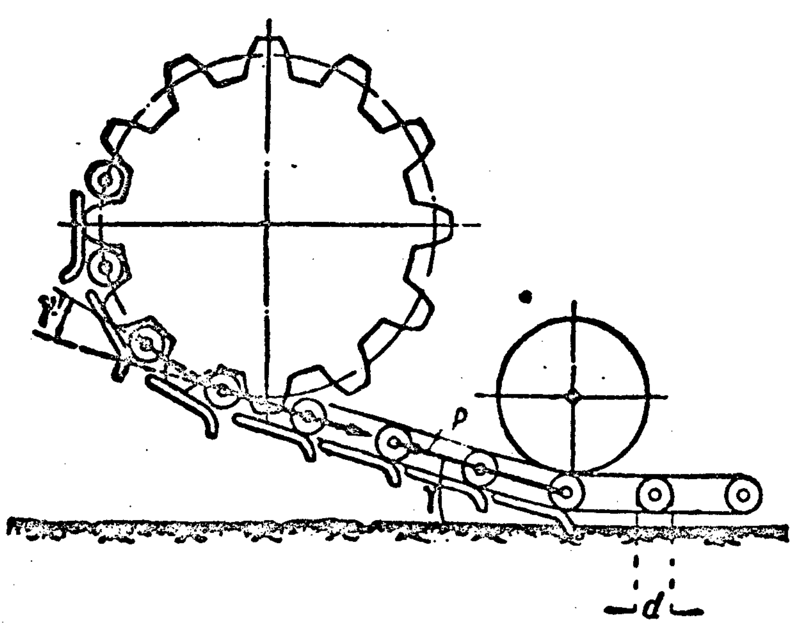


Figure 9. Friction in the Joints

c) By the degree of track tension: the value T, the friction in the track joint, can be expressed by the following formula (see Fig. 9):

$$T = \mu P \frac{d}{2 \cdot 100} \gamma \text{ mkg},$$

in which μ is the friction coefficient of the joint, P is the tension of the track link per kg, d is the diameter of the track bolt (in cm), γ is the cut-off angle of the link in comparison with the following track link (in degrees);

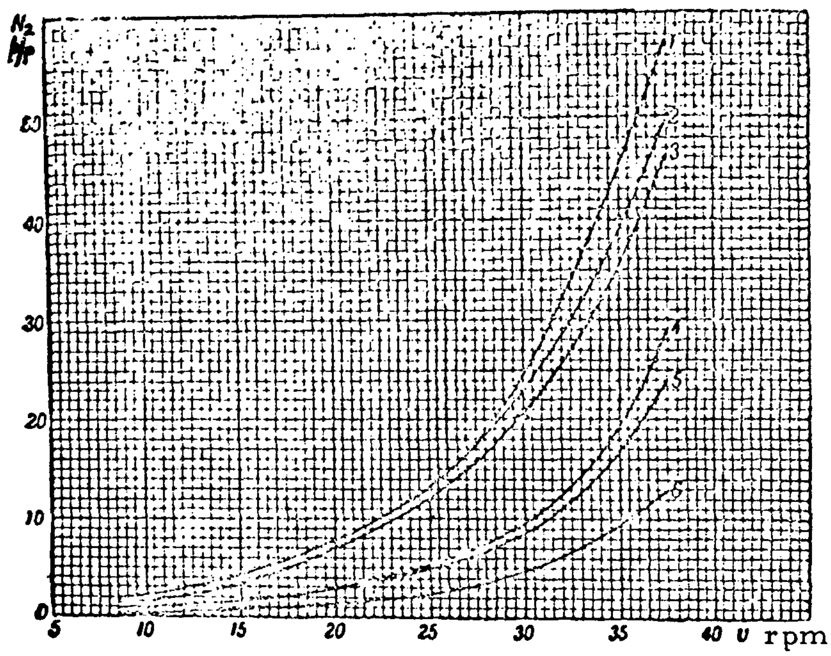


Figure 10. Graphic Representation of Tests on the Aberdeen Proving Grounds

1 = two sprocket wheels, dry track
2 = steel idler wheel
3 = the same idler wheel with rubber tire
4 = same, where the bolt bearing is lubricated
5 = same, with a new drive sprocket
6 = the drive sprocket is replaced by an idler wheel with rubber tires.

d) By the length of the track. The longer the track with otherwise equal preconditions, the greater the power loss;

e) By the design of the track and the manner in which it meshes with the drive sprocket;

f) By the operating condition of the track (whether the track links are dry or well-lubricated) and by the amount of wear on the track joints.

Fig. 10 gives a graphic representation of tests on the Aberdeen proving grounds.

This demonstrates the influence of the many variables on the efficiency of the track.

The test equipment is shown in Fig. 11.

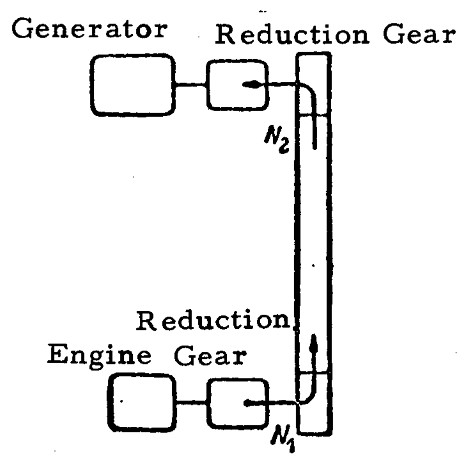


Figure 11. Test Arrangement for the Determination of Track Efficiency

Above: Current Generator -- Transfer Gear
Below: Engine -- Transfer Gear

The engine turns the track by means of a reduction gear and transmits to it the power N_1, which is converted into electrical energy in the current generator. If a power of N_1 HP is directed to the track and only N_2 is taken off by the current generator, then the loss in the track is $N_G = N_1 - N_2$.

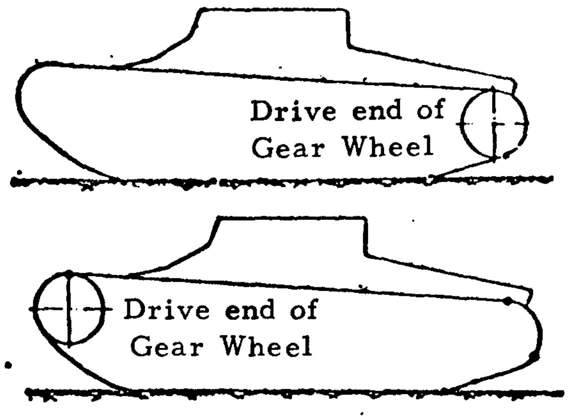


Figure 12. The Influence of Drive Sprocket Arrangement on Track Efficiency. The deflection points caused by load are indicated by dots.
Above: Drive Sprocket

It is not recommended to lubricate the joints of the track links during travel because dirt, sand and dust stick to the oiled surfaces and are ground in, although tracks oiled during the tests increased in efficiency.

The track joints would wear out more rapidly due to lubrication.

With regard to the loss increase due to friction in the track joints, the drive sprocket arrangement -- whether in the front or in the rear -- is important.

The number of joint deflections under load and thus the occurrence of friction forces are less when the drive sprocket is in back than when it is in front (Fig. 12).

According to measurements on the Aberdeen proving grounds (Fig. 10), the relationship between the track efficiency η and tank traveling speed v has been expressed by curves:

$$\eta = \alpha e^{bv^c}$$

$$\eta = \alpha + bv + cv^2.$$

6. Coefficients of Movement Resistances

It has already been mentioned that the movement resistance of the tank acts in direct ratio to the adhesion weight of the tank.

In determining these coefficients, the variables are also determined on which this resistance against movement depends.

For this reason we wish to take a closer look at the power loss in the running gear, insofar as it is related to the travel of the tank.

The power loss in the running gear is made up of the following factors:

a) The loss of power caused by unwinding of the track, which depends on power efficiency transmitted to the track and is practically independent of the weight of the vehicle;

b) The loss in the road wheels of the tank hull as they roll along the track, which is dependent on the weight of the tank;

c) The loss due to vertical sinking into the earth's surface (forming a track in the ground), which is also dependent on weight.

Since the loss caused by unwinding of the track (point a) is to be regarded as the mechanical coefficient of the efficiency, now the power losses must be studied which are presented under points b) and c).

The loss due to the rolling of the road wheels along the track at otherwise equal preconditions depends on the road wheel tires. The friction coefficient with the rolling of rubber on steel is twice as great as the friction coefficient when steel is rolled on steel.

It is even greater when rubber is rolled on rubber (Panhard - Kegresse - Hinstin tracks).

According to the theoretical studies of Professor B. O. Betschitzki, the friction coefficient of rolling in general is expressed by the following formula:

$$f = k\frac{r}{H}\sqrt{\frac{1}{E_K} + \frac{1}{E_0}\frac{Q'}{b}}.$$

r is the radius of the wheel, E_K is the spring temper of the wheel, H is the level of power consumption, E_O is the spring temper of the support, Q' is the load of the wheel, b is the width of the rubber tire, k is the invariable coefficient.

Consequently, from the standpoint of reducing rolling resistance of the road wheel on the track, it is more practical to use a steel rim than a rubber tire. In the case of a rubber tire, however, the rolling phenomena occurs more sparingly, because during tank travel any bumps or jolts which occur are damped and travel noise is decreased.

In addition, road wheel resistance is influenced by its diameter.

From the literature concerning rolling of a wheel we know that the resistance against the rolling motion of the wheel is less, the larger the wheel (the road wheel).

From this point of view it is more practical to use wheels of larger diameter in a tank running gear.

Nevertheless it must be considered that when installing road wheels of larger diameter, the resistance against rolling decreases somewhat; at the same time, however, the mobility of the tank on terrain is reduced, because the specific pressure on the earth's surface is increased and thus the power loss due to sinking into the ground because the number of contact points with the earth's surface is smaller (Figs. 13a and 13b).

Road wheels of large diameter, but in double rows, have been installed in the newest German tracked towing vehicles (see Fig. 13c).

Without regard to these designs, the named kinds of power losses caused by vertical sinking into the ground stand in direct relationship to the specific pressure q and the characteristics of the roadway.

Without regard to the rolling friction, gliding friction occurs between the road wheels and the track teeth.

In addition, reference should be made to the friction on the track bolts, likewise friction on the road wheel axles (to be sure, this friction is very slight, because the wheels run on bearings, the friction coefficient of which is approximately 0.01).

In addition, the friction on the locations of running gear suspension and the friction on the springs is to be taken into account.

Since the types of losses which have no relation to the weight of the tank but are only stipulated by the transmitted power efficiency are regarded by us as coefficients of the tank net efficiency, it is natural to presume that the movement resistance of the tank is determined by the power loss of the running gear and the loss caused by the direct countereffect of the earth's surface.

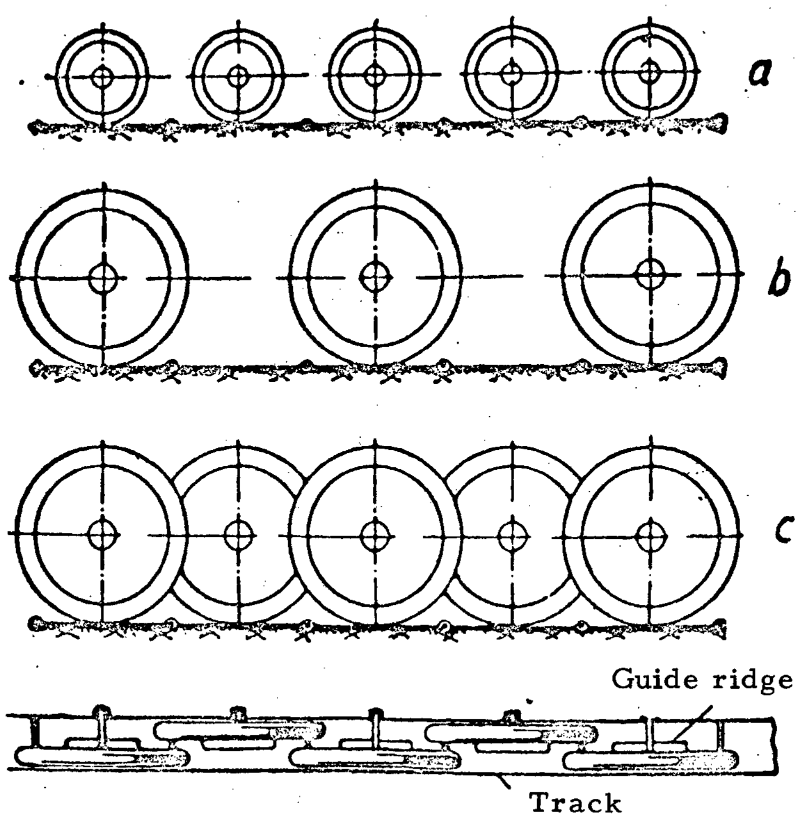


Figure 13. Influence of the Road Wheel Diameter on the Mobility of the Tank
1. Leading edge (track guide) 2. Track

In other words, we assume that the other resistances which oppose the forward movement of the tank appear as an exterior force on the earth's surface.

If we are to go by data in the literature, there are test stands in German industry (Fig. 14) with which the efficiency is determined with various running gear designs at certain speeds and tank weights.

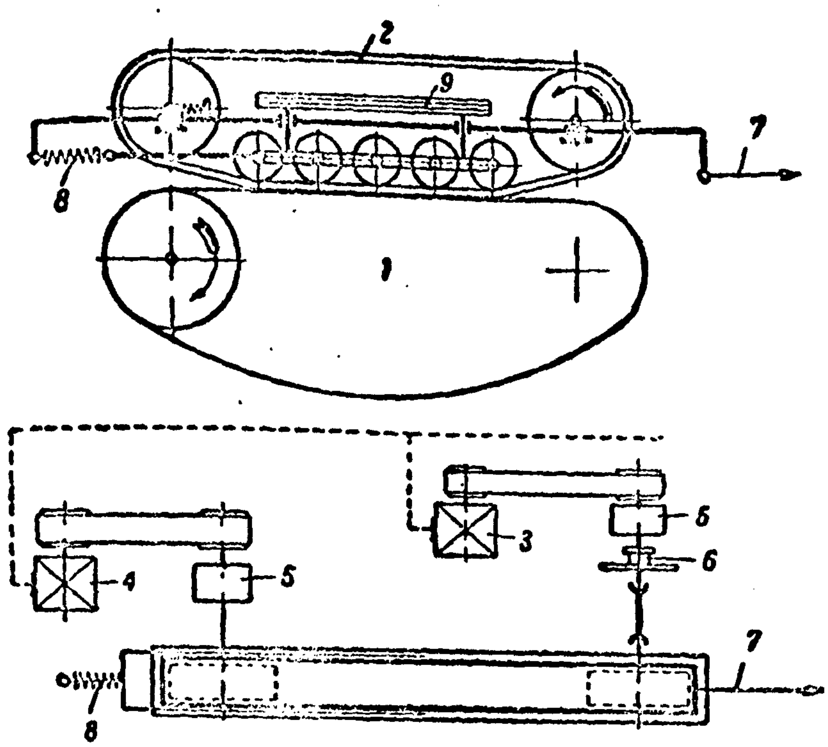


Figure 14. Test Stand

1 - Test Stand; 2 - Track to be tested; 3 - Electromotor; 4 - Current Generator; 5 - Transfer Gear; 6 - Rotating Output Recorder (Dynamograph) 7 - Tractive Effort Recorder to the Torque Gauge (Torsiograph); 8 - Recording Equipment for the resistance of the Road Wheel Slide; 9 - Load of the Slide.

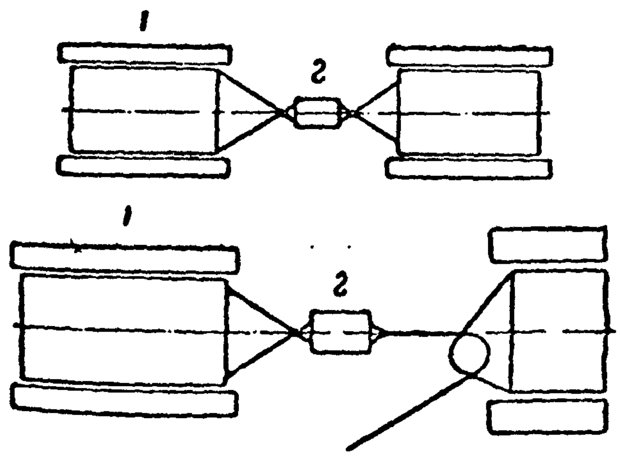


Figure 15. Determining the Coefficient f with the Towing Method

Table 4.

Size of the Coefficient f for Wheels and Track Suspension Systems

Type of Ground	Tractor with wheels	Tractor with tracks
Hard ground with grassy overgrowth	0.02 - 0.03	---
Arable soil	0.06	---
Soft sandy soil...............	0.12	0.10
Dry sandy soil...............	0.07 - 0.08	0.07
Loose sandy soil	0.30 - 0.40	0.10 - 0.15
Firm sandy soil	0.08	---
Dry grass on firm soil	0.10	0.065
Stubble field, harvested with a mowing machine	0.09	---
Meadow, mowed with a mowing machine	0.10	0.08
Dry meadow.................	0.13 - 0.14	---
Autumnal arable soil (frozen)..	0.15	---
Freshly plowed land..........	0.20 - 0.30	0.10 - 0.12
Asphalt street	0.015	0.03
Paved street	0.040	0.06
Country road	0.080	0.06 - 0.07
Swamp	0.30	0.10
Railroad wheels on tracks	0.004	---
Streetcar wheels on tracks	0.008	---
Icy road	0.02 - 0.023	0.03 - 0.04
Deep mud	0.20	0.10 - 0.15

Examining the experimentally-determined coefficients, it is possible to make a choice between the various track and suspension systems.

The towing method to determine the resistance coefficient for tanks is used a great deal.

With this method, a single tank (Fig. 15) (with the engine shut off and the steering clutches disconnected) is towed over the test distance with an output recorder (dynamograph) 2.

At a given movement of the tank, the recording instrument indicates the resistance coefficient of the tank independent of the weight.

It is to be taken into account that the coefficient determined in this manner indicates only the power loss in the idling track, together with the loss caused by sinking into the ground at low speed.

Table 4 contains the values of the resistance coefficient which were determined with the described method.

For the purpose of determining the nature of the adhesion coefficient φ we now wish to consider the effect of the track on the earth's surface.

During travel the tank presses the surface plates or grips of its track into the earth's surface and forms a "line of teeth marks" (Fig. 16). It moves forward on the gear tooth system in this manner.

In the process, the track lug tends to crush, cut and send the dirt flying (Fig. 17).

In addition, friction has its effect between the surface of the track link and the earth's surface.

As the horizontally directed forces increase, so does the compressive load on the ground, until the line of teeth marks is destroyed.

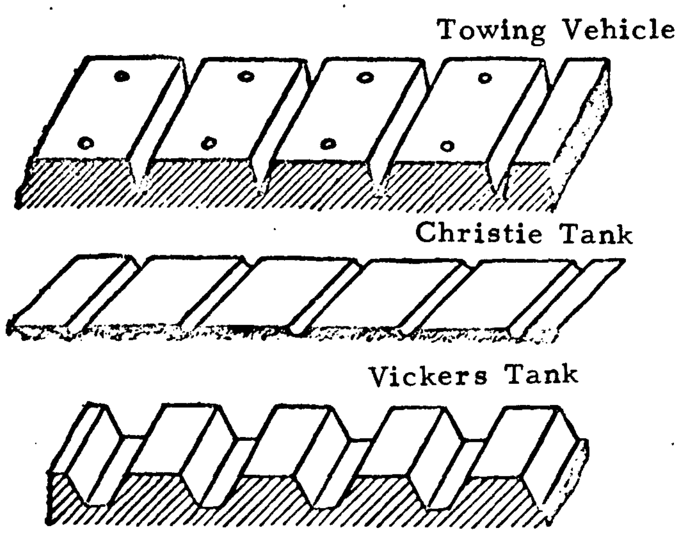


Figure 16. Base Plates

If the track link were smooth and had no grips, then the adhesion on the earth's surface would be stipulated by the force of friction when the track began to slide along the earth's surface and the adhesion or adherence coefficient would have to be equated with slip.

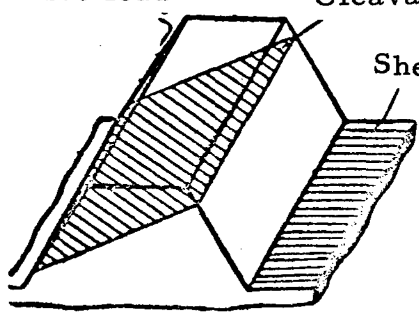


Figure 17.

The maximum tractive effort is achieved when the track glides for a short distance on the ground, without really adhering, whereby the percentage of this slippage is dependent on the nature of the ground; the track design also plays a certain role.

In its forward movement the tank cannot completely utilize the tractive effort by ground traction. For this reason, we distinguish the maximum coefficient of ground traction φ from the coefficient φ_i presently laid claim to, the relative value of tractive effort on the ground to the adhesion weight of the tank:

$$\varphi_i = \frac{P}{Q} \tag{41}$$

From this it follows that the static friction coefficient is equal to the travel resistance coefficient f_o.

It can be determined which maximum possible value the coefficient of ground traction can attain. It is known that a tank in uniform travel on particularly firm ground can overcome inclinations up to 45°. Consequently, the tractive effort developed on the ground can be expressed in the following formula:

$$fG \cos \alpha + G \sin \alpha = P,$$

while on the other hand the propulsive force equals ground traction.

$$\varphi G \cos\alpha = f G \cos\alpha + G \sin\alpha.$$

Consequently,

$$\varphi = f + \operatorname{tg}\alpha \qquad (42)$$

or when α = 45%:

$$\varphi = 1 + f.$$

This means that the maximum traction coefficient sometimes attains a value larger than 1.

In the design of a tank, usually the angle of inclination α_{max} is given. Proceeding from this angle, the calculation of the power plant is conducted.

Since a smooth chain has sufficient traction only on some sufficiently firm types of ground, when the track is designed an attempt is made to impart to it good ground traction by means of a tooth construction. Of course this must not happen at the cost of its use on firm ground at high speeds.

Grips must not be mounted too closely together in a row, because otherwise shearing resistance of the holes left in the ground is exceeded.

When the grip is lengthened up to a certain height, ground traction is increased and thus the traction coefficient as well; but at the same time this makes travel more difficult on roads at high speeds.

In order to obtain a true value of the maximum adhesion coefficient, two methods may be used:

1. The tank is towed with its brakes firmly applied.
2. The tank to be tested tows a trailer connected with an output recorder.

The first-named method corresponds with the method for determining the travel resistance coefficient.

The dynamometer is read with the brakes firmly applied until the tracks are blocked.

The maximum value indicated on the recording instrument, after dividing by the adhesion weight of the tank, indicates the value of the coefficient φ.

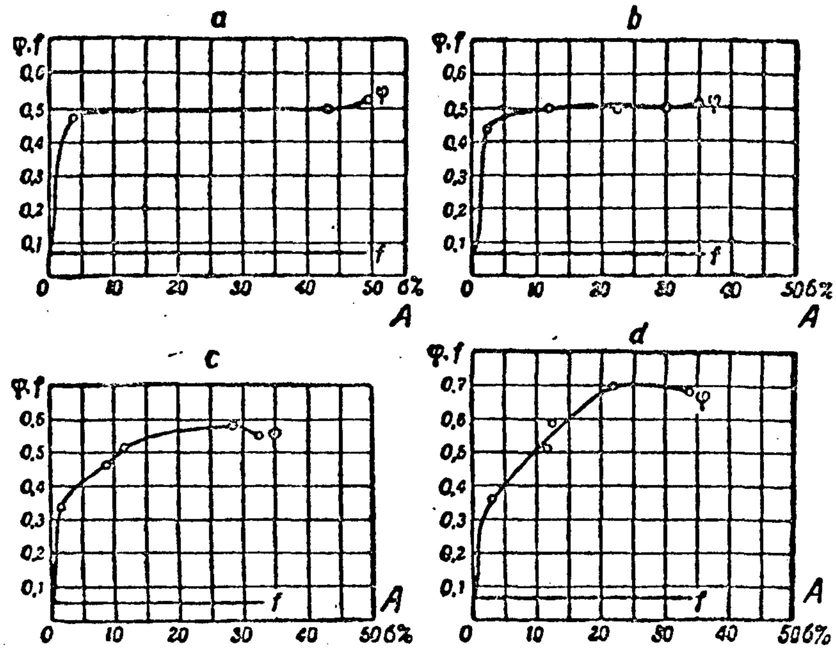


Figure 18. Graphical Illustration of the Coefficients φ and f.

a. Wet mud b. above, slightly dried mud c. Lower down, wet mud d. Wet mud on the surface

A. Initial Thrust

It cannot be asserted that this method is particularly exact because the preconditions for the activity of the earth grips which push dirt in front of them differ considerably than when under normal forward movement.

When determining the maximum value of the adhesion coefficient according to the second method, the tank takes over the role of a towing vehicle and the dynamometer takes over the role of a towed load.

The tank being tested starts up and pulls the dynamometer along behind it.

The dynamometer (dynamograph) is turned on under uniform forward movement and the load on the towing hook is increased until the track slips on the earth's surface without achieving forward motion.

The maximum value of tractive output determined on the towing hook is taken as a basis of the calculation.

The adhesion coefficient φ is determined according to the following formula:

$$R_{max} = P_{max} = \varphi G = fG + P_k$$

according to the formula

$$\varphi = \frac{P_k}{G} + f \qquad (43)$$

Here P_k is the value indicated by the tractive effort recorder and f is the coefficient of travel resistance. This coefficient is determined according to the above-mentioned method.

The values of the maximum adhesion coefficient for the various ground types are compiled in Table 5, and in Fig. 18 we find the graphic illustrations of engineer P. P. Lebendenko based on tests conducted with the "Communard" towing vehicle on wet sandy loam soil.

Table 5.

Adhesion Coefficient φ for a Towing Vehicle with Tracks and Wheels

Ground Type	Wheeled	Tracked
Paved street	0.6-0.7	0.6-0.8
Dry and compacted country road	0.6-0.7	0.6 (without skids) 0.8-0.9 (with skids)
Soft sandy road	0.3-0.5	0.6-0.7
Deep mud	0.2-0.5	0.5-0.6
Loose sand	0.1-0.3	0.45-0.55
Dry grassy ground with firm substratum	0.9-1.0	1.0-1.1
Mowed meadow	0.65	0.7-0.9
Freshly plowed field	0.5-0.5	0.6-0.7
Firmly compacted (rolled) snow	0.5	0.7-0.8
Icy road	0.1 and less	0.2-0.4 (without grips) 0.5 (with grips)
Compacted highway	---	0.7 (without grips) 1.2 (with grips)
Virgin soil (unplowed land)....	---	0.1-0.3 (without grips)
Swamp		Until moisture has been pressed out

7. Gliding, Slipping and "Grinding" of the Tank

It has already been mentioned that in tank travel there is a stress exerted in the horizontal direction in addition to the vertical pressing down of the earth's surface. For this reason, bulges and depressions (cellules) appear in the ground which are somewhat longer than the grips are wide (Fig. 19).

This phenomenon is designated as "slip" and "is measured according to the loss of speed".

This so-called "slip" identifies the adhesion properties of the track in the various earth types. Consequently, by "slip" we mean the course of the track surface on the ground pushed back by deformation and pressure on the ground caused by the track grips opposite the direction of travel.

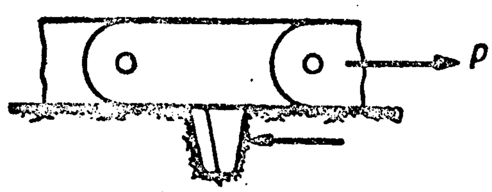


Figure 19. Enlarged Grip Indentation due to Slip

In "slip" the distance covered by the tank is smaller than the distance covered by the track in its relative movement.

In the case of tracked vehicles, the amount of "slip" can be determined as follows.

We are assuming a distance s covered by the tooth construction of the drive sprocket moving "without slip" during which time it has revolved n times. In other words

$$s = z\,l\,n$$

n is the total number of revolutions of the drive sprocket, z is the number of track links which roll over the gear wheel in one revolution, l is the length of the track link.

Now we load the tank on the towing hook. The total resistance against forward movement of the tank increases and becomes greater than in the former case. For this reason a greater tractive effort on the earth's surface is required.

The track begins to "slip" because the earth's surface must be compressed for the purpose of absorbing increased tractive effort.

Consequently, the drive sprocket on the tank will no longer perform n revolutions over the same distance, but rather n_1, thus more than n.

The theoretical length of the distance (s_1) is

$$s_1 = z\,l\,n_1.$$

In other words the absolute value Δ_s of "slip" can be expressed by an equation having the following form:

$$\Delta s = s_1 - s = z\,l\,(n_1 - n).$$

The portion of "slip" ζ on the track movement is

$$\zeta = \frac{\Delta s}{s_1} = \frac{z\,l\,(n_1 - n)}{z\,l\,n_1} = 1 - \frac{n}{n_1} = 1 - \frac{s}{s_1} \qquad (44)$$

Sometimes the value $(1 - \zeta)$ is also designated as the glide coefficient:

$$\eta_\zeta = 1 - \zeta \qquad (45)$$

The power efficiency which is expended uselessly by slip, takes the following form:

$$N_\zeta = N_z\,\eta_g\,\zeta \qquad (46)$$

η_g = efficiency of the track, ζ = glide or slip coefficient, N_z = the power given off by the drive sprockets.

It can readily be imagined that the glide coefficient can be increased up to 1 with a gradual increase of load on the towing hook; this means that the tank will not move from the spot despite the turning of its tracks.

Engine power can be sufficient for movement in this case as well.

With this track design, the nature of the ground offers us no possibility of transmitting the required power to the earth's surface. The ground will yield and be torn up; consequently any forward movement will prove to be impossible.

8. Specific Pressure

The value of the named ground pressure which is calculated on 1 cm^2 of the contact surface between the track and the earth's surface, stands in direct relationship to travel resistance.

The deformation of the earth's surface and thus also travel resistance are greater, the larger the specific pressure.

In addition, specific pressure exerts a considerable influence on the mobility of the tank on land.

In this regard it is possible to distinguish between different types of specific pressure:

a) the static kind -- when the tank is stopped,
b) the dynamic kind -- when the tank is moving.

In addition, it is possible to distinguish between mean and actual specific pressure.

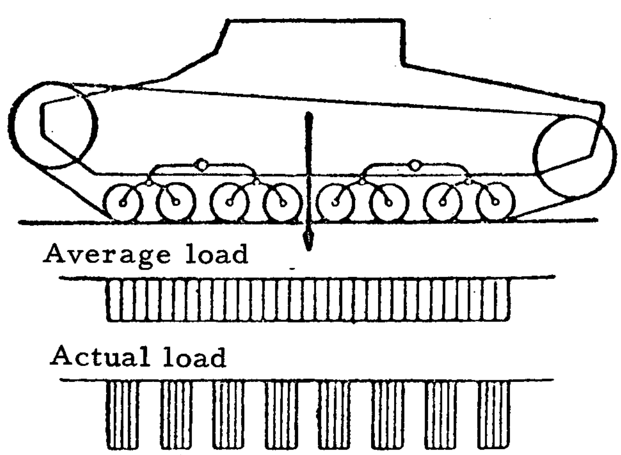


Figure 20. Specific Pressure on Solid Ground

Specific average load (q_o) designates that load exerted on a unit of supporting surface of the tank under the assumption that the weight of the support surface of the tank is equally distributed.

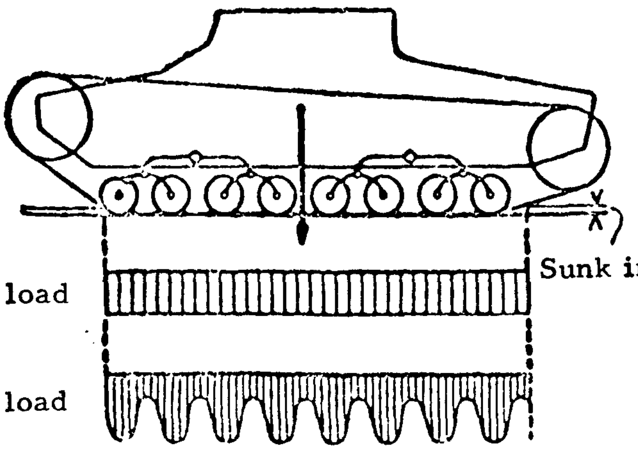


Figure 21. Specific Pressure on Soft Ground

It is: $$q_0 = \frac{G}{F} \text{ kg/cm}^2 \qquad (47)$$

Here G is the weight of the tank and F is the supporting surface.

The specific pressure on the length of the track can be graphically represented in the form of a rectangle (Fig. 20).

The average specific pressure changes in measure as the track sinks into the earth, since when it is sinking in the length of the supporting surface increases (Fig. 21).

The specific pressure given in the instruction manuals is always the average specific pressure.

Usually this pressure is given at "zero" (i.e. when the tank is standing on solid ground, for example on asphalt) and when sinking in at 100 mm - q_{100}.

Table 6 gives the values of the specific average pressure with various types of vehicles and suspension systems under movement.

It is to be noted here that the specific average pressure has nothing to do with the mobility of the tank on terrain, because the load is really transmitted onto individual points of the track.

Since the track is not really a rigid single unit, the load is absorbed only at individual points and is only transmitted back to the ground by those links supported by the road wheels. Those track links not subjected to load transmit no vertical load onto solid ground.

Table 6

Specific Average Pressure

Type	Specific Pressure	Remarks
Man on one leg	0.56	---
Man on snow shoes	0.03-0.036	Length of snowshoe 3 m; supporting load 90kg
Horse without rider	1.12	On 3 legs
Horse with rider	2	On 3 legs
Passenger car	1.26	With 6 seats
Truck	1.4-2	Sunk in 2 cm
Patrol Tank	2.3	Front wheels twin axle
	5.6-6.2	Rear wheels twin axle Double tires
	2.85-3.15	Rear wheels triaxial
Towing vehicle (wheeled)...	4.5-10	Sunk in 2 cm
Tracked towing vehicle	0.3-0.5	---
Landgoing vehicle	0.03-0.05	---
Tank	0.2-0.9	---
Light	0.2)	
Medium	0.4-0.5)	According to English data
Heavy	0.6-0.9)	

On the other hand, on soft ground they are connected with the loaded links and thus transmit a certain load onto the ground. But it is smaller than in the case of track links loaded by the road wheels (Figs. 22 and 23).

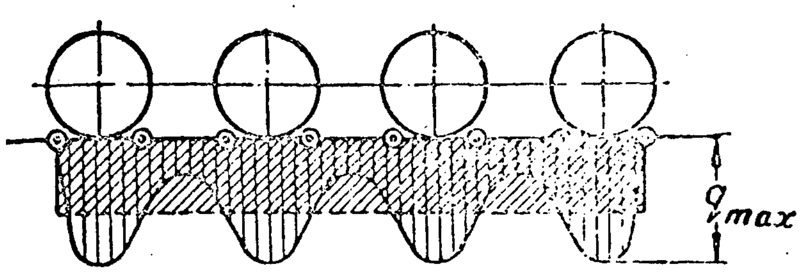


Figure 22. Actual Specific Load on Soft Ground

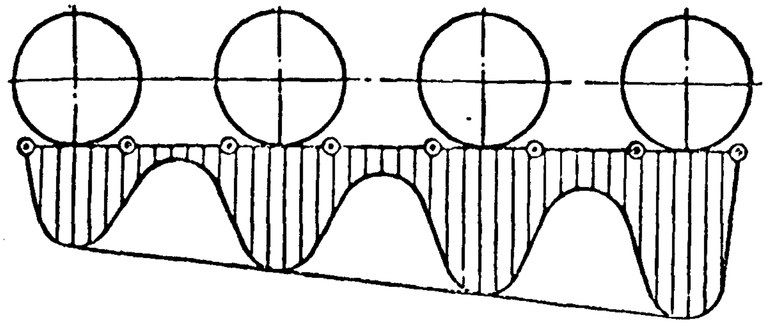


Figure 23. Actual Specific Load on Soft Ground

If the specific average pressure is determined with the following formula,

$$q = \frac{G}{n l b}, \tag{48}$$

in which n is the number of road wheels, l is the length of a track link and b is its width, then the values of the average pressure will approach nearer to reality.

This formula applies only when the specific average loads are distributed according to the law of the rectangle, if the suspension is designed accordingly and is in neutral position.

In the case of another center of gravity, taking into account the special nature of the suspension during travel, the actual average pressure is expressed in another way because the loads of the road wheels deviate from each other (see Section IV "Suspension of the Tank").

In order to determine the mobility of the tank we must consider the specific maximum pressure; consequently, it seems more correct to use the following formula:

$$q_{max} = \frac{Q_1}{k l b} \tag{49}$$

In this formula: Q_1 is the load on the most heavily loaded track link, k is a coefficient, which is dependent on the uneveness of the road and which decreases the contact surface of the track link with the earth's surface, l is the length of the track link and b is its width.

If the track is equipped with projections (grips) and travel is on hard ground so that the track link does not actually sink in, the supporting surface can be calculated according to the following formula:

$$F = k_1 n k l b \tag{50}$$

n is the number of grips which come into contact with the ground, k_1 is the coefficient of reduction of each grip, l is the length of the grip, b is the width of the grip, and specific pressure in this case can be expressed as

$$q_{max} = \frac{Q_1}{F}$$

The coefficients k and k_1 must be determined experimentally.

9. Determination of the Minimum Travel Speed of a Tank

Section 4 gives a formula (37) to determine the required output of a tank engine N_d at maximum travel speed v_{max}.

On the other hand, when designing a tank, a maximum angle of inclination α_{max} is required when the tank develops the full power of its engine and travels in a uniform manner.

The required tractive effort P_{max} under these conditions equals travel resistance

$$P_{max} = f\, G \cos \alpha_{max} + G \sin \alpha_{max} = \varphi\, G \cos \alpha_{max}.$$

When the values of N_d and P_{max} are determined, the required minimum travel speed is calculated by the following formula:

$$v_{min} = \frac{270\, \eta\, N_d}{P_{max}}.$$

The information given here and in the preceding sections offer us the possibility of solving the main problems of calculating tractive effort.

a) According to the given technical preconditions for the construction of a tank it is possible to determine the required power output of the engine N_d and the minimum travel speed of the tank v_{min};

b) The maximum speed and the maximum surmountable slope can be calculated for the finished tank (N_d and v_{min} are known).

1. Example

The following is given: the weight of the tank 8800 kg, slopes to be encountered during travel at maximum speed, up to 4%; the maximum angle of inclination 40°, the maximum travel speed v_{max} = 34 km/h.

The required power of the engine N_d and the minimum travel speed of the tank v_{min} are to be determined

$$N_d = \frac{P\,v_{max}}{270\,\eta}\,;\quad P = (f + i)\,G\,,$$

in which $f = 0{,}04,\ i = 0{,}04$ and $\eta = 0{,}75$

$$N_d = \frac{0{,}08 \cdot 8800 \cdot 34}{270 \cdot 0{,}75}\,;\quad N_d = \text{ approx. } 115 \text{ HP}$$

$$v_{min} = \frac{270\,\eta\,N_d}{P_{max}}\,,$$

$$P_{max} = f\,G\cos\alpha + G\sin\alpha\,,$$

in which $f = 0{,}06,\ \cos\alpha = 0{,}76,\ \sin\alpha = 0{,}64.$

$$v_{min} = \frac{270 \cdot 0{,}75 \cdot 115}{8800\,(0{,}06 \cdot 0{,}76 + 0{,}64)} = \frac{270 \cdot 0{,}75 \cdot 115}{8800 \cdot 0{,}686}\,;\quad v_{min} = \text{ approx. } 3.9 \text{ km/h}$$

2. Example

We are to determine the maximum speed of a Vickers-Armstrong tank, model B, year 1931, on a highway with slopes up to 4% as well as the minimum traveling speed of this tank.

In the first part of Heigl's pocket book we find the following data on this tank.

Engine output at 2000 rpm 80 HP, maximum climbing ability α max = 45°, weight of the tank 8 t.

$$v_{max} = \frac{270\,\eta\,N_d}{P}\,,$$

in which $f = 0{,}04$ and $i = 0{,}04$

$$v_{max} = \frac{270 \cdot 0{,}75 \cdot 80}{8000 \cdot 0{,}08}\,;\quad v_{max} = 25{,}3 \text{ km/h}.$$

$$v_{min} = \frac{270\,\eta\,N_d}{P_{max}}\,,$$

$$P_{max} = (f\cos\alpha + \sin\alpha)\,G\,,$$

in which $f = 0{,}06$, $\cos\alpha = 0{,}7$ and $\sin\alpha = 0{,}7$

$$v_{min} = \frac{270 \cdot 0{,}75 \cdot 80}{8000 \cdot 0{,}75} = 2{,}7 \text{ km/h.}$$

10. The Total Coefficient of Travel Resistance

In order to be able to fully evaluate the dynamic properties of the tank, it is necessary to use the "dynamic identification".

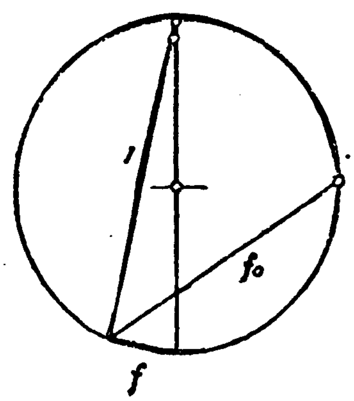


Figure 24. Illustration of the Formula $f_o = f \cos\alpha + \sin\alpha$

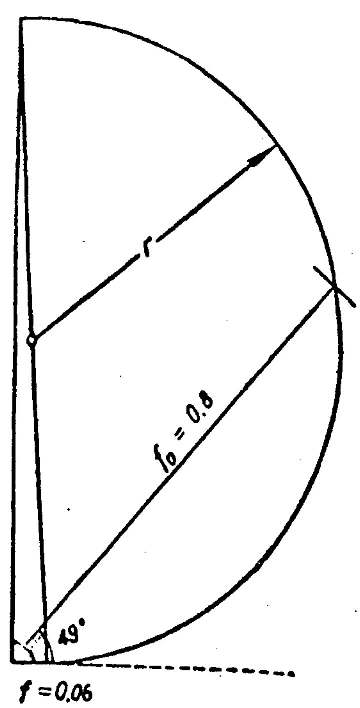


Figure 25. Graphic Solution of the Problem; f_o and f are given, α is to be determined.

When performing calculations on tanks it is frequently sufficient to begin with the total coefficient of travel resistance.

$$f_0 = f\cos\alpha + \sin\alpha.$$

The expression f_0 represents the equation of the circle with the diameter

$$r = {}^1/_2 \sqrt{f^2 + 1} \text{ dar.}$$

This formula is graphically presented in Figure 24.

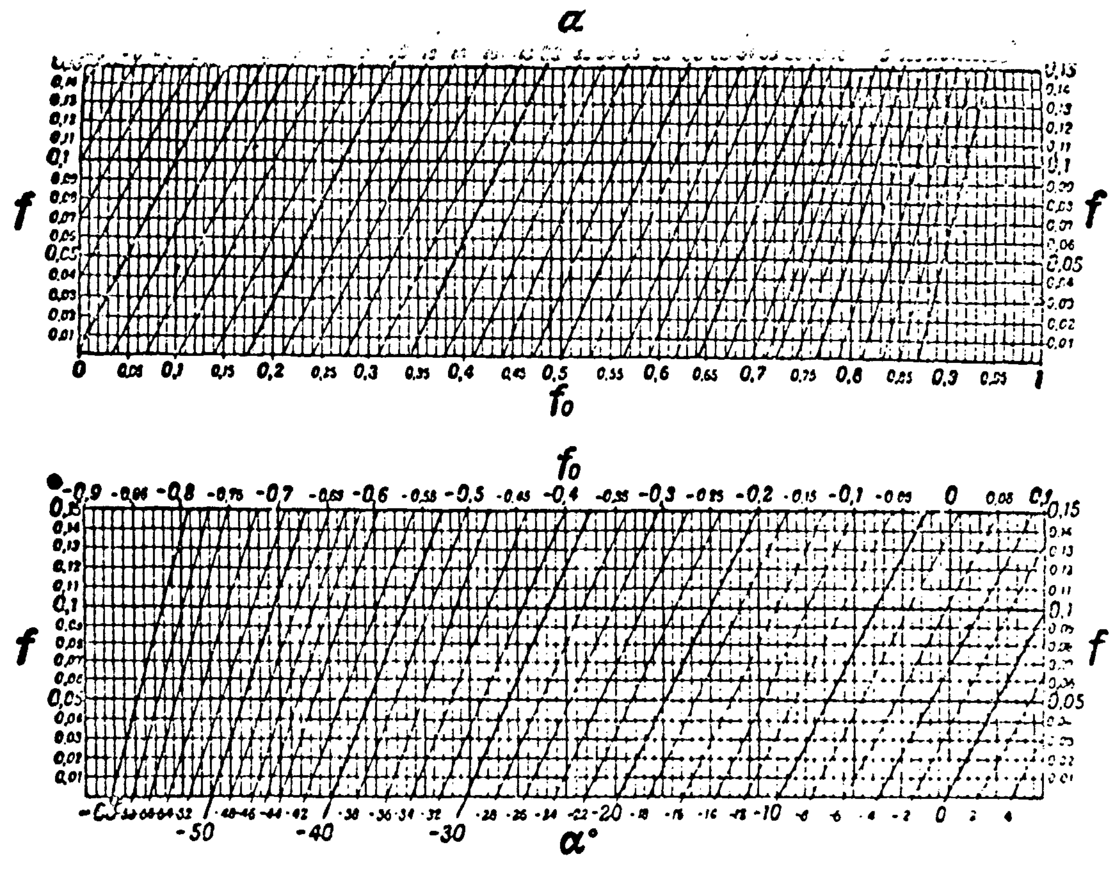


Figure 26. The Nomograph for the Solution of the Equation

$$f_0 = f\cos\alpha + \sin\alpha$$

The same expression can be represented in the form of a nomograph. Finally, the solution can proceed analytically.

Since mostly the calculation is conducted with f_0, the advantage of this calculation should be studied here.

We will presume that the coefficients f_o and f are known and that the angle of inclination α is to be determined.

In order to illustrate this more clearly, we will take the numerical values $f_o = 0.8$ and $f = 0.06$ as a basis.

The first method is the graphic one.

We shall draw a right angled triangle with the short sides 1 and $f = 0.06$ (Fig. 25). We shall draw an arc over the hypoteneuse of this triangle as the diameter. The diameter of this circle is $r = 1/2\sqrt{1 + f^2}$.

We shall bring f_o into intersection with this circle from the apex of the right angle. The angle sought between f and f_o is

$$\alpha = 49^o$$

The second method is the analytical one

$$f_0 = f \cos \alpha + \sin \alpha.$$

After substituting the value $\sin \alpha = \sqrt{1 - \cos^2 \alpha}$ we obtain a quadratic equation according to the solution of which we determine the value of the angle.

The third method is based on the nomograph (Fig. 26).

The graphic representation presented above is intended to determine the angle of inclination, the one below is intended to determine the gradient of slope. On the vertical one we find the value $f = 0.06$, on the horizontal one we find the value $f_o = 0.8$.

From its point of intersection we follow the diagonal line to the upper horizontal line and in this way find the value of $\alpha = 50^o$.

Although the solution according to this method was found accurate and rather rapid, it must be admitted that the first named method (the graphical one) is more favorable, because it affords us the opportunity of checking the correctness of our solution for adhesion conditions.

We already know that the preconditions for travel without "slip" are to be seen in the consideration of the following inequality:

$$\varphi \cos \alpha > f_0 = f \cos \alpha + \sin \alpha,$$

or

$$\varphi < f + \mathrm{tg}\, \alpha.$$

Consequently, in case we plot Section AD = φ (Fig. 27a) from point A (Fig. 27) in the direction f = AC, Section CD = tg α_{max} and angle DB'C = α_{max} represents the maximum adhesion value.

In case α is smaller than α_{max} or if Section AF is smaller than AD, the tank can take the slope; if Section AB is larger than AD (Fig. 27b), the tank will slip or slide.

The total coefficient f_o can be given on the tracks on the basis of the available tractive effort P_d. This tractive effort is calculated according to the output of the engine and according to traveling speed:

$$f_0 = \frac{P_d}{G},$$

$$R_d = \frac{270\,\eta\,N_d}{v}.$$

In this case it is usually designated as the "dynamic factor" and expressed by the letter D.

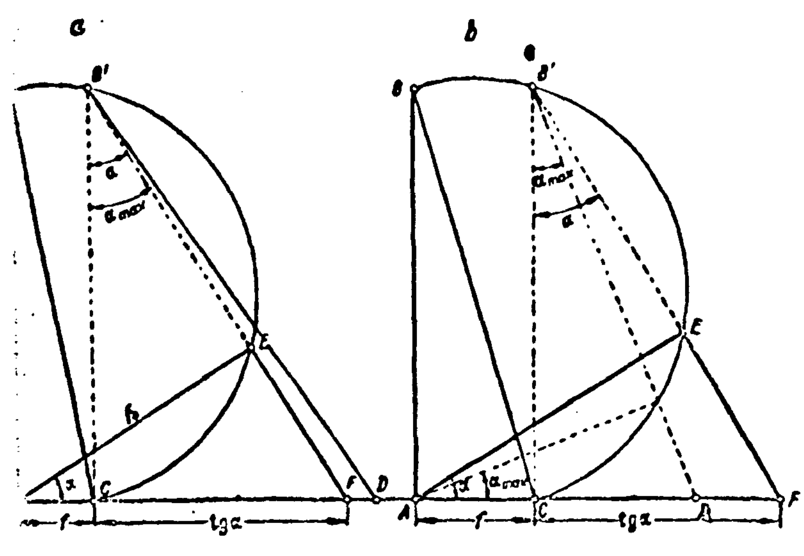


Figure 27. Graphic Solution of the Relationship between f, f_o, α and φ.

In this manner, if the total coefficients of traveling resistance for the engine D and for the resistance f_o as well as the adhesion coefficient φ are known, it is possible to calculate the type of tank movement as well as its climbing ability.

11. Speed Range and Selection of Intermediate Speeds.

The ratio between the maximum and minimum speed of a tank is designated as the tank's speed range.

$$d = \frac{v_{max}}{v_{min}}$$

We now wish to determine the theoretical value of the tank speed range.

Traveling resistance can be determined by the following formula:

$$R_0 = A + B v^2 \tag{51}$$

In this formula $A = f_0 G$ is the coefficient and is limited by the variable preconditions for forward movement.

The air resistance coefficient $B = kF$ is determined according to the tank dimensions.

We will presume that after giving extensive consideration to the preconditions for tank travel it was established that the total coefficient of resistance f_0 changes within the following limits.

$$f_1 \leq f_0 \leq f_n.$$

Then when traveling at f_n (the most difficult preconditions for travel α_{max}) traveling resistance is

$$R_n = f_n G$$

(we will neglect air resistance because it is so slight).

Correspondingly, in case travel occurs on the lowest limit f_0 (travel at maximum speed -- v_{max}):

$$R_1 = f_1 G + k F v^2_{max}.$$

The corresponding efficiencies of resistance are:

$$N_n = f_n G v_{min};$$
$$N_1 = f_1 G v_{max} + k F v^3_{max}.$$

N_n and N_1 must be equal because under travel they correspond with the full power efficiency of the engine; this means that

$$\frac{f_1 G v_{max} + k F v^3_{max}}{f_n G v_{min}} = 1$$

from which it is clear that at traveling speeds below 50 km/h

$$\frac{v_{max}}{v_{min}} = \frac{f_n}{f_1}$$

If we assume that at minimum speed at α =45

$$f_n = f \cos\alpha + \sin\alpha = 0{,}1 \cdot 0{,}7 + 0{,}7 = 0{,}77,$$

and at maximum speed

$$f_1 = f + i = 0{,}06 + 0{,}02 = 0{,}08$$

on the highway, we finally obtain the tank speed range

$$d = \frac{v_{max}}{v_{min}} = \frac{0{,}77}{0{,}08} = \text{approx. } 10$$

Thus the maximum speed of a tank is approx. 10 time greater than its minimum speed.

In reality, however, such speed ranges occur very seldom.

On the average, the numerical values of d fluctuate within the following limits:

Passenger car	3 - 4
Truck	5 - 7
Commercial towing vehicle	2.7 - 3
Tank	6 - 10

Now we wish to determine the tractive effort preconditions for the operation of a tank engine.

As has already been stated, the traveling resistance of a tank can be expressed at certain preconditions:

$$R_0 = f_0 G + K F v^2$$

and the resistance output N_R as follows

$$N_R = f_0 G v + K F v^3.$$

Taking into consideration the circumstances that the traveling speed of the tank stands in relation to the rotational velocity rate n (if there is no slip) in the following ratio:

$$v = \frac{z l n}{60 i_0},$$

we obtain the expression for the resistance output (considering air resistance) in the following equation:

$$N_R = f_0 G \frac{z l}{60 i_0} n + K F \left(\frac{z l}{60 i_0}\right)^3 n^3 \qquad (52)$$

This output must correspond to engine power as well

$$N_d = \frac{N_R}{\eta}.$$

When travel resistance increases, carburetor position remaining the same, the rotational speed of the drive sprocket begins to decline (if i_o does not change) and thus the engine rotational velocity declines as well.

As is clear from the engine characteristic lines (Fig. 28) an increase of engine torque occurs in addition to a decline of engine rotational velocity in the intermediate range from n_o to n_1.

If with changed preconditions for the travel of a tank, the new engine rotational velocity n_i lies in the intermediate range from n_o to n_1, then the phenomena of automatic regulation takes over, and the new engine rotational velocity will be the compensating rotational velocity.

However, in case no automatic regulation occurs for the trip with a change of the preconditions (because travel resistance is too great) and the rotational velocity n_i is smaller than n_1, then the engine will stop if no successful attempt is made to bring the engine back up to a constant rotational velocity using artificial means.

This type of engine balancing to a steady rotational velocity when travel resistance changes is achieved with the gear shift by changing the ratio between the engine and the drive sprockets.

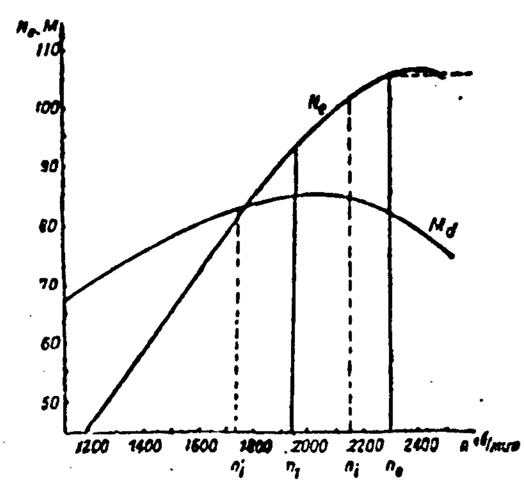


Figure 28. Limits of Engine Automatic Regulation

In this regard the question arises which method should be followed in selecting the gear ratio, i.e. in other words how the intermediate range of speeds should be classified in the individual ranges so as to achieve in this manner the so-called transmission gradation.

12. Arrangement of the Slippage Transmission according to the Rules of the Geometric and Arithmetical Series.

In addition to the usual requirements placed on the slippage transmission of a tank, another desirable feature is an acceleration up to the required speed in the shortest possible time T, and over the shortest possible distance.

Since this time span T usually does not occur, during acceleration of the engine the maximum possible power output will not be generated.

There is the following equation for the accelerated travel of a tank

$$d\left(\frac{\delta m\, v^2}{2}\right) = d\,[(P-R)\,s],$$

from which we obtain the following results:

$$T = \frac{\delta m}{P-R}\,(v - v_0).$$

assuming that P and R are constant values.

It is evident from this equation that the greatest possible tractive effort must be achieved, i.e. the greatest possible output must be available for the acceleration of the engine in order to keep time span T as short as possible.

We will assume that the tank has achieved its maximum speed, for example v_3 with the appropriate ratio (Fig. 29).

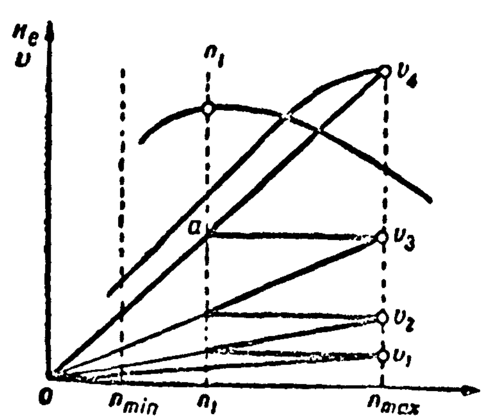


Figure 29. Graphic Illustration of a Transmission Gradation according to a geometrical Series.

Thus the engine reaches its maximum rotational velocity of n_{max} which should not be exceeded, taking into account the working life of the engine (because otherwise the engine will undergo too much wear, or else the crankshaft and the connecting rod bearings will incur damage).

In this way the maximum, at n_{max} the highest possible engine power is achieved.

We will assume that we must generate a very high traveling speed for the preconditions of the trip.

After the gears are shifted, the rotational velocity of the drive shaft will become smaller by almost the amount of the transformation ratio, than the rotational speed of the engine. We will assume that the angular velocity of the crankshaft is ω_m when the clutch is engaged and the angular velocity of the drive shaft is ω_A, where ω_m is larger than ω_A.

Then when the main clutch is engaged and when the angular velocity of the drive shaft and of the output shaft is equalized, the angular velocity of the crankshaft will be reduced to a certain speed of

$$\omega_1 \; (\omega_1 < \omega_m)$$

while the speed of the driven shaft will be increased to

$$\omega_1 \; (\omega_1 > \omega_A)$$

After the angular velocities have been equalized they are increased again until the propulsion moment equals resistance moment.

In order that acceleration to n_{max} occurs in the same time span in all gears it is necessary that the rotational speed of the engine n_1, at which acceleration begins, be the same in all gears. That means that the ratio of each of the aforementioned speeds is at a constant value in relation to the next speed, i.e.

$$\frac{v_1}{v_2} = \frac{v_2}{v_3} = \ldots \frac{v_n - 1}{v_n} = \frac{n_1}{n_{max}} = q = \text{const.}$$

This ratio shows us that the speed values are grouped in geometric progression.

When the ratio between the minimum speed and maximum speed is known, i.e. the so-called speed range

$$d = \frac{v_{max}}{v_{min}} = \frac{v_n}{v_1},$$

then this can be determined as follows for the purpose of determining the number of intermediate speeds:

$$\frac{v_n}{v_n - 1} \frac{v_n - 1}{v_n - 2} \cdots \cdots \frac{v_2}{v_1} = \left(\frac{1}{q}\right)^{n-1} = \alpha^{n-1}.$$

Consequently,

$$d = \alpha^{n-1}$$

and thus

$$\alpha = \sqrt[n-1]{d}, \qquad (53)$$

in which each of the speeds is ascertained according to the formulas

$$v_2 = v_1 \alpha; \ v_2 \alpha = v_1 \alpha^2 = \cdots = v_{n-1} \alpha = v_1 \alpha^{n-1}$$

It is difficult to present the rule here without deviations, because it is not possible to precisely select the number of teeth on the gear wheels.

Figure 30 gives a clear illustration of the phenomena which occur in geometrical progression when shifting a transmission.

The forward motion begins in the first gear. After full speed has been reached in this gear (Point 4), we shift to the next gear (Point α). In this way the rotational velocity of the engine reaches n_1 (according to gradation 2).

Then by applying the gas, acceleration is achieved in the 2nd gear until full speed has been reached v_2, whereupon we shift to the third gear, until we have reached a speed of v_4.

The relationship between the engine rotational velocity and the gradually increasing tank travel speed is expressed by a straight line. It goes through the origin of the coordinates with the slope K:

$$v = k\,n,$$

where

$$k = \frac{z\,l}{16{,}7\,i_0}$$

i_0 is the ratio of force transmission in the appropriate gear.

Taking into account the graphic illustration of velocity gradation in geometric progression it can be determined that the maximum speed ranges deviate sharply from each other, while the lower speeds lie close together.

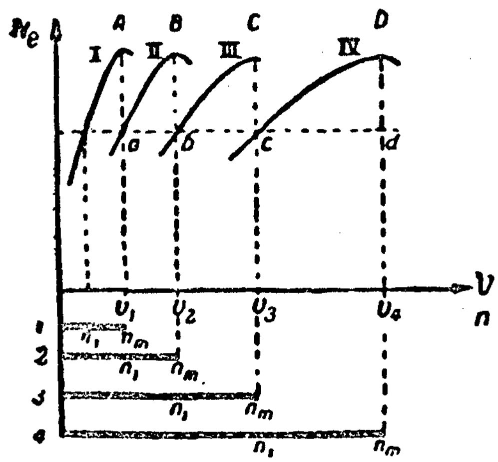


Figure 30. Graphic Illustration of Shifting Levels with Geometric Gradation of the Gears

For this reason, in case we travel at greater speed, a transition must be made to a lower speed at a relatively smaller increase of resistances, in which the average speed is generally reduced.

For this reason transmission gradation in geometric progression can only be recommended for the tank which operates with a large power reserve.

If the line of smallest shifting engine speed tilts slightly to the right (by the angle α), then as Fig. 31 shows, the space between the high speeds will become somewhat smaller which causes average speed to increase.

To be sure, the shifting rotational speeds will be different (Points 1, 2 and 4). Consequently the difficulties when shifting will be different as will the demand for engine power.

On the other hand, if the known line tilts to the left, then the intervals between the speeds will be even greater.

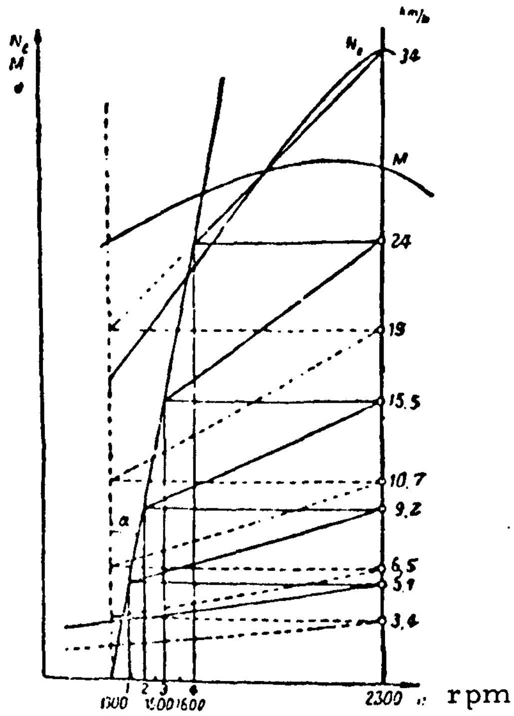


Figure 31. Change of Shift Gradation with the Slope of the Rotational Speed Line

We achieved a greater mutual approach of the speeds by using the arithmetical progression. Our task was to make the differences between the two adjacent gears to a constant value. Then

$$v_n - v_{n-1} = v_{n-1} - v_{n-2} = \cdots = v_3 - v_2 = v_2 - v_1 = \alpha$$

If all equations are compiled such as

$$v_n - v_{n-1} + v_{n-1} - v_{n-2} + \cdots - v_3 + v_3 - v_2 + v_2 - v_1 = v_n v_1 = (n-1)\alpha$$

we obtain

$$\alpha = \frac{v_n - v_1}{n-1} \qquad (54)$$

The value α is designated as origin of the progression.

The maximum speed of a tank is v_{max} = 34 km/h, the minimum speed v_{min} is 3.4 km/h.

Our intention is to install a five-speed transmission in arithmetical progression into the tank. Now the numerical value of the intermediate speeds is to be determined.

$$\alpha = \frac{v_{max} - v_{min}}{n - 1}; \ \alpha = \frac{34 - 3{,}4}{4} = \frac{30{,}6}{4} = 7{,}65$$

$$v_5 = 34 \text{ km/h}; \ v_4 = 26{,}35 \text{ km/h}; \ v_3 = 18{,}7 \text{ km/h}; \ v_2 = 11{,}05 \text{ km/h}; \ v_1 = 3{,}4 \text{ km/h}.$$

Figure 32 gives a graphic illustration of the gear shift levels in arithmetical progression.

Studying the graphic illustration it can be seen that the high ratios lie close to each other and that the engine rotational speeds at which shifting is necessary are greater at higher gears than at the lower ones; this means that acceleration occurs more rapidly at the high ratios.

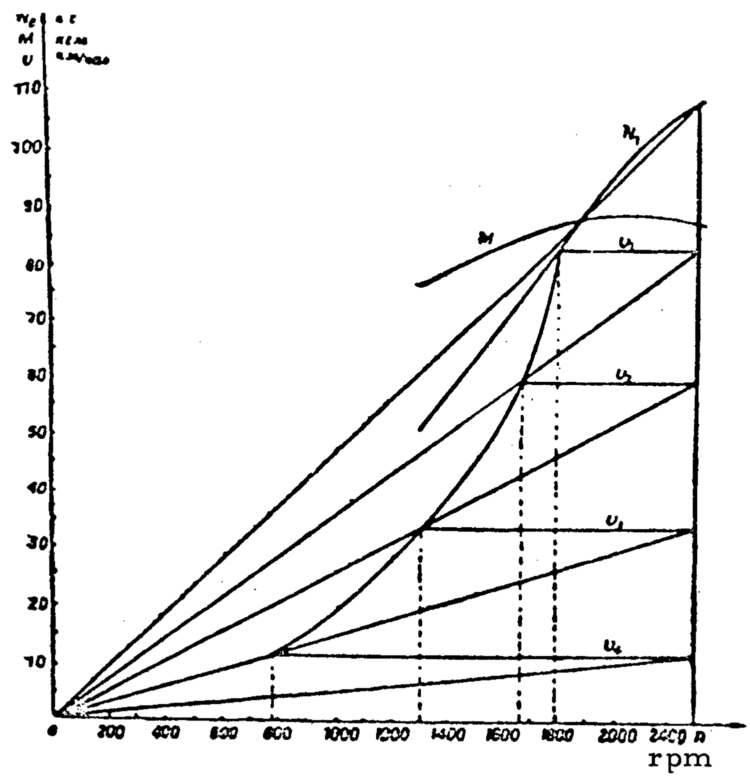


Figure 32. Graphic Illustration of the Ratios Gradated in Arithmetical Progression

In this regard the gradation in arithmetical progression is much more advantageous.

However, acceleration occurs more slowly at the lower ratios and in addition, the engine may stall as a result of the decline of rotational speed when shifting is made to a lower gear.

In selecting the gear ratios we assume that the shifting will be performed rapidly enough that the traveling speed does not decline rapidly.

To be sure, tank traveling speed does decline during shifting and acceleration begins not at the full speed of the previously shifted gear, but at another, lower speed.

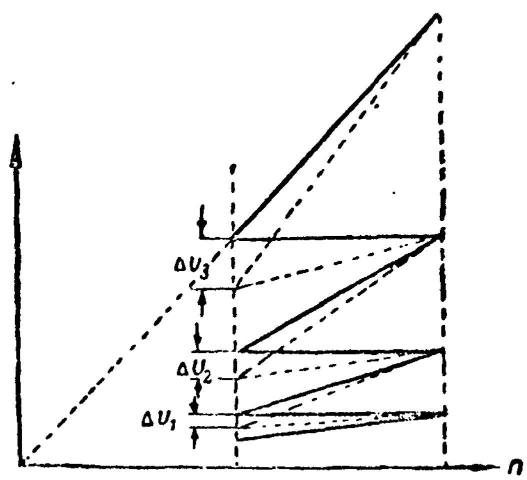


Figure 33. Graphic Illustration of Gear Shifts considering the Decline of Travel Speeds when shifting to the next gear.

In other words, after shifting from v_i to v_{i+1}, v_{i+1} is smaller than v_i at acceleration because of the effect of traveling resistance R and of air resistance R_w during shifting, at which time tractive effort is lost.

For this reason, according to the information quoted earlier

$$\frac{v_i + 1}{v_i} < \frac{n_{max}}{n_i}.$$

In other words the graphic illustration of speeds has the appearance shown on Fig. 33.

If the graphic illustration is studied closer it is clear that when traveling speed is reduced during shifting instead of a series of speeds in geometric progression, a series of speeds is achieved at which the lower speeds are located closer together.

Such a method can be of real interest when making calculations for a tank which is to be used in the mountains.

Whoever is interested should refer to the study of the military academy for motorizing and mechanization (WAMM) written by its dean, A. S. Anatow, entitled "The Calculation of Tractive Effort" (Moscow 1936).

But it must be remembered that it is difficult to set up such a calculation because velocity decline will not be uniform in the different gears.

The following equations apply for the shifting process in the time span during which the engine is disengaged.

$$d\left(\frac{\delta m v^2}{2}\right) = d(R s);$$

$$\frac{\delta m}{2} 2 v \, d v = R \, d s;$$

$$\delta m \int_{v_1}^{v_2} v \, d v = R \int_{s_1}^{s_2} d s.$$

In which: δ is the coefficient allowed by the rotating mass, v is the initial speed, R is the sum of the resistances which have an effect during the shifting period, s is the distance covered during shifting.

This means that the decline of speed $\Delta v = v_1 - v_2$ stipulated by the distance covered

$$\Delta s = s_2 - s_1$$

and by resistance

$$R = f_0 G + k F v^2$$

Both influences will be different when shifting gears to different speeds.

We usually calculate that the shifting procedure takes about 3 s.

But this number is given for a vehicle; if it is to be applied to a tank, it must be checked.

The distance in which shifting must be carried out is, according to the data of English Automobile periodicals, approximately the total length of the vehicle and cannot be applied in the case of tanks.

The matter of the determination of the gear ratio in the shift gear stands in close relationship to this information.

From the formula for the speed of the tank

$$v = \frac{zln}{16{,}7\, i_0}$$

it is to be concluded that

$$i_0 = \frac{zln}{16{,}7\, v} = \frac{A}{v}$$

in which i_0 is the transformation ratio from the engine up to and including the drive sprockets.

Consequently, the following applies for the selection ratios of the tank.

$$i_n = \frac{A}{v_n}; \quad i_{n-1} = \frac{A}{v_{n-1}}; \ \ldots i_2 = \frac{A}{v_2}; \quad i_1 = \frac{A}{v_1}.$$

On the other hand the total conversion ratio of force transmission of the tank from the engine to the drive sprockets usually consists of several ratios, which include the variable ratio in the gear shift i_i, the invariable main drive i_n (usually a bevel gear pair) and the final drive i_b.

In other words $\quad i_0 = i_k\, i_n\, i_b.$

Consequently the conversion ratio in the gear shift is calculated according to the formula

$$i_k = \frac{zln}{16{,}7\, v\, i_n\, i_b} \tag{55}$$

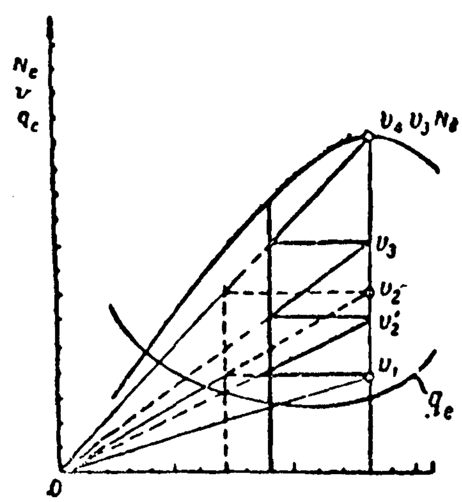


Figure 34. The Number of Gears in the Gear Shift

The influence of the numerical value of the constant gear ratio on the quality of a motorized vehicle has been thoroughly studied already.

It must only be remembered that the total gear ratio of 4-5 must be higher for tanks. For this reason we install a final drive between the bevel gear pair and the drive sprocket.

In conclusion, something should be said concerning the number of stages in the gear shift.

From the engine characteristic lines it is clear (Fig. 34) that the narrower the boundaries within which the rotational velocity changes, the closer the engine approaches its maximum power. The fuel consumption q per HP declines the shorter the time span of acceleration.

On the other hand it becomes much more difficult to increase the number of gears in the gear shift.

Usually tank gear shifts do not have more than 4-5 gears.

13. Determination of the Time Span t and of the Acceleration Course when Shifting into Higher Gears and when Braking.

$$\delta m \, i \, dv = \pm \sum P \, ds \tag{a}$$

in which δm is the assumed mass of the tank, v is the traveling speed of the tank, $\sum$ P is the sum of the forces which effect the tank, d s is a road element.

The sign of the sum of forces is stipulated by its direction.

When traveling with the engine disengaged, the sum of the forces $\sum$ P will be exerted opposite the direction of travel (Fig. 35a), i.e. it will have a minus sign and be expressed in the formula

$$\sum P = f_0 G + k F v^2 \tag{56}$$

When traveling with the brakes applied, braking force R_r (see Fig. 35b) is added to the right side of the equation.

$$\sum P = f_0 G + k F v^2 + R_r \tag{57}$$

Finally, the following equation is valid in accelerated travel with running, engaged engine:

$$\sum P = P_d - f_0 G - k F v^2 \tag{58}$$

Equation (a) can now be solved as follows:

$$ds = + \frac{\delta m v dv}{\sum P}$$

or

$$ds = \frac{M v dv}{A + B v^2}.$$

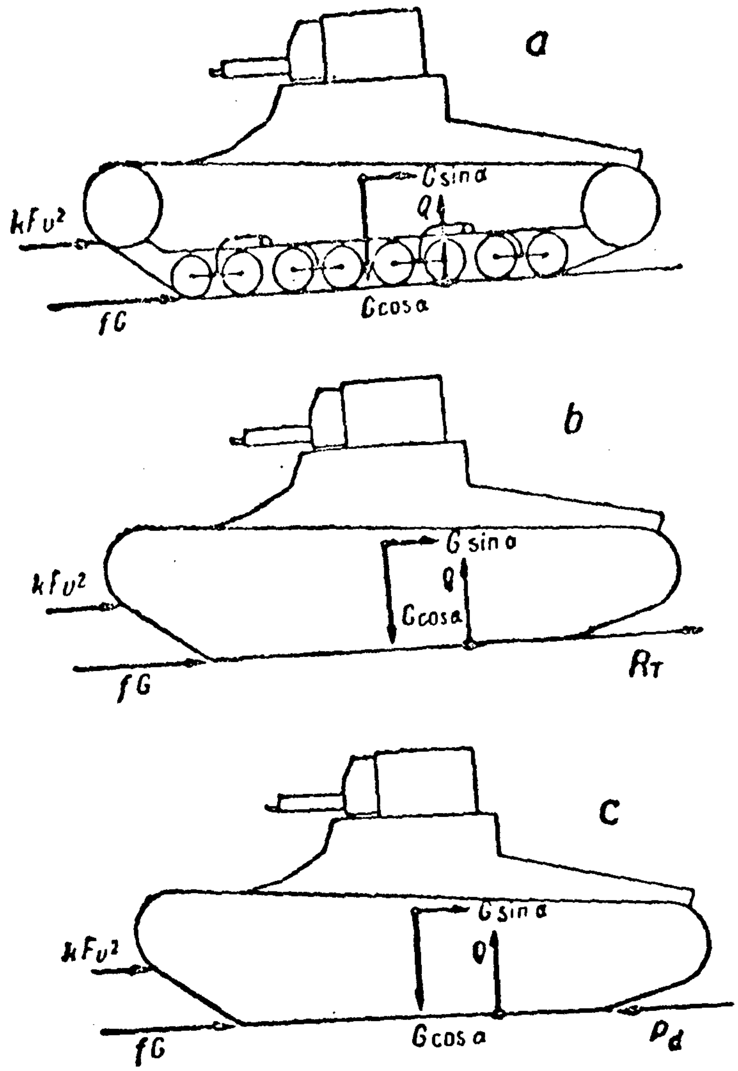


Figure 35. Various Cases of Force Play

Here the coefficients A and B are selected according to the formulas (56) and (57) and (58)

$$A = f_0 G;$$
$$A = f_0 G + R_v;$$
$$A = P_d - f_0 G \text{ and}$$
$$B = k F.$$

After integration we will obtain the following formula:

$$s = \pm \frac{M}{2B} ln (A + B v^2) \pm C.$$

After determining the constants from the integration range t = 0 and v = v_o, we will obtain the following formula:

$$s = \pm \frac{M}{2B} ln \frac{A + B v^2}{A + B v_0^2}. \tag{59}$$

or

$$\frac{A + B v^2}{A + B v_0^2} = e^{\pm s \frac{2B}{M}}$$

which results in the final speed

$$v = \sqrt{\frac{(A + B v_0^2) e^{\pm s \frac{2B}{M}} - A}{B}} \tag{60}$$

If v and v_o are known, the path of movement s can be determined in this manner.

If this theorem of momentum is used, the time of acceleration and of braking can be determined

$$M \, dv = \pm (A + B v^2) \, dt$$

and the following;

$$dt = \frac{M \, dv}{A + B v^2};$$

$$t = \pm \frac{M}{\sqrt{BA}} \text{arc tg } v \sqrt{\frac{B}{A}} + C.$$

C is determined from the boundary conditions v = v_o and t=0

$$C = \mp \frac{M}{\sqrt{BA}} \text{arc tg } v \sqrt{\frac{B}{A}}$$

and

$$t = \frac{M}{BA} \left(\text{arc tg } v \sqrt{\frac{B}{A}} \mp \text{arc tg } v_0 \sqrt{\frac{B}{A}} \right) \tag{61}$$

If the force of resistance is disregarded, we obtain the following equations (assuming that B = 0)

$$\pm ds = \frac{M}{A} v\,dv;$$

$$\pm s = \frac{M}{A}\left(\frac{v^2 - v_0^2}{2}\right) \qquad (62)$$

$$v = \sqrt{\frac{2As}{M} \pm v_0^2}. \qquad (62a)$$

and further

$$M\,dv = \pm A\,dt$$

$$t = \pm\frac{M}{A}(v - v_0). \qquad (62b)$$

In deriving the expressions for s, v and t, we have assumed that A was a constant size; that is not exactly true.

As already mentioned, the resistance against movement is evidently stipulated by speed. The precise designation is unknown. For this reason we will assume that R is a constant number at the given preconditions, this number having no relationship with speed.

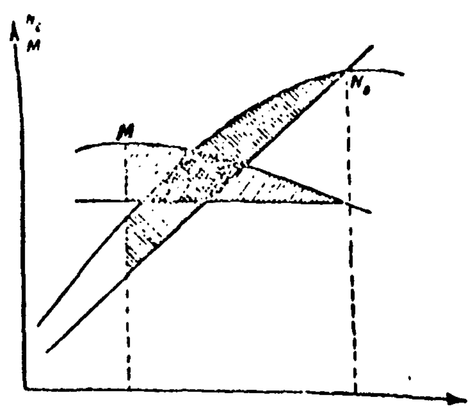


Figure 36. Engine Characteristic Calculating According to the Simplified Formula

For the case where the tank travels with running, engaged engine, we are assuming in calculating according to the formulas (62) that tractive effort is constant, i.e. we are calculating that torque is constant. In other words, we are assuming that engine power N_e changes with rotational velocity according to the law of the straight line (Fig. 36).

As is clear from the graphic illustration, with this type of calculation the power efficiency within the shaded area is not taken into account.

This remark concerning the constant size of tractive effort relates to the case of the self-regulating engine (Fig. 37).

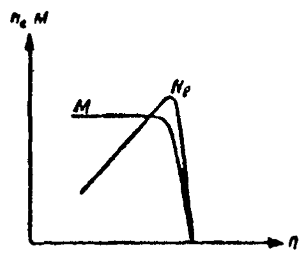


Figure 37. Self-regulating Engine

The ratio of M_d at maximum rotational velocity to torque on the lower limit of the gear shift fluctuates between 0.5 and 1.5.

Engines with rotational velocity limits are frequently utilized in tanks.

The straight-lined part of the curve $N = f(n)$ is utilized. M_d is straight and parallel to the abscissa and then jogs sharply downward (Fig. 38).

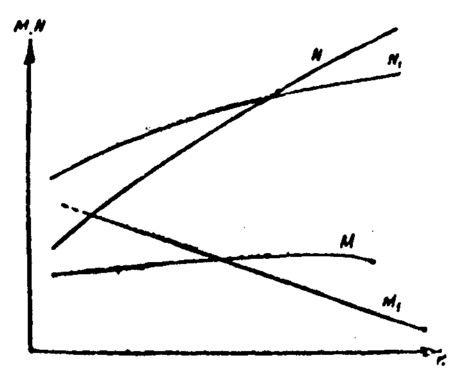


Figure 38. Characteristic Line with the Engine Throttled

Of course it is easier and faster to calculate acceleration according to formulas (62) than to use the one generally applied in vehicle construction.

For this reason, if the characteristic of the engine seems to allow it, this formula should always be used.

The shortcoming of the described method, however, lies in the fact that it is not suitable for general usage. A much more generally usable method is the one suggested by E. A. Tschudakov; to determine the path and time span of acceleration we use the dynamic characteristic $D = f(\varphi)$.

Since the force of air resistance can be neglected in the case of tanks, its dynamic factor can be expressed in the following formula:

$$D = \frac{P_d}{G}$$

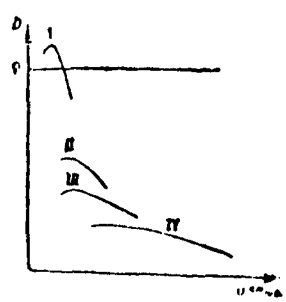
whereby P_d is the tractive effort of the engine.

Since the tractive effort in regard to the engine for travel is either equal to the tangential countereffect of the ground or is larger than it $(P_d\ (f\cos\alpha + \sin\alpha)\, G = f_0 G)$ a comparison of the dynamic factor with the so-called summary coefficient of travel resistance f_0, enables us to assign a value of

$$D - f_0 = \pm \frac{\delta}{g} b$$

for the amount of tractive effort reserve.

This tractive effort reserve can be utilized to overcome increased resistance at acceleration.



$$D = f_0 \pm \frac{\delta}{g} b.$$

Figure 39.

Since the maximum value of tractive effort is the adhesion tractive effort $P_\varphi = \varphi G$, the part of the dynamic characteristic which lies above the straight line φ is not usable.

The accelerations of the tank can be determined utilizing the dynamic characteristic:

$$b = (D - f_0)\frac{g}{\delta}.$$

According to this formula we can graphically illustrate the tank acceleration b + (v) (see Fig. 40).

The acceleration time can be determined on this basis

$$b_v = \frac{dv_T}{dt}$$

according to which

$$dt = \int_{v_1}^{v_2} \frac{1}{b_T} dv_T$$

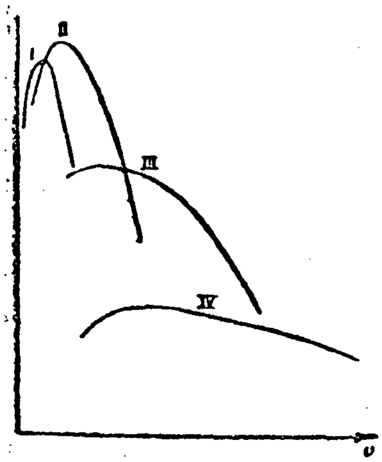


Figure 40. $b = f(v)$

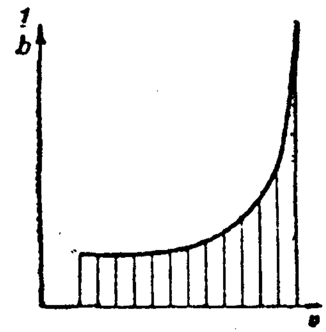


Figure 41. $\frac{1}{b} = f(v)$

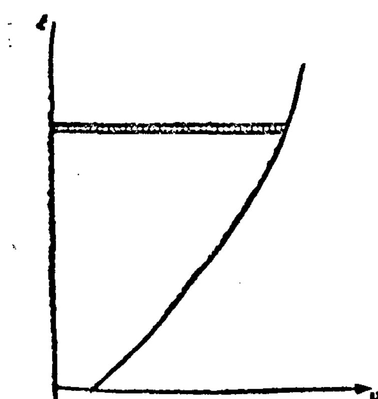


Figure 42. $t = f(v)$

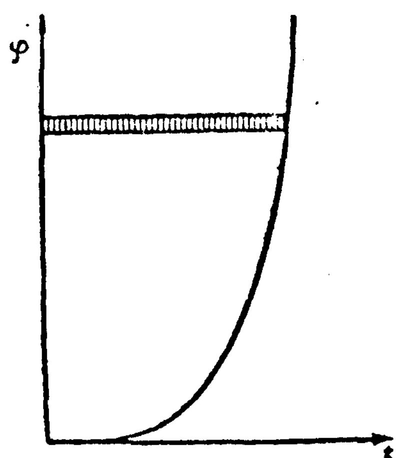


Figure 43. $\varphi = f(t)$

Since this integral cannot be solved analytically, because this kind of analytical relationship between 1 b and v is unknown, this integral must be solved graphically by planimetering the individual surfaces of the graphic illustration 1 b = $f(v)$ (Fig. 41) and plotting the results on the graphic illustration t = $f(v)$ (Fig. 42).

According to this graphic illustration we can finally determine the path of acceleration

$$s = \int_{t_1}^{t_2} v\,dt$$

by integrating. (see Fig. 43).

14. General Conclusions

The following can be determined as the result of information presented in Section I concerning the procedure for calculating tractive effort:

1. The required power output of the engine N_d is determined according to the given maximum speed, according to the height differences of the terrain and the nature of the ground.

2. The required minimum velocity v_{min} is determined according to the preconditions for ground adhesion and the maximum angle of inclination α_{max}.

3. The determined speed range v_{max}/v_{min} is graduated in arithmetical progression. Only when a large power reserve is available to the engine should the geometric progression be utilized.

Cases may occur in which a part of the speed follows the selected graduation while another part drops out of the graduation because it has proven necessary to achieve certain speeds, for example the speed at which one wishes to travel with the artillery or the infantry column.

4. On this basis the gear ratios were determined from which we can derive the basic design dimensions and ascertain the dynamic properties of the tank.

5. Using simplified formulas we determined the paths and the time span for acceleration of the tank as well as the angle of inclination which may be surmounted in different gears.

After the basic values of the engine have been determined, everything is checked according to the method of the dynamic factor.

6. All other problems were solved: the average speed when passing, the weight of the trailers which can be towed under various preconditions.

Section II.

STABILITY

1. Basic Concepts and Definition of the Concepts

In the calculation of tractive effort it was shown that in order to achieve a normal manner of travel (without slip and without the engine dying out), the following condition must prevail:

$$P_\varphi \geq P_d \text{ and } P_d \geq R.$$

The tank may fail to move forward if the tracks "grind" in position or if the engine stalls. It can also occur, however, when the tank loses its equilibrium and tips over.

This section deals with the calculation of stability of tanks at various positions of travel and also gives the method according to which a tank can be evaluated from the standpoint of stability.

Using the relationships derived in this section, we can evaluate the efficiency of the tank when surmounting artificial and natural obstacles (ditches, low walls, slopes and inclinations), since stability is a precondition for this. The load on the individual parts of the track and suspension for various traveling conditions is determined.

By stability we mean the ability of a stopping or moving tank to maintain or to retain its original position as soon as the exterior forces, which have caused it to lose its original position, cease to exert their effect.

Of all the types of stable positions of the tank, only a few can be taken into account as necessary for continuing travel.

Thus the evaluation of stability will depend on how long the tank remains in the position being considered.

Tip position is understood to be the position of the tank (the angle of rotation of the tank to the axis of tilt being considered) at which, even when the effect of the tipping exterior forces has ceased, continues its turn (return) back to the previous direction and thus cannot return to its original position.

In order to illustrate clearly the concept of axis of tilt, we disregard the movement of the tank in a line parallel to the road, since this movement exerts no influence on stability.

In addition, we neglect the vibrations, insofar as we do not use differential equations in this regard.

Therefore, the axis of tip is also the axis of instantaneous rotation of the actual movement.

This observation is necessary because in a turn the tank can exhibit several successive axes of rotation due to its suspension design. Consequently it is possible that the axis of rotation is located at several positions of the tank.

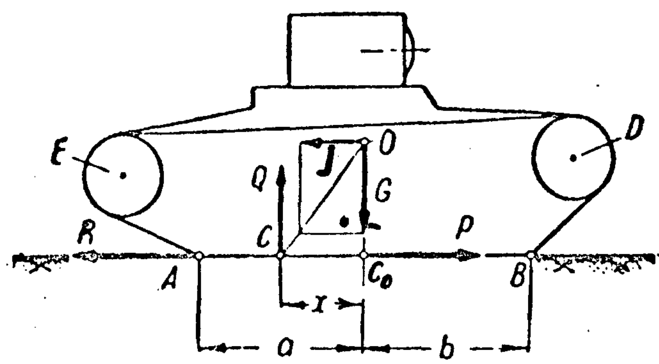


Figure 1. Forces Which May be Exerted on the Tank During Travel.

Before we proceed to a further explanation of the concepts and expressions, we wish to come to a better understanding of Fig. 1, which shows a schematic drawing of a tank, the track and suspension of which forms a unit with the hull.

The following forces are exerted on the tank: tractive effort P, the force of resistance against vibrations R, the force of inertia J and the normal countereffect of the ground Q.

The other designations A, B, D and E are the axes of tip which are to be considered.

C is the metacenter (intersection point of the resultant force of the earth's surface) on the surface of the drawing, C_o is the initial position of the metacenter, O is the center of gravity, a and b are the distances between the plumb lines through O and the possible axes of tilt and x is the shift of the center of pressure.

The amount of shift N of the metacenter from its original position will be considered as a measure of stability which corresponds to the position of the tank in the horizontal plane in rest position.

As long as the metacenter lies between the axes of tilt A and B being considered, the tank is stable. However, as soon as the center of pressure coincides with the most outer axis, the tank begins to lose its equilibrium and begins to tip.

If we wish to determine the stability of a tank, this means finding the position of the metacenter. Two problems may arise in determining stability in this manner:

1. From the given forces and moments which exert an influence on the forces, the position of the metacenter must be determined.

2. According to the known metacenter, the factors (forces, angle of terrain, accelerations, etc.) must be determined which exert an influence on its position.

We now wish to introduce the concept of original stability.

The tank has an original stability if O lies vertically above C_O.

The original stability does not necessarily have to be the maximum stability.

For example it is possible that the shift of the metacenter increases and not decreases the original stability of the tank, if the center of gravity is shifted from the geometrical center point of the support surface to one side or the other.

We will cite examples to illustrate the concept of "original stability".

Figure 2 illustrates a tank in rest position on a horizontal plane.

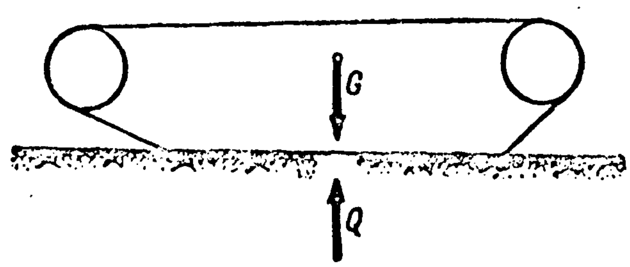


Figure 2. Tank in Rest Position.

Here the results of all forces -- the force of weight G will coincide with the normal lines to the plane of travel.

The tank exhibits its original position of equilibrium.

On Fig. 3 the resultants of the weight G, of the resistance on the towing hook R_k and the force of inertia J intersects with the plane of travel, i.e. vertical below O, at point C_o.

The shift of the center point of pressure x equals zero, the tank is in its original position of equilibrium.

It is necessary to distinguish between the static and the dynamic stability of a tank.

By static stability we mean the ability of the tank to retain the prescribed position when exterior forces and moments are exerted on it, the strength of which increase up to a certain limit without causing accelerations and then become constant again.

In this case the shift of the instantaneous center or the ability of the tank to maintain its prescribed position has nothing to do with the amount of moments and forces.

By dynamic stability we mean the ability of the tank to resume its original position when the exterior factors (forces), which attain an arbitrary, final value when they are exerted, cease.

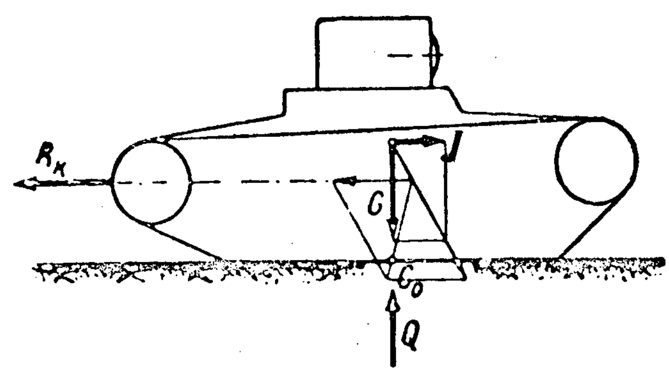


Figure 3. The Shift of the Pressure Center Point is Zero.

In this case the tank will turn in the direction desired without being influenced by whether or not the exterior forces have ceased to exert their influence. Consequently, without regard to the direct effect of the exterior forces, the inertia will be a function of movement, which the exterior forces have exerted during their period of effect.

2. Influence of the Design of Track and Suspension on the Inertia of the Tank

In order to obtain an idea of the influence of track and suspension design on the stability of a tank, we now wish to take up the consideration of tanks which have the same chassis and road wheels, but different types of roller bearings, i.e. a rigid (Fig. 1), an individually sprung (Fig. 4a) and a bogie wheel suspension frame (Fig. 4b).

In the case of a tank with a rigid suspension, the axes being considered for tipping will be axes A and B (Fig. 1). That means that the instantaneous center in this case will move within the limits of section AB, without putting the tank in danger of tipping.

Thus in order to put the tank in danger of tipping, the resultants of all forces and moments being exerted on the tank must proceed in such a way that they do not intersect with section AB.

We will observe almost the same thing when the tank is equipped with individual road wheel springs (Fig. 4a).

The difference is due to the fact that because of the sprung support of the last road wheel (when the tank tips at axis A or B,) distance BD or AE will not be equal to the corresponding distances as when rigid support prevails.

As a result, the inertia in the case of individual springs of the road wheels when tipping at axis E or D will be somewhat smaller than with the rigid concept.

We will observe a considerable difference of the distance between the axes of tilt being considered in the first interval of time in the case of bogie wheel suspension frames (Fig. 4b).

If the road wheel rocker arm is short, we can regard axes A and B as axes of rotation.

Section AB of the swing arm type is considerably smaller than the corresponding section in the case of the tank with spring or rigid suspension.

But this fact cannot be the sufficient basis for a conclusion as to the degree of inertia of this or any other tank.

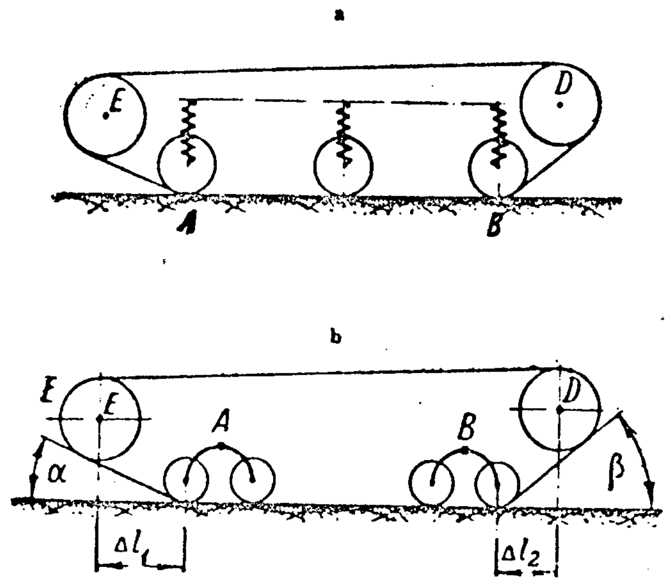


Figure 4. Individual Springs and Rocker Arm Springs

Disregarding the narrowing of the interval between axes A and B, the tank with swing arm suspension is often much more stable than both the first-named tanks.

Inertia is determined by the angles of the lines going from the center of gravity through all possible axes of rotation and the line perpendicular to the plane of travel.

As Figure 5 shows, when the center of gravity is low (O_2), the swing arm suspension gives the tank much more stability than both the other aforementioned types of suspension.

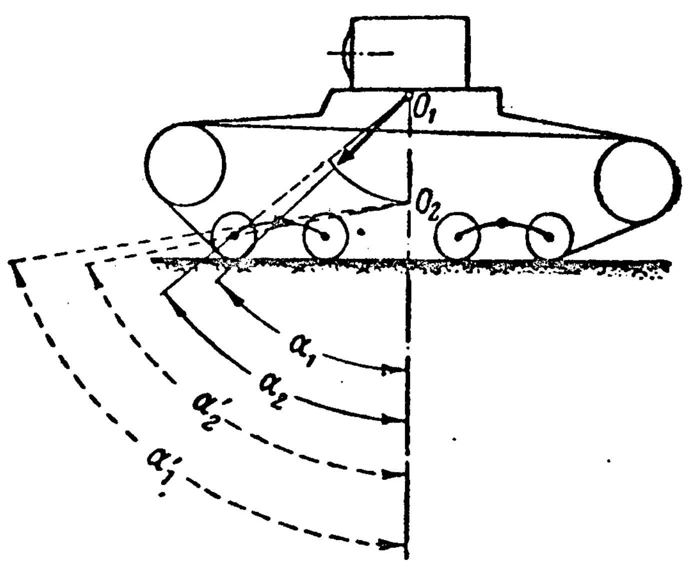


Figure 5. Angle of Stability with Individual Springs and Swing Arm Springs.

The greater the angle α between the straight lines from the center of gravity through the point of wheel support and the perpendicular line, the greater is the inertia of the tank (at otherwise equal preconditions).

We wish to designate this angle as the angle of tank stability.

3. Determination of the Inertia of a Tank on a Plane in General

We assume that we are dealing with a rigid roller bearing.

The exterior forces and moments being exerted are as follows:

G = the weight of the tank,
G sin α = the components of tank weight parallel to the roadway,
G cos α = the components of tank weight vertical to the roadway,
R = the resistance against slip of the tank as a result of the deformation of the ground under the influence of adhesion weight,
R_k = the force of resistance on the towing hook,
P_J = tractive power,
Q = the vertical reaction of the ground,
M_J = the total moment of inertia of all rotating individual parts whose plane of rotation coincides with the longitudinal surface of the tank.

$$M_J = J_1 \varphi_1'' + J_2 \varphi_2'' + \cdots J_n \varphi_n''.$$

J is the polar moment of inertia of the rotating individual parts and φ'' is the angle of acceleration.

The following are included in the individual rotation parts of the tank with steering clutches:

1. Two drive sprockets,
2. Idler wheels,
3. n' road wheels,
4. n track supporting rollers,
5. Two large gear wheels of the final drive,
6. Two small gear wheels of the final drive,
7. Two steering clutches,
8. The main driving gear wheel..

If the transformation ratio of the final drive is designated as i_b and the polar moments of inertia and the radius of all named individual parts in the same series is designated as

$$J_1, r_1; J_2, r_2; \cdots J_8, r_8$$

we obtain:

$$M_J = \frac{dv}{dt}\left(\frac{J_1}{r_1} + \frac{J_2}{r_2} \cdots + \frac{J_5}{r_5} - \frac{J_6}{r_6 i_b} - \frac{J_7}{r_7 i_b} - \frac{J_8}{r_8 i_b}\right).$$

in which dv/dt is the linear acceleration of the tank.

The last three members of the equation have been selected with minus signs because the direction of rotation of the individual parts is different from all the others.

Disregarding the forces and moments which are exerted on the tank, it is necessary to refer to the geometric differences which influence the position of the tank. These include:

x = the shift of the instantaneous center,
α = the angle of inclination of the road,
γ = the angle between the towing hook resistance and the horizontal plane,
h = the point at which gravity and the force of inertia are exerted,
h_k = the point at which the force of resistance is exerted on the towing hook.

With the given preconditions, we can solve one of the two problems.

1. To determine the stability of the tank at the given preconditions (x is to be determined).

2. Determine the preconditions for tipping the tank.

We wish to take up the solution for each problem individually.

Before we solve the first problem, we wish to establish which values are known and which are to be calculated.

Using the preconditions for the stability of a tank as a point of departure we can write down the three equations of equilibrium and determine the three unknown values in this way.

Obviously the two unknown values can be X and Q, while the third unknown value will be one of the following:

$P, \; J, \; R_k$ etc.

In accordance with the problem at hand, the values to be determined may be

Q and J

Then the values being considered for the given preconditions will include the following:

α = the angle of inclination
η = the travel coefficient of the tank
G = the weight
$R_k \operatorname{tg} \gamma$ = the vertical components of resistance on the towing hook
R_k = the horizontal components of resistance on the towing hook.
δ = the coefficient of mass increase
f = the coefficient of travel resistance
q = the traction coefficient
P = tractive effort

We will assume the tractive power to be unknown and uniform at each conversion without considering the number of revolutions.

The required equations are:

$$\sum X = R_k + J + G \sin \alpha + f G \cos \alpha - P = 0 \qquad (1)$$

From this equation we determine the value of J and, because dv/dt is known, the value of M_J.

$$\sum M = x G \cos \alpha - h (G \sin \alpha + J) - h_k R_k - (a_k - x) R_k \operatorname{tg} \gamma - M_J = 0. \qquad (2)$$

When we know the value of M_J and J, then we can calculate the value of x.

If x is equal to or smaller than a , the tank is stable; but if x is larger than a , the tank begins to tip.

We should refer to the fact that from the physical standpoint, x will never be greater than a , because the metacenter does not lie behind the axis of tip.

In order that the given preconditions be taken into account, care must be taken that the following relation is maintained:

$$P \leq \varphi Q \qquad (3a)$$

If P is larger than φ Q, the tank will either slip or the track will grind.

We wish to determine the vertical countereffect of the ground Q according to the following equation:

$$\sum Z = G\cos\alpha + R_k \operatorname{tg}\gamma - Q = 0 \qquad (3)$$

If we solve the second problem, i.e. determine the moment of tip, then this means that we have found an answer to the question at what angle of tilt or slope, which tractive effort on the hook, which acceleration when starting up or braking, etc.; we obtain the following value

$$x \geq a$$

From this data it follows that of all the unknown values which give the position, no more than three variables can be determined.

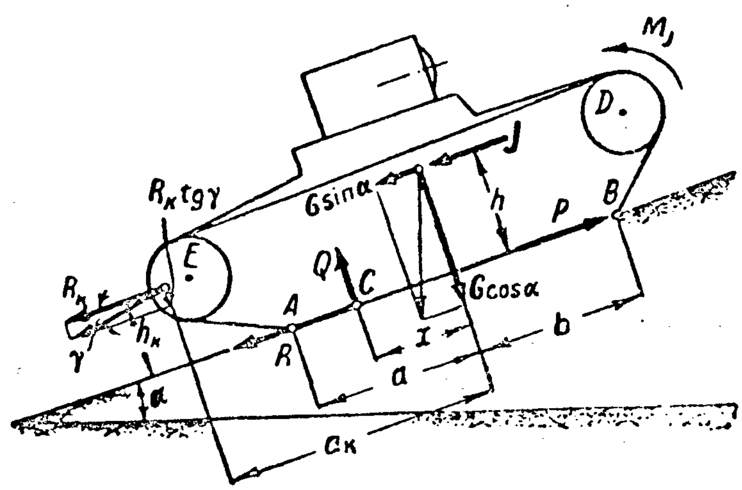


Figure 6. Forces being exerted on the tank with rigid roller bearings when the tank is traveling in a non-uniform manner.

The number of the various combinations is as large as triple the number of variations.

Among the unknown values which present preconditions for tipping, at which x = a, are:

$$\sphericalangle\, \alpha,\ P,\ Q.$$

Among the given values are:

$$G;\ R_k;\ f;\ J;\ M_J;\ \alpha;\ h;\ \eta;\ \gamma;\ h_k \text{ and } L.$$

As before we turn to three equations:

$$\sum X = 0.$$

The unknown values α and P are contained in this equation

$$\sum Z = 0$$

With the unknown values: Q and α

$$\sum M = 0.$$

In which α is unknown.

If we solve the last-named equation in reference to the angle α and substitute into the first and second equation, we will determine the value of P and Q, i.e. the preconditions under which x = a.

Care must be taken that $P \leq \varphi Q$.

All special cases can be solved with the above quoted equations in which we assume a series of values with zero.

Since it is simpler, however, to write down individually the equations for each special case, we will not take up a general solution.

4. Durability of a Tank on a Slope at Uniform Traveling Speed

When the tank is to surmount an inclination with an angle of α (Fig. 7), the shift of the tank metacenter is determined according to the following equation:

$$x = h \operatorname{tg} \alpha \tag{4}$$

The maximum value of the angle of inclination at which the tank maintains its equilibrium is determined from the following inequality:

$$\alpha_{max} \leq \operatorname{arc\,tg} \frac{a}{h}.$$

The surmounting of an inclination with an angle α requires more precise calculations which we will conduct later on.

We are assuming that the tank has been subjected to a short but sharp jolt when surmounting an inclination with an angle of α because the road wheel has struck an uneven place and thus the tank has begun to tip. We now wish to consider the possible consequences more closely.

When the tank turns on the front or the rear, it can exhibit two different rotational axes, either at points B and D or at points A and E.

It is also possible for the tank to get into the following position: when it begins to turn on axis A, it stops after moving around angle γ because the turn around axis E causes an increase of the tipping moment and a reduction of the moment of equilibrium.

Consequently, only a partial decline of stability occurs with a movement around axis A. On the other hand, the tank can lose its equilibrium completely when moving around axis E.

If the angle of stabilization in comparison to axis A is equal to or greater than the angle of stabilization at axis E (after moving around angle γ), then the tipping movement of the tank in the first point of time (axis A) will already have begun to tip completely.

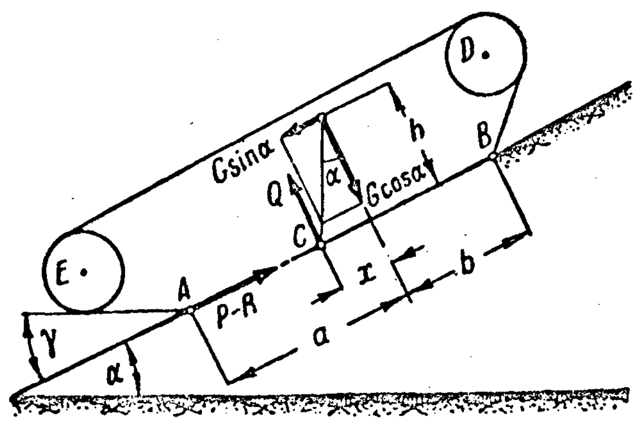


Figure 7. The forces being exerted on the tank under uniform traveling speed.

Here we wish to consider a case in which the angle of stabilization is smaller at axis A than the stabilization angle at axis E, that means

$$\alpha_A < \alpha_E$$

Two different positions can result under these preconditions:

1. As soon as the tank has made a complete turn around angle φ up to angle γ , it stops until something else causes it to move out of this position.

2. Due to the law of inertia the tank will continue its turn in the previous direction until weight line has crossed over axis of turn E.

As is clear from the established preconditions, in this case we will study the dynamic inertia of the tank.

In order to be able to determine which of the probable cases will occur in regard to the tank position, the following method must be used:

It is necessary to determine the difference of the effect of the tipping and of the stabilizing moment when the tank turns around angle γ at axis A.

We designate the distance of axis A from the center of gravity with the letter ϱ and write down the formula for the effect of both moments.

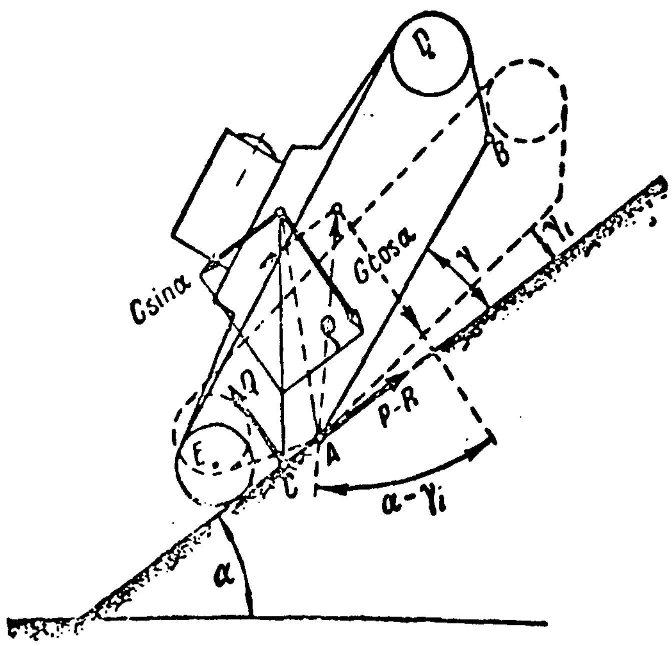


Figure 8. Turn Around Axis A

This involves (Fig. 8) the effect of the tipping moment

$$W_{kp.} = M_{kp.}\,\gamma = \int_0^{\gamma} \varrho \cos(\alpha - \gamma_i)\, C \sin\alpha \, d\gamma_i =$$

$$= \varrho C \left(\frac{\sin 2\alpha}{2} \sin\gamma - \sin^2\alpha \cos\gamma + \sin^2\alpha \right)$$

and the effect of the stabilizing moment:

$$W_{st.} - M_{st.}\,y = \int_0^{y} \sin(\alpha - y_i)\, G \cos\alpha\, d y_i =$$

$$= \varrho G \left(\frac{\sin 2\alpha}{2} \sin y + \cos^2\alpha \cos y - \cos^2\alpha\right).$$

The difference of the effects is:

$$A\,W = W_{kp} - W_{ST} = \varrho G (1 - \cos y)$$

Consequently, after the tank has turned around angle y, according to the law of inertia it will continue its rotation this time already around axis E (Fig. 9).

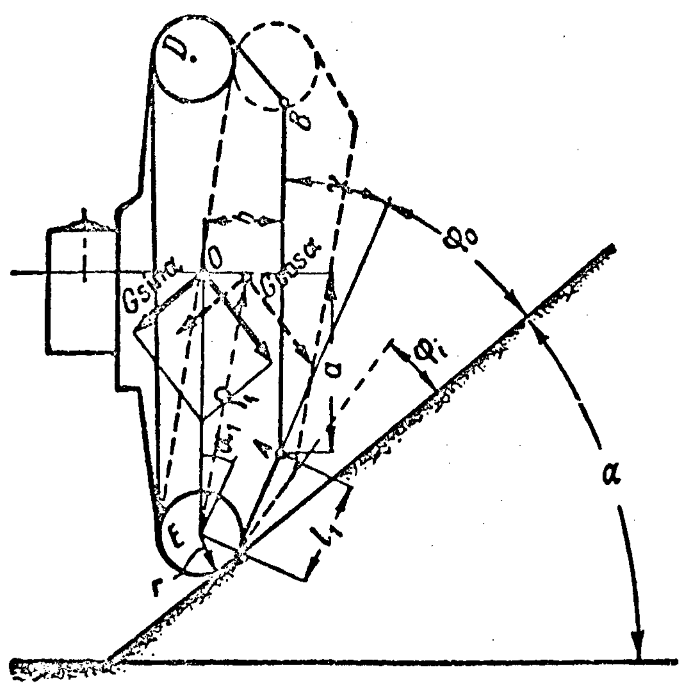


Figure 9. Turn Around Axis E

In order that the vertical line of the center of gravity does not cross over axis E, it is necessary that the difference between effects $A\,W_1$ of the stabilizing and tipping moment opposite axis E be greater than $A\,W$ when the tank turns at axis φ_0.

To calculate the following formula we will write down

$$A\,W_1 = W_{1\,st} - W_{1\,kp}.$$

The tank angle of rotation at which the weight line passes over axis E can be determined according to the following equation:

$$\varphi_0 = 90° - \alpha - \alpha_1.$$

Angle α_1 can be determined from the following formula:

$$\operatorname{tg}\alpha_1 = \frac{a\sin\gamma + h\cos\gamma - r}{l_1 + a\cos\gamma - h\sin\gamma}.$$

If we designate the distance ϱ between axis E and the center of gravity, and the momentary value of the angle of rotation opposite axis E as φ_1 (Fig. 9) we obtain:

The effect of the stabilized moment:

$$W_{1st} = \int_0^{\varphi_0} \varrho_1 \sin(\alpha + \varphi_1)\, G\cos\alpha\, d\varphi_1 =$$
$$= \varrho_1 G\left(\frac{\sin 2\alpha}{2}\sin\varphi_0 - \cos^2\alpha\cos\varphi_0 + \cos^2\alpha\right).$$

The effect of the tipping moment:

$$W_{1kp} = \int_0^{\varphi_0} \varrho_1 \cos(\alpha + \varphi_i)\, G\sin\alpha\, d\varphi_i =$$
$$= \varrho_1 G\left(\frac{\sin 2\alpha}{2}\sin\varphi_0 + \sin^2\alpha\cos\varphi_0 - \sin^2\alpha\right).$$

The difference of the impulses:

$$A\,W_1 = W_{1st.} - W_{1kp.} = \varrho\, G\,(1 - \cos\varphi_0) \tag{6}$$

Consequently the tank will not tip over if $\varrho_1(1-\cos\varphi_0) > \varrho_1(1-\cos\gamma)$, it will lose its stability, however, if

$$\varrho_1(1-\cos\varphi_0) < \varrho(1-\cos\gamma).$$

On the basis of these assertions it is not difficult for us to arrive at the following conclusion: the deeper this axle of the front or rear wheels with an unchanged mutual distance of these wheels, the greater will be the inertia of the tank.

5. Transverse Stability

The migration of the metacenter is determined according to the following equation (Fig. 10):

$$y = h\operatorname{tg}\beta \tag{7}$$

The tank tips whenever the metacenter lies outside the support, the boundary case is:

$$y = \frac{B}{2}$$

or

$$y = \frac{B}{2} = h\operatorname{tg}\beta_{max},$$

which results in

$$\beta_{max} = \operatorname{arc\,tg}\frac{B}{2h}$$

If the angle of inclination is $\beta <$ arc tg B/2h, the tank is stable, but if $\beta > \beta_{max}$, the tank will tip.

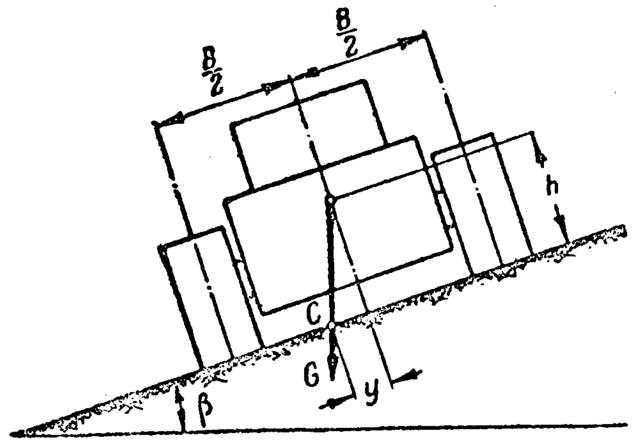


Figure 10. Tank in Sloping Position

In order that the tank not slip, the following inequality must prevail

$$G \sin \beta \leqq \mu G \cos \beta,$$

or

$$\mu \geqq \operatorname{tg} \beta \tag{8}$$

in which μ is the friction coefficient of lateral slip.

6. Stability of a Tank on a Slope

If the tank is traveling on a plane which forms angle α to the horizontal plane, the weight of the tank may be divided into two components: That parallel to the plane of movement $G \sin \alpha$ and that vertical $G \cos \alpha$ (see Fig. 11a).

The first component may or may not coincide with the longitudinal axis of the tank (which runs through the center of gravity).

If it does not coincide (Fig. 11b), the weight is distributed on three coordinates: On the one vertical to the plane of movement $G \cos \alpha$, parallel to the longitudinal axis of the tank $G \sin \alpha \cos \psi$ and parallel to the transverse axis (which also runs parallel through the center of gravity of the tank) $G \sin \alpha \sin \psi$.

ψ is the angle which is formed by the longitudinal axis with the line of the strongest travel resistance on the plane.

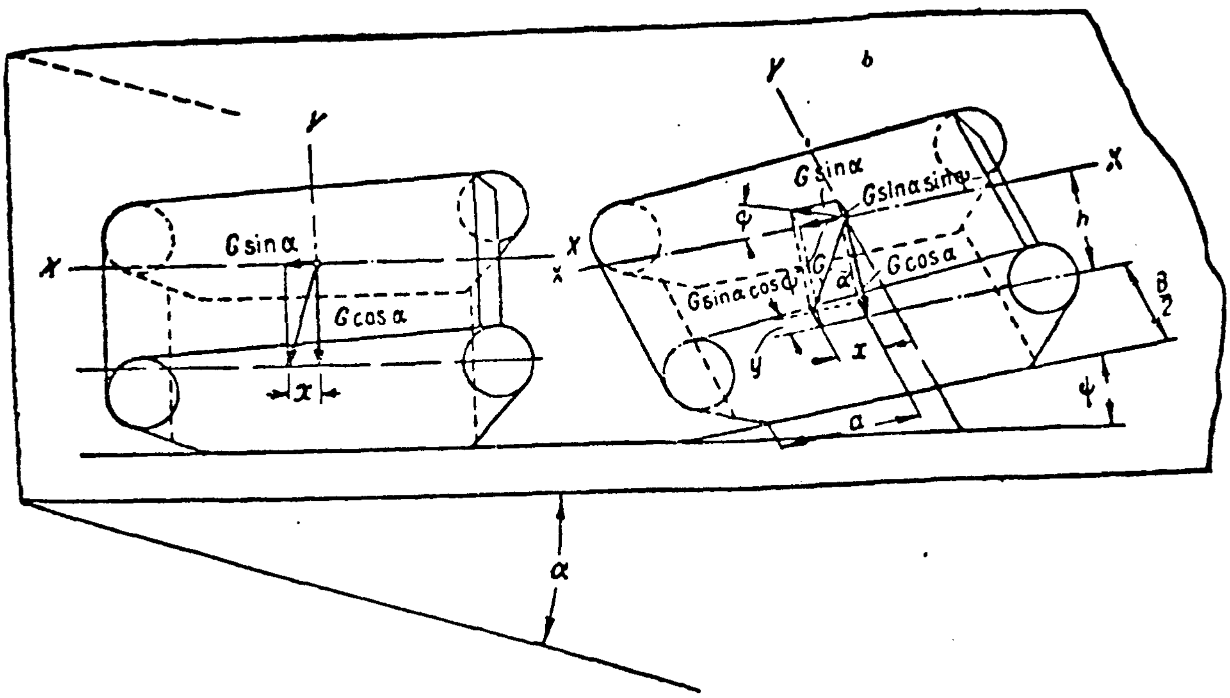


Figure 11. Tank on a Slope

The last-named position of the inclined plane to the tank is usually designated as a slope.

As we see, a shift of the pressure center point in two directions occurs on the slope.

Consequently on a steep slope, which the tank cannot surmount despite traction on the ground and engine tractive effort and stability, an attempt must be made (usually with snow shoes) to surmount this inclination in such a way that the tank turns downward at an angle ψ.

In this case we will determine the shift of the metacenter according to X and Y from the equations:

$$x = h \operatorname{tg} \alpha \cos \psi \tag{9}$$

and

$$y = h \operatorname{tg} \alpha \sin \psi \tag{10}$$

If we square both equations, we will obtain the following equation:

$$\operatorname{tg}\alpha = \frac{1}{h}\sqrt{x^2 + y^2}.$$

The maximum value of angle α is found when we introduce the maximum values for X and Y into the equation: $X \leq a$ und $Y \leq B_2$

$$\operatorname{tg}\alpha_{max} = \frac{1}{h}\sqrt{a^2 + \frac{B^2}{4}} \tag{11}$$

The angle of inclination corresponds to the angle of rotation around which the tank must be turned

$$\psi = \operatorname{arc\,tg}\frac{B}{2a}$$

The circumstance must also be considered that the vertical countereffects of the ground will effect the tracks in a non-uniform manner if ψ does not equal zero, i.e. we obtain

$$Q_1 = \frac{G}{2}\left(\cos\alpha + \frac{2h}{B}\sin\alpha\sin\psi\right)$$

and

$$Q_2 = \frac{G}{2}\left(\cos\alpha - \frac{2h}{B}\sin\alpha\sin\psi\right).$$

Accordingly the resistances against movement of each track will be non-uniform, consequently the tank may turn involuntarily (unintentionally).

As we recognize, three moments are exerted on a tank when traveling on a slope; two of them are tipping moments

$$M_y = h\,G\sin\alpha\sin\psi$$

and

$$M_x = h\,G\sin\alpha\cos\psi$$

and one turning moment, the last-named moment being expressed in two different forms: Up to the beginning of "slip" of the track with the smallest load it is a changing moment

$$M_z' = \frac{B}{2}f(Q_1 - Q_2)$$

and from the beginning of "slip" of the least loaded track on, since there is no possibility of equalizing the turning moment by the tractive effort of the track:

$$M_z'' = \frac{B}{2}[G\sin\alpha\sin\psi - 2Q_2(\varphi - f)].$$

In this formula the final moment is already considered after the equalization.

"Slip" will occur if $\varphi Q_1 < f Q_2 + \frac{G}{2} \sin\alpha \cos\psi$

or

$$\varphi < \frac{f \operatorname{ctg}\alpha - \frac{2h}{B} f \sin\psi + \cos\psi}{\operatorname{ctg}\alpha - \frac{2h}{B}\sin\psi}.$$

In order to prevent an accidental turning of the tank during travel at an angle $\psi \neq 0$, the track with the least load must be braked.

So that all data might be applicable, care must be taken as in the previous cases to adhere to the following conditions:

$$G \sin\alpha \cos\psi \leq G \cos\alpha\, \varphi,$$
$$G \sin\alpha \sin\psi \leq G \cos\alpha\, \mu,$$

or in other words:

$$\operatorname{tg}\alpha \cos\psi \leq \varphi,$$
$$\operatorname{tg}\alpha \sin\psi \leq \mu.$$

EXAMPLE:

Given are: $a = 1{,}5\ m;\ h = 0{,}8;\ B = 2\ m;\ f = 0{,}06.$

To be determined are: α_{max}, ψ, M_z, as well as Q_1 and Q_2 to be compared (equalized) are: α_{max} at $\psi \neq 0$ with α_{max} at $\psi = 0$.

SOLUTION:

1. $$\operatorname{tg}\alpha_{max} = \frac{1}{h}\sqrt{a^2 + \frac{B^2}{4}} = \frac{1}{0{,}8}\sqrt{1{,}5^2 + \frac{4}{4}} = 2{,}25.$$

From this results $\operatorname{arc\,tg} 2{,}25 = 66° = \alpha_{max}$.

2. At $\psi = 0,\ y = 0$

$$\operatorname{tg}\alpha = \frac{1}{h}\sqrt{a^2} = \frac{a}{h} = \frac{1{,}5}{0{,}8} = 1{,}9;\quad \operatorname{arc\,tg} 1{,}9 = 62° = \alpha'_{max}.$$

3. $$\operatorname{tg}\psi = \frac{B}{2a} = \frac{2}{3} \approx 0{,}7;\quad \operatorname{arc\,tg} 0{,}7 = 35° \doteq \psi.$$

4. $$Q_1 = \frac{G}{2}\left(\cos 66° + \frac{1{,}6}{2}\sin 66° \sin 35°\right) \approx 0{,}4\,G.$$

5. $$Q_2 = 0.$$

6. $$M_z = \frac{2}{2}(G \sin 66° \sin 35)) \approx 0{,}52\,G.$$

7. Pitching Moment of the Tank When Braking

Determining the stability of a tank when starting up (accelerating) is completely "useless" because an actual acceleration, no matter how powerful the engine might be, never causes the tank to tip.

On the other hand, sudden sharp braking at a certain tank traveling speed and at a (certain) coefficient of ground traction, can cause the tank to tip.

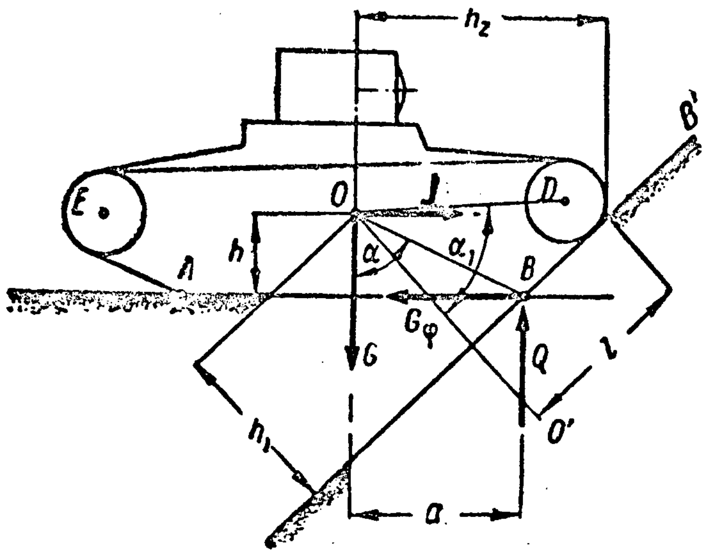


Figure 12. Tipping When Braking

We now wish to determine the tank traveling speed before braking and the coefficient of ground traction insofar as either may cause the tank to tip with sudden braking.

The tank can tip over at axis B (see Fig. 12) when the force of inertia reaches

$$J = \frac{Ga}{h} \leq G_\varphi$$

This results in the following coefficient of ground traction which can cause the tank to tip:

$$\varphi \geq \frac{a}{h}.$$

If the inertia of the tank when turning on axis B is designated by the angle α and when turning on axis D is designated by the angle α_1, then if $\alpha_1 > \alpha$

$$\varphi \geq \frac{l}{h_1}.$$

If the experimentally determined value of the coefficient φ is smaller than the one determined according to the equations, then before the tank tips it will begin to slip.

In the course of completely tipping over the center of gravity would increase from h to h_2. If we disregard power loss on the drive sprocket axle then the lift will be

$$W_1 = G(h_2 - h).$$

Consequently, before tipping over the tank must have a weight which is expressed by the following equation

$$\delta = \frac{m v^2}{2 \cdot 3{,}6^2} = G(h_2 - h).$$

The result is that

$$v = 16\sqrt{\frac{h_2 - h}{\delta}} \tag{12}$$

v is expressed in km/h and h is expressed in m.

8. Stability when Turning in an Inclined Position

We will assume that C is the centrifugal force of inertia, β is the angle of slope and v the speed of the center of gravity on the tank.

We now wish to calculate the value of force C at which the tank tips (Fig. 13).

We have the following equation:

$$(C + G\sin\beta)h - \frac{B}{2}G\cos\beta;$$

from which results:

$$C = G\left(\frac{B}{2h}\cos\beta - \sin\beta\right) \tag{13}$$

The relationship between the turning radius and the speed of the center of gravity at which tipping occurs, is expressed in the following formula:

$$R_0 = \frac{m v^2}{C}.$$

To avoid any slip it is necessary that

$$C + G\sin\beta \leq \mu G\cos\beta.$$

At β = 0 formula (13) can be written as follows:

$$C = \frac{B}{2h}G \tag{13a}$$

We now wish to determine the influence of the steering gear design in relation to speed and turning radius at which the tank tips.

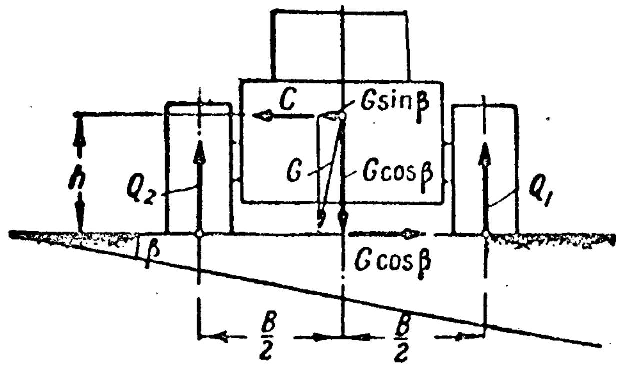


Figure 13. Forces Exerted on the Tank when Turning on a Slope

In this we will assume that in both cases the engine rotational velocity, the transformation ratio to the drive sprocket, as well as the number of links which go around the rim of the drive sprocket, are equal.

It should be recalled that there are two types of steering gears: the simple differential and the steering coupling.

In the case of the simple differential, the path radius of the center of gravity at which tipping occurs is expressed in the following formula:

$$R_0 = \frac{m v^2}{C}.$$

v is the speed of the tank center of gravity which is as large as the speed of the same point with movement in a straight line.

In the case of the steering coupling, the path radius of the center of gravity at which tipping occurs is determined from the following equation:

$$R_2 = \frac{B}{2} + \frac{m}{2C} v_2^2 \pm \sqrt{\frac{m}{2C} v_2^2 - \frac{mB}{C} v_2^2}$$

v_2 is the speed of the tank moving in a straight line. When a change is made to circular travel, it becomes the speed of the outer track. R_2 is the path radius of the outer track.

We now wish to determine the relationship between the radii of the tipping movement and use as a point of departure the precondition that the speeds which occur before turning tanks with a simple rotating steering gear and with the clutch gear are equal and that engine rotational velocity remains constant.

In order for the tank to tip over in a turn due to centrifugal force, it is necessary that C demonstrate a definite value which is unrelated to the design of the steering gear, i.e. that C be a constant value.

Consequently,
$$C = m\frac{v_d^2}{R_d} = m\frac{v_b^2}{R_b},$$

i.e.
$$\frac{v_d^2}{v_b^2} = \frac{R_d}{R_b}.$$

The index "d" relates to the rotating steering gear and index "b" refers to the clutch gear; in both cases v and R refer to the course of the center of gravity.

If we designate the increasing speed of the tank before it turns as v, we can write the following equations:

$$v_d = v_1;$$

$$\frac{v}{v_b} = \frac{R_b + \frac{B}{2}}{R_b}.$$

If we square the equations, the result is

$$\frac{v^2}{v_b^2} = \frac{v_d^2}{v_b^2} = \frac{\left(R_b + \frac{B}{2}\right)^2}{R_b^2} = \frac{R_d}{R_b}$$

the result is that

$$R_d = \frac{\left(R_b + \frac{B}{2}\right)^2}{R_b} = R_b + B\left(1 + \frac{B}{4R_b}\right) \qquad (14)$$

If, however, we begin by disregarding the fact that C is a constant value and that $R_b = R_d$, then we can determine the relationship between the speeds before the turn at which tipping occurs at the given R by the following equation

$$v_b = v_d\left(1 + \frac{B}{2R}\right).$$

In all cases treated here, the maximum value of C must not be greater than μQ.

9. Surmounting Obstacles

Tanks can overcome obstacles in three ways:

1. with the use of special equipment;
2. using its power;
3. utilizing the geometrical dimensions, the position of the center of gravity, ground traction and the tractive effort of the engine.

Here we wish to study the latter method.

a) Trenches

A trench can be crossed by a tank at two basically different tank positions, namely:

a) when the center of gravity lies outside the support for a certain period of time;
b) if the center of gravity does not go beyond the support or is located between two support points.

In the first case the trench can be crossed only by utulizing the weight of the tank before it sinks back into the ground.

We do not wish to deal with this case.

In the second case we must have some idea of the influence of the following values in regard to trench width:

1. the dimensions of the tank;
2. the position of the center of gravity;
3. the type of ground;
4. the track and suspension system.

Before we go any further it seems fully clear that when the road wheels are on rigid bearings the trench can only be crossed if its width is not greater than the smaller of the dimensions a and b (see Fig. 14).

We will assume that all dimensions a are smaller than the corresponding dimensions b.

In the case of a trench whose edges are not compressed, we can assume that $B = a_3$.

In reality the edge of the trench can cave in because of the drive sprocket or the idler wheel, thus widening the trench. Thus for reasons of safety it must be assumed when crossing a trench that $B = a_2$.

Of course the maximum width of the trench which has been crossed will be smaller, the greater the difference between a_1 and b_1.

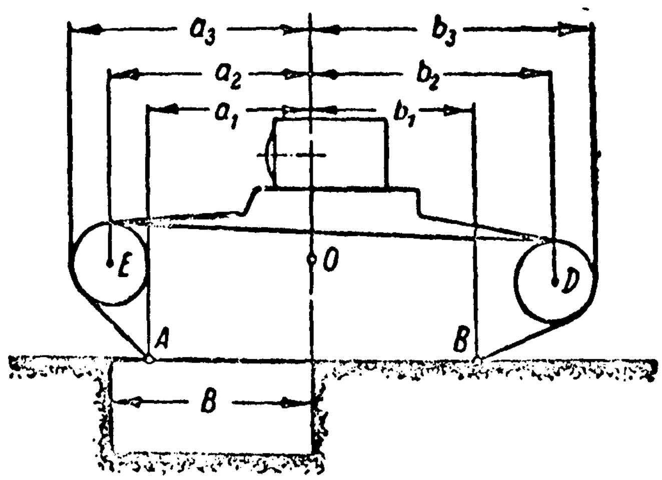


Figure 14. Crossing a Trench

Consequently, the concentric grouping of the center of gravity v will be most favorable.

When traveling uphill and downhill, the trench width to be crossed at $a_1 = b_1$ will decline by the value $h \text{ tg } \alpha$ where h is the height of the center of gravity and α is the angle of tilt or inclination.

The influence of the spring tension (elasticity) of the suspension will be expressed in the fact that the tank begins to tilt, thus moving the center of gravity lower as soon as the foremost road wheel goes over the edge. Consequently, as soon as the center of gravity begins to sink, the tank hull tilts somewhat and the center of gravity is lowered further.

Consequently, any arbitrary elastic support impairs the precondition for crossing a trench, if they are compared to a rigid suspension.

b) Vertical Wall

If the question is to be answered how a tank surmounts a vertical wall, main attention must be devoted to the method of describing it while maintaining the purpose, however, that the conclusions reached approach reality and form a basis for knowing the tank design so as to be able to judge its ability to surmount the wall.

The methods of solving this problem will be different according to whether the tank is regarded as being a rigid body rather than equipped with springs.

In the case of rigid tank suspension, climbing a vertical wall becomes a deflection of the rigid body at right angles.

If the tank has a track and suspension with springs, then a solution of the problem is more difficult.

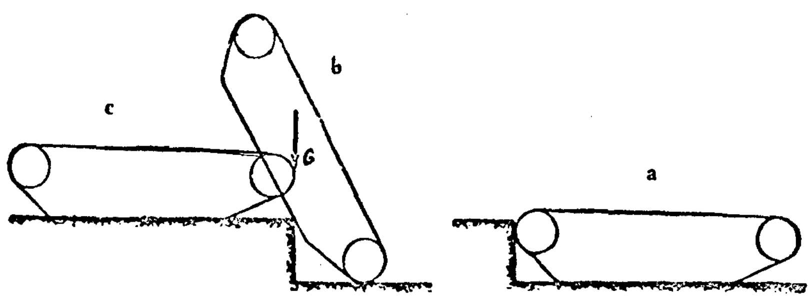


Figure 15. Climbing a Vertical Wall

We will classify the process of climbing a vertical wall into two parts:

1. From the moment that the idler wheel comes in contact with the vertical surface of the wall (Fig. 15a) until the metacenter touches the wall (Fig. 15b).

2. From the moment that the vertical line from the center of gravity comes into contact with the surface of the wall, up to the instant in which the tank reaches the surface of the ground (Fig. 15c).

In order to illustrate more clearly the essential points of this phenomenon, we will consider the behavior of a rod moving toward a right-angled obstacle.

We are dealing with two wheels A and B, which rotate freely on axles connected by a rod. We will assume that the rod moves uniformly at right angles with the rod, whereby one side of the angle equals H and the other is arbitrarily large.

Now we wish to consider the manner of movement more closely.

The movement of the rod is made up of two types of movement; one continuous (toward the horizontal and vertical plane) and a rotational movement around the center of gravity. It can be replaced by a rotational movement around the momentary center point M. We find this center point at the point of intersection of the vertical lines to the speed vectors of wheel rim points A and B.

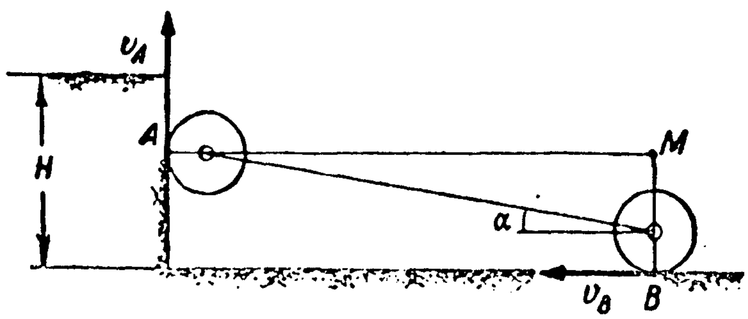


Figure 16. Movement of a Rod Toward a right-angled Obstacle.

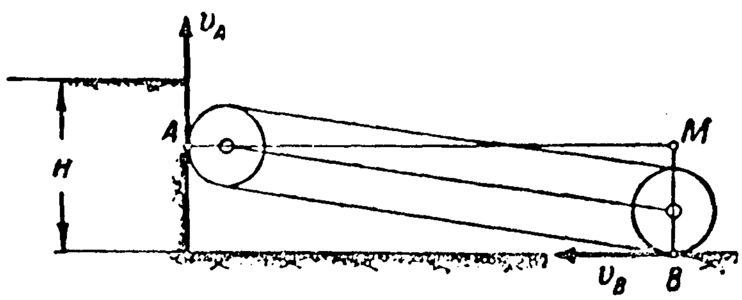


Figure 17. Movement of a Track Toward a right-angled Obstacle.

From the drawing we obtain the following equation:

$$v_A = \overline{MA\omega}, \quad v_B = \overline{MB\omega},$$

in which ω is the angular velocity.

v_A and v_B are the rotational speeds of the point on wheel rims A and B.

Then:

$$\frac{v_A}{v_B} = \frac{\overline{MA}}{\overline{MB}} = q.$$

Of course the value of q will change with a change of the angle of rotation. Consequently, $q = f(\alpha)$.

As we see, the rotational speeds (with the exception of MA = MB) are unequal; for this reason their rotational velocities of the moving rod at an angle will be unequal.

Now we wish to put a track (Fig. 17) on wheels A and B.

According to the conclusions already drawn, shift velocities v_A and v_B are not equal. The system is connected by a band (track), consequently the rotational velocities must be equal, therefore $v_A = v_B$.

Due to the slip movement in points A and B, the equality of the speeds no longer prevails.

In our case, this phenomenon is only possible at point A because, if this slip at point A does not take place and v_A is larger than v_B, the resistance force against slip of point B will push the track from the wall and preclude any forward motion.

The slip velocity at point A coincides with the direction of movement of this point.

We wish to study the forces on the tank during this movement in order to construct the first time interval.

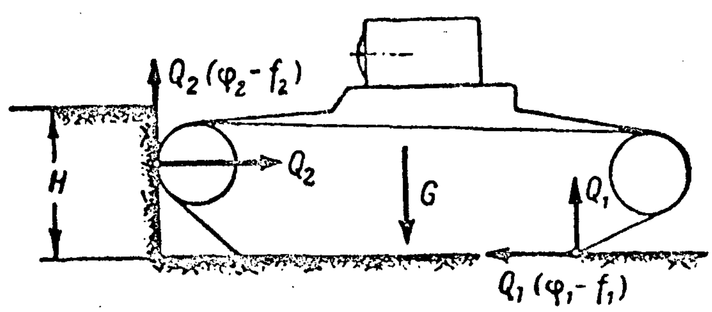


Figure 18. Forces being exerted when climbing a vertical wall at the beginning of the phenomenon.

The forces being exerted on the equipment are (Fig. 18):

Q_1 and Q_2 are the vertical reactions of the ground.
$Q_1(\varphi_1 - f_1)$ and $Q_2(\varphi_2 - f_2)$ are the tangential reactions of the earth.

In order to maintain the given speed of the equipment on the obstacle it is necessary that the moment of the forces M_1 opposite any arbitrary point in clockwise direction be greater than the moment in counterclockwise direction.

As is clear from Fig. 18, it is not difficult to maintain the named ratio of values between M_1 and M_2 if force $Q\,(\varphi_2 - f_2)$ is directed upward. This is impossible, however, with the presence of slip velocity which is also directed upward.

We will pose the question in the following form: "Under what conditions is a change of direction of force $Q_2\,(\varphi_2 - f_2)$ toward the opposite side in the realm of possibility?"

A precondition is that the point of contact of the track with the ground must have an upward directed velocity.

We now wish to show how this is achieved.

The speed of slip is just as great as the difference between the speeds (Fig. 17):

$$v_{sl.} = v_A - v_B = (AM - BM)\,\omega.$$

It is clear that the speed of the track, if it is drawn downward at point A with a speed of v_{gl} will equal zero at the point opposite the earth's surface so that, instead of the upward directed $Q_2\,(\varphi_2 - f_2)$ we obtain the value $Q_2\, f_2$.

If, however, the rotational speed of the track is larger than v then force $Q_2\,(\varphi_2 - f_2)$ will be directed upward (opposite track movement). This is what we wanted.

Consequently we achieve a change of direction of force $Q_2\,(\varphi_2 - f_2)$ by pulling the track downward at point A at a speed greater than v_{gl}.

The mentioned track movement is achieved by slip at a speed of $v_b > v_{gl}$.

The amount of slip is expressed by the following equation:

$$\zeta = \frac{r_{sl.}}{r_{sl.} + r_B} + \frac{r_A - v_B}{r_A} = 1 - \frac{v_B}{v_A} = 1 - \frac{\overline{BM}}{\overline{AM}} \tag{15}$$

If, for example, in the first moment $\dfrac{\overline{BM}}{AM}$ 0.1 then the required slip becomes $\zeta = 90\%$.

Consequently, slip is a precondition for an upward directed force $Q_2 (\varphi_2 - f_2)$. This slip will occur in equal measure as forward motion is reduced, i.e. in which the angle of rotation of the tank is greater.

In most cases the first time interval is not the boundary case when climbing a vertical obstacle, but the second time interval, when the line of the weight coincides with the vertical plane of the wall.

Before the vertical line from the center of gravity touches the wall, the tank forms an angle with the horizontal plane in which the engine may die out because it is overloaded or because slip occurs, probably for the last named reason.

We are assuming that the vertical line lies on the center of gravity in the vertical plane. We now write down the equation for the wall height H to be climbed as a function of the tank (Fig. 19).

$$H = L \sin\alpha + h_0 \cos\alpha + r - \frac{h}{\cos\alpha} \qquad (16)$$

As we see, $H = f(\alpha)$.

In order to determine the maximum value for H, the function must be checked according to the maximum and minimum value. The formula α is unsuitable for calculating and leads to a result which does not conform to reality because in most cases the value α_{max} determined according to the study is larger than the α_{max} of ground traction.

To calculate H_{max} it is sufficient to substitute the maximum surmountable angle of inclination of the tank into the equation $H = f(\alpha)$ because the tank movement in the moment at which it reaches the horizontal surface can equal the climbing movement.

Experience has shown us that the calculated H_{max}, apart from certain differences of ground traction during climb over the wall, comes very close to reality.

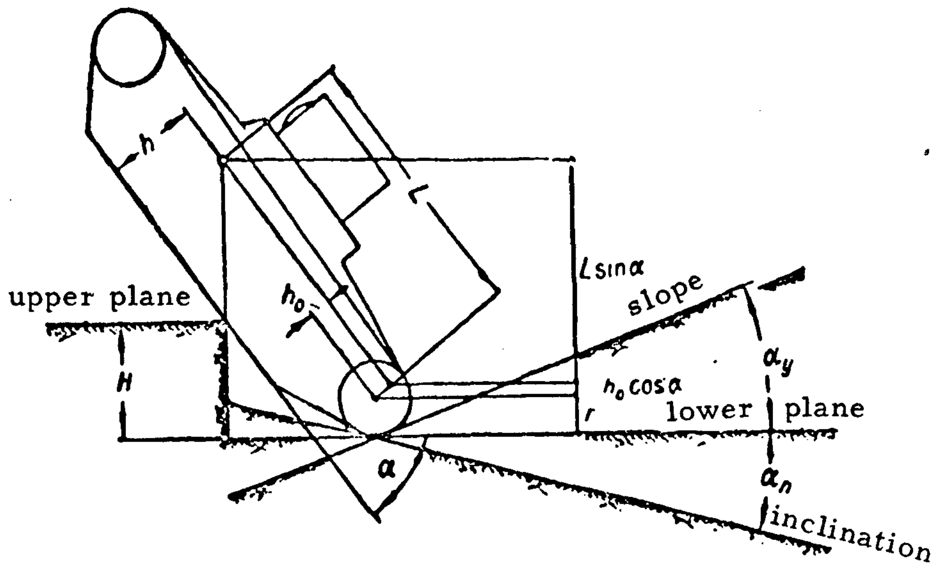


Figure 19. Conclusion of the first time period when climbing the vertical wall.

1. Upper surface of ground. 2. Slope. 3. Lower surface.
4. Inclination

In $H = f(\alpha)$ we notice the following:

1. The nearer the center of gravity lies to the bow and the deeper it lies, the higher is the slope to be surmounted (relative to stability).

2. In the overwhelming number of tanks α_{max} is larger than $\alpha_{1\,max}$ and the greater ground traction of the track, the larger is $\alpha_{1\,max}$ and the higher the wall to be climbed over.

3. The height of the wall to be climbed is in no way related to which wheel is in front; the idler wheel or the drive sprocket.

4. The height of the front wheel axle exerts only a slight influence on the height of the wall to be climbed.

c) Influence of Track and Suspension Design on Wall Height

We wish to consider two track and suspension systems more closely; the road wheel with individual springs and the swing arm suspension. The relationship between wall height and the type of suspension becomes especially clear in the first time interval.

We wish to consider the suspension with road wheels having individual springs (Fig. 20). 'Its influence on wall height is determined in first order by the height of two points, that is the axis of the steering wheel (we are assuming that the drive sprocket is in the rear).

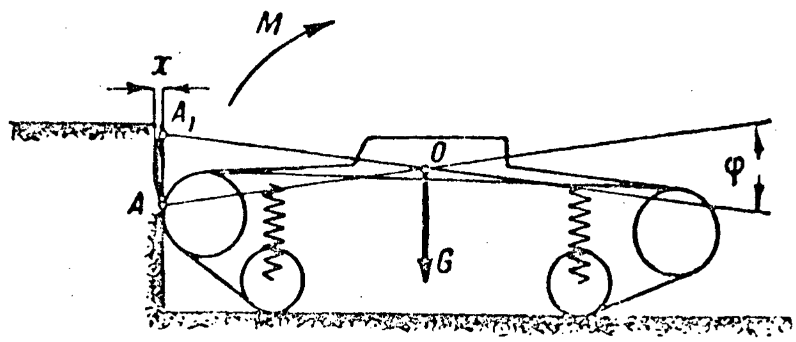


Figure 20. Tank with road wheels having individual springs, climbing over a vertical wall.

We wish to call attention to the following characteristics stipulated by the track and suspension:

a) We do not know the velocity direction of two points on the hull according to which we can determine the center point of the actual turn for the hull.

b) The rear support point (without considering the tank movement in the horizontal plane) cannot serve as center point of the turn as long as the springs are not fully compressed or a part of the weight is transmitted from the drive sprocket to the ground.

The center point of the turn coincides with the center point of the system which generally does not coincide with the center of gravity but lies to the left or to the right of it.

In order to determine the nature of the influences exerted on the track and suspension without using differential equations, we used the following method.

We are assuming that the center of gravity is located higher than the axis of the idler wheel.

Moment M is affecting the equipment. The tank begins to turn around the system center point and therefore describes an arc AA_1 if the wall can be compressed (since the error is only insignificant, we can assume that the system center point coincides with the center of gravity).

Doubtlessly the resistance against shift at point A will become stronger; in addition, however, it is necessary that the tank center of gravity not be shifted because otherwise the idler wheels dig into the ground making it impossible to climb the wall above the idler wheel axle.

For this reason the tank will slip up to 100% during its turn at angle φ, which corresponds to arc AA_1.

We are assuming that the wall will not give.

Under these circumstances, the center of gravity will be moved toward the rear while the tank hull is turning at angle φ, in spite of shifting into first gear, and will exhibit lag during travel.

The probable result of this will be that the normal reaction of the wall will drop to zero and that the tank will not be able to climb the wall above the height of the idler wheel axle without the aid of special equipment. The tank tractive effort during the first time segment will also be variable and at many points in time can be completely equalized by forces of inertia.

From this it is clear that it is in no way necessary that the idler wheels dig into the earth and a 100% slip occurring or that the tank rolls back during the turn of its hull around the angle φ.

We will observe a similar phenomenon in the case of the suspension arm support, if they are placed on the tank hull below the tank's center of gravity.

In the second time segment the use of road wheels with individual springs will work in such a way that distance h from the vehicle floor to the center of gravity is reduced at braking by the additional stroke distance of the drive sprocket if the tank center of gravity lies in the vertical plane.

If the tank is equipped with suspension arms, this reduction of distance will be effected by utilizing slack in the track and by compression of the springs.

The following can be said in reaching a conclusion on this information:

1. In the case of an elastic suspension it is very difficult to surmount the wall in the first time interval if the center of gravity lies higher than the idler wheel axle.
2. In comparison with rigid suspension, the height of the wall to be climbed is greatly increased if the tank has road wheels with individual springs, but will be increased by an insignificant amount if the tank is equipped with a suspension arm system.

d) <u>The Tank Climbing a Vertical Wall on Sloping or Inclining Terrain.</u>

Without considering the type of track and suspension, the system center point, the axis of the system frame, the outermost support point on upward sloping terrain, will lie below the point at which the idler wheel comes into contact with the vertical wall (Fig. 21); consequently the wall height to be surmounted will be higher in the first segment of time than on a horizontal surface, other preconditions being equal.

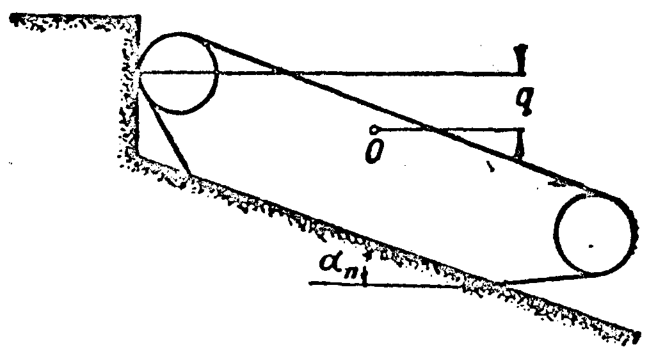


Figure 21. Climbing a Vertical Wall on Upward-sloping Terrain

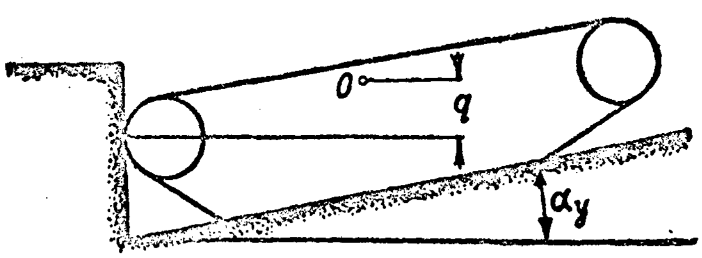


Figure 22. Climbing a Vertical Wall on Downward-sloping Terrain

In the second time segment the picture changes completely; in the same angle of inclination of the tank floor opposite the horizontal plane, the tank exhibits a lower support point in comparison with the first case. Consequently, the surmountable height of the wall will also be lower than if the wall were being climbed from a horizontal surface.

On slopes (Fig. 22) the turning center point lies higher than the contact point of the idler wheel with the vertical wall; consequently the height to be climbed in the first time segment will usually not be higher than the idler wheel axis.

In the second time segment the wall to be climbed from downward sloping terrain can be much higher than an approach from upward-sloping terrain, if the upward sloping and downward sloping angle is the same.

Consequently, we come to the following result:

1. From upward sloping terrain the height of the wall to be climbed will be less in the second time segment.
2. In the case of downward sloping terrain the wall height to be climbed will be limited by the first time segment (in first gear).
3. The wall height to be climbed by the tank on upward and downward sloping terrain is less than on a horizontal plane, all other preconditions being equal.

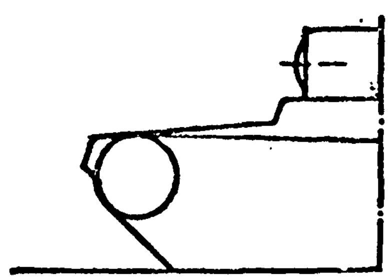


Figure 23. The Tank Juts in Front of the Idler Wheel

In conclusion we wish to state the following:

In order to be able to protect the drive sprockets and the driven wheels against fracture when breaking through a building wall or a walled obstacle, it seems advisable to fasten the wheels on the tank hull in such a way that they will not jut beyond the armor plating (Fig. 23).

This will involve one or two methods of installation:

1. When traveling at high speed the tank hull will completely destroy the wall or its upper edge and surmount the obstacle in this way. This explains the preference of mounting the wheels on the front as suggested above.

2. If the tank is traveling in first gear, the tangents of the wall reaction will be directed downward and the tank will not be able to climb the wall if its hull lies on the obstacle.

e) Knocking Over Trees

On combat missions or in the performance of its duties the tank may have to pass through a forest (or will encounter an artificial obstacle); in this case the power output of the tank is most important.

We wish to take up the task of knocking down a tree, without considering the weight of the tank.

In this case, force T is exerted on the tree at height H (see Fig. 24).

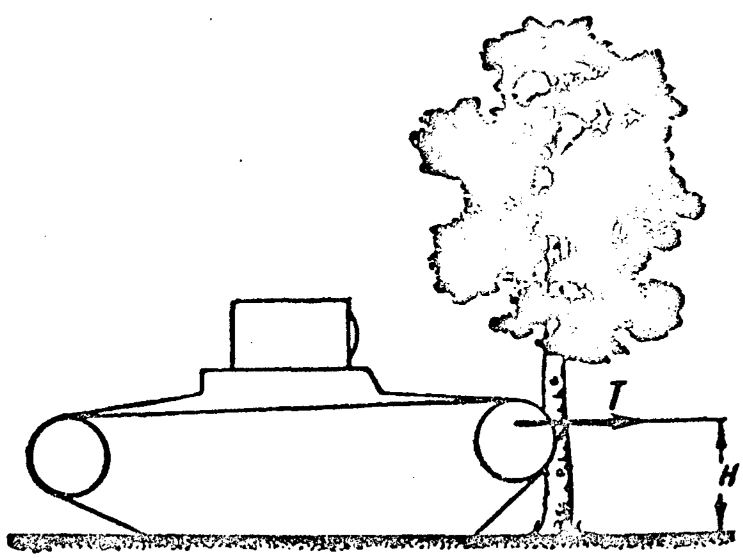


Figure 24. Knocking Over a Tree at Low Speed

When this force is exerted, we can count on two results:

a) The tree will be knocked over by its roots being loosened;
b) The tree will be broken off by bending moment TH in the cross section at which the size ratio TH/W reaches its maximum value.

H is the height between the point at which force T is exerted and the breaking point.

W is the resistance moment of the cross section.

It would be pointless to give further study to the first phenomenon (tearing out by the roots) since this has no direct relationship to the diameter of the tree, but is stipulated only by the nature of the ground, the season and the tree type.

In regard to the second case, it is very important to learn to what extent the possibility exists for the tank to raise the height (the "lever arm") of the bending moment, or the point at which its force is applied to the tree.

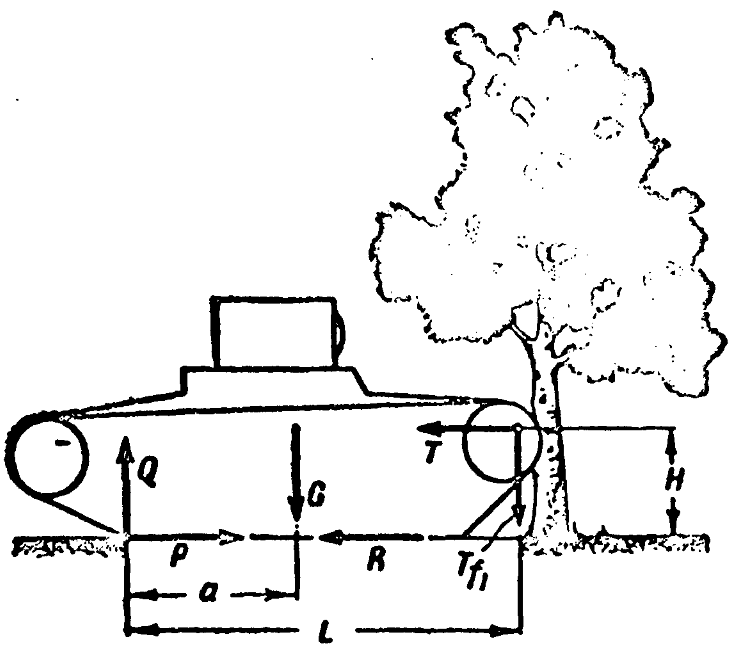


Figure 25. Forces being exerted on a tank when knocking over a tree.

If we compile the forces which are exerted on the tank in this case and then set up the appropriate equations (see Fig. 25) we obtain:

$$\sum M = T\,H - Ga - T_{f_1}\,L = 0;$$
$$\sum Z = P - R - T = 0$$

or assuming that the tractive effort of the engine equals that of ground traction

$$T = Q(\varphi - f);$$
$$\sum Z = Q - G - Tf_1 = 0.$$

f_1 is the friction coefficient of the longitudinal slip on the hull against the tree and f is the coefficient of resistance of the tree against movement.

If we solve all equations with the common purpose of eliminating T and Q, we reach the result that the tank will begin to raise the contact point of the hull with the tree

$$\varphi - f = \frac{a}{H - f_1 L + a f_1}$$

Now we wish to consider to what extent this is possible.

We assume that

$$H = 0.7\text{ m}; \; L = 2; \; a = 3\text{ m and } f_1 = 0.1$$

and thus

$$\varphi - f = \frac{a}{0.7 - f_1 (L \quad a)} \approx 0.7$$

But this in no way corresponds to the true value of the named coefficients q and f and thus it is utterly impossible for the tank to slip upward on the tree if we exclude the case that the tree trunk is bent.

In this manner the moment being imposed on the tree is

$$M - T\,H_k$$

in which H represents the height of the wheel above the ground.

With this method the tree falls forward in the tank's direction of travel.

f) Knocking over Trees at High Speed using the Weight of the Moving Tank.

Without becoming involved in the calculation of the phenomenon described, the calculation being so very involved, we will only attempt a description to reach the necessary conclusion.

We will deal with two possibilities:

1. Knocking over trees with a low crest;
2. Knocking over trees with a high crest.

We will assume that a tank traveling at speed v comes into contact with its hull against a tree trunk with a low crest.

If the tree breaks off at the point of contact, the trunk will certainly fall backward onto the tank roof (Fig. 26).

This phenomenon is controlled by the fact that the inert resistance prevents the tree from falling forward since the crest remains where it is while the point of contact moves forward together with the tank, causing the tree to fall backward.

In this case we are dealing with a double-braced beam and a shock load between the support points.

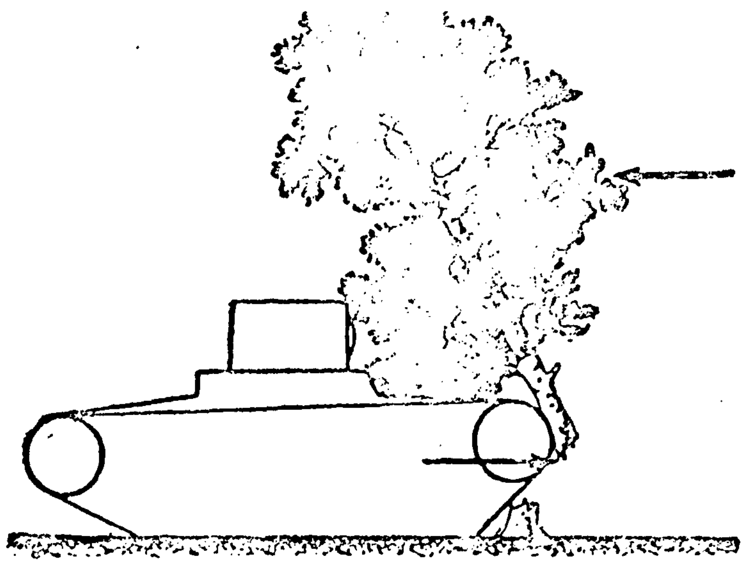


Figure 26.

When the tank collides with the tree trunk at speed v, the crest being high (Fig. 27), a somewhat different phenomenon will occur: here again we are dealing with a beam on two supports with dynamic load between the support points; but in this case the second support point (air resistance) is thrust upward in comparison with the first case.

Because of this fact the maximum value of $M - h/W$ canalso occur in the other cross sections because in relation to W, the tree height is smaller. This occurs although the maximum bending moment is appliedat the point of contact.

The final result is that the tree breaks at two points; the top falls backward while a piece of the trunk falls onto the front of the tank.

In both cases the tree top falls backward and breaks the radio antenna.

In conclusion we wish to refer to the fact that light tanks at full speed can break down trees with a diameter of 30-35 cm without having to fear damage to the hull or other parts.

In the case of medium tanks we can calculate with diameters of 50-55 cm.

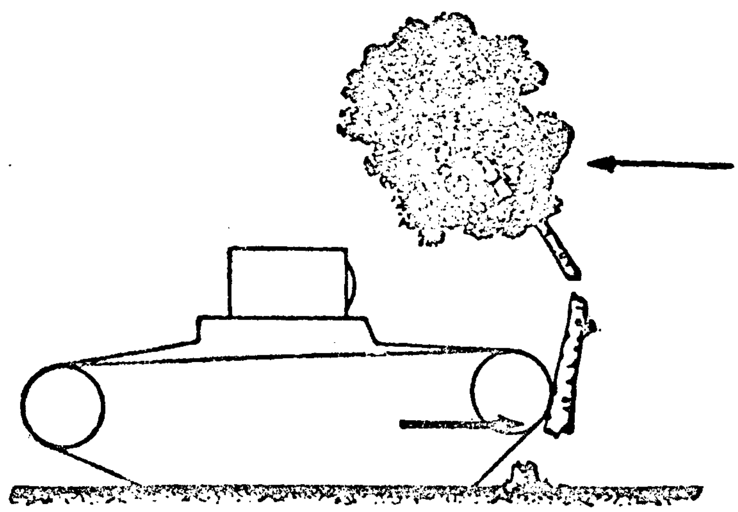


Figure 27.

Section III.

THEORY OF THE TURNING MANEUVERS OF TANKS

1. Turning of Tanks

By and large the turning of tanks can be reckoned with surface movements.

A line will always be present which represents the momentary axis of turn. The projection of this axis on the plane of motion is called the tank's center point of turn.

According to the type of turn, the center point of turn will either shift and form a crooked line (curve) or will remain at one point if the turn is performed uniformly.

The movement of each track individually can be regarded in accordance with resistances which appear during movement; these can be classified into two types:

a) Forward movement together with the pole at which the speed corresponds with the direction of the track at any point in time, i.e. in which it forms the speed of the turn, and
b) the turning of the tracks in contrast to the mentioned pole.

During forward movement, resistance will be expressed in the compression of the earth's surface, i.e. the deformation in the vertical direction and is determined according to the coefficient of resistance f against moving in a straight line.

Resistance against the turn will be expressed in the slip of the track on the earth's surface, tearing up and casting out of the earth's surface in the transverse direction.

The direction of the forces in this case can be assumed to be vertical to the longitudinal axis of the track.

Accelerations of track movements during the turn, i.e. the changes of its speed is characterized by the type of reaction on the steering gear.

The speeds will be non-uniform: the track which develops the maximum speed v_2 will be designated as outside one, the one which develops the less speed will be designated as the inside one.

The poles of the track rotational movement, whose absolute speeds correspond with the turning speeds of the tracks, can be designated as the center turning points of the tracks (Fig. 1, points A and B).

The center turning point of the tracks will not always be located on the transverse axis of the tank. It will shift during the turn from the transverse axis to one side or the other. Its grouping in each individual case will depend on the movement of the tank performing the turn.

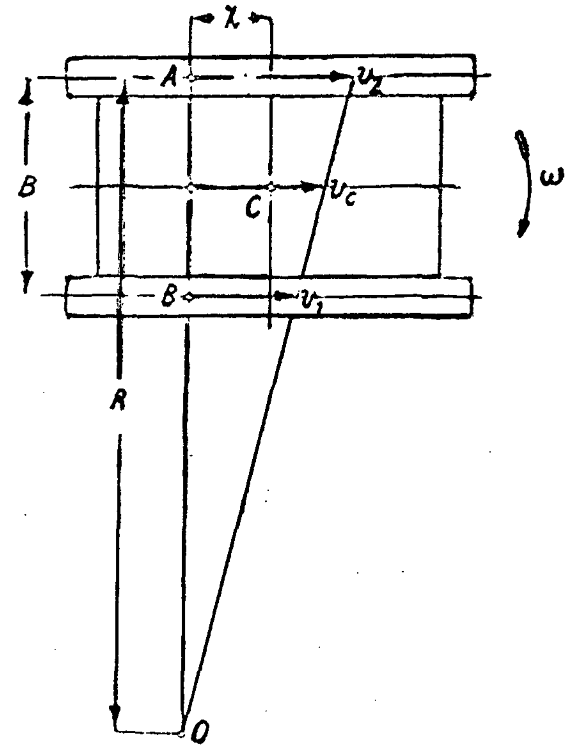


Figure 1. Position of the Center Point of Turn on the Tank

The position of the momentary center point of the turn on the one hand will depend on the position of the center point of track rotation, and on the other hand on the speeds v_2 and v_1 of the outside and inside track.

Fig. 1 shows the position of the tank center point of turn at the intersection of the line drawn through the center point of track turn and the line which passes through the ends of the speed arrows of the outer and inner track.

The distance between the tank center point of turn and the axis of the outer track is designated as the radius of the turn R.

The angular velocity ω of the turn in the case of the tank and both tracks will by and large be equal.

If we know the speeds of the outer track v_2 or the speed of the inner track v_1 and the radius of turn, then the angular velocity of the turn ω can be determined:

$$\omega = \frac{v_2}{R} = \frac{v_1}{R - B} \qquad (1)$$

If the speeds v_2 and v_1 are known, the radius of turn R and the angular velocity ω can be determined according to formula (1).

We will designate the speed on the intersecting point of the radius of turn and the tank longitudinal axis as mean velocity and introduce the expression v_c for it. In the special case, this speed can also be the center of gravity v_o.

On the basis of the formula

$$\omega = \frac{v_c}{R - \frac{B}{2}}$$

as well as formula (1), we obtain the formula for the relations between the speeds

$$v_1 = v_2 \frac{R - B}{R} \qquad (2)$$

$$v_1 = v_c \frac{2(R - B)}{2R - B} \qquad (3)$$

$$v_2 = v_c \frac{2R}{2R - B} \qquad (4)$$

$$v_c = \frac{v_1 + v_2}{2} \qquad (5)$$

A tank with steering coupling begins to turn when the steering clutch is disengaged, i.e. when the track is freed from the transmission of forces and the brake belonging to this steering clutch is applied.

A turn will result in that direction in which the steering clutch is disengaged and the track braked.

If the engine rotational speed remains the same during the turn as before the turn, then in the case of steering clutches the speed of the outer track v_2 will remain the same as when the tank was traveling straight ahead.

On the other hand, the speed of the inner track will be reduced in relation to braking power. Also, the speed of movement of the center of gravity will be reduced.

During the turn, the speeds v_2 and v_1 as well as the radius of turn R will have a constant value.

A tank with a simple rotating steering gear can be turned by braking one of the two rear axle shafts. A turn will result in that direction in which the rear axle shaft was braked.

If, when turning a tank with a simple rotating steering gear, engine rotational velocity remains the same as before the turn, then it can be said on the basis of the equation for planetary gears $2n = n_1 + n_2$ and according to formula (5) that an average speed of the tank remains unchanged, so that the total of its speeds remains a constant value the entire time.

This means that the increased speed of the outer track will be as great as the reduction of speed of the inner track.

If conditions make a turn more difficult, the tank will turn by utilizing its mass. The engine rotational velocity as well as average speed will be reduced to a degree dependent on the operating of the engine.

The reduction of average speed in this case will occur simultaneously with the speed reduction of the outer and inner track whereby the relation between these speeds, which can be determined according to formula (5) will remain constant.

During movement the turning point of the tank will not change position. The radius of turn R, the speeds of the outer track v_2 and the inner track v_1 will likewise remain constant.

2. Exterior Forces and Moments Exerted on the Tank During a Turn

a) The Tank Turns on Flat Terrain

The determination of forces required for turning is rather involved when we consider all forces being exerted during the turn.

In order to study the essential points of these phenomena, insofar as this is possible and in order to be able more easily to determine the forces required for turning, we will first take up the simplest case of a turn on a level section of road at a uniform angular velocity, i.e. with a constant radius of turn.

We will proceed on the following assumptions:

a) We will take as a basis the centrifugal force which passes through the tank around its axis during the turn, and not through its center of inertia, and which equals zero;

b) We will assume that the tank center of gravity intersects the axes of symmetry of the tracks' support surfaces;

c) Finally, we will assume that the vertical pressure is distributed uniformly on the tracks' support surfaces.

Later on we will take up the influence of centrifugal force in more detail. For the present we will only state that influence of centrifugal force can be neglected if the traveling speed of the tank is low and if the tank performs the turn with a large radius.

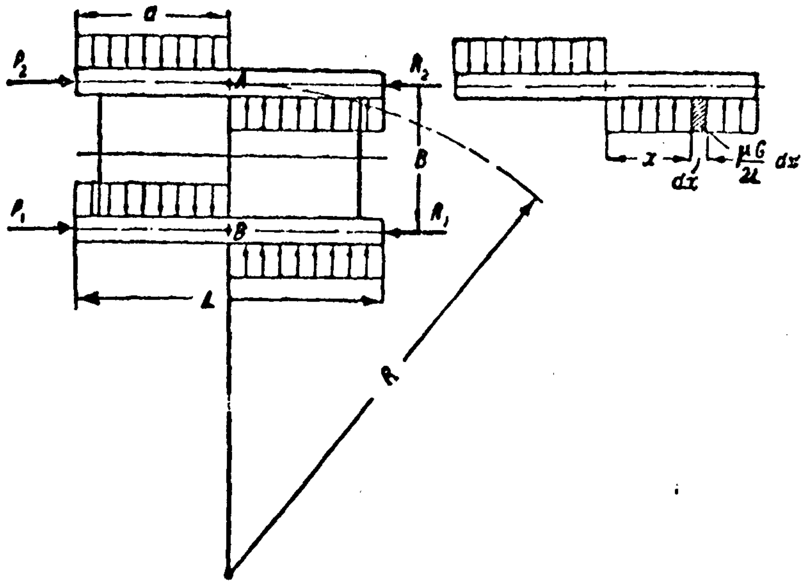


Figure 2. Forces being Exerted on the Tank During Turn

Figure 2 schematically depicts the turning tank and the forces being exerted on it.

P_2 is the tractive effort of the outer track; this tractive effort is directed in the direction of forward movement.

P_1 is the tractive effort of the inner track in the same direction.

R_2 and R_1 of the tracks' resistance to forward movement. In the case of the above mentioned presumptions

$$R_2 = R_1 = \frac{fG}{2},$$

in which f is the coefficient of resistance to forward movement.

The rectangles on Fig. 2 depict the forces of resistance of the ground against the turning of the tracks. In other words this is the reaction of the ground on the tracks, i.e. the reactions caused by the friction of the track on the ground and the throwing out of earth through the edges of the track links and grips.

These forces work against the effort of the tracks to turn.

The resistance forces against turn are determined solely by the weight of the tank, other preconditions being equal. Consequently, they can be expressed by the weight and the coefficient of turn resistance (μ).

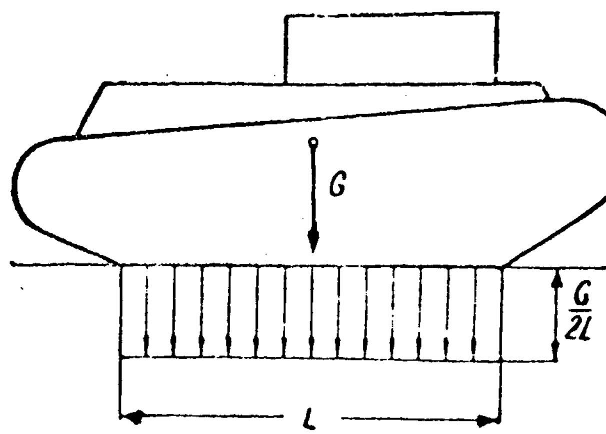


Figure 3. Uniform Track Load

In the aforementioned assumptions, one-half of the tank weight G/2 is imposed on each track. In the case of the load on the support surfaces of the tracks (Fig. 3) which we assumed to be uniform, the specific pressure on the longitudinal unit of the supporting part of the track would be G/2L. Consequently, the specific force of resistance of μG/2L would be exerted on the longitudinal unit of the track.

We now wish to write down the equations of force equilibrium and use the equations of the moments in regard to the turning center point of the outer track A (Fig. 2) and in regard to the turning center point of the inner track B:

$$P_2 + P_1 - R_2 - R_1 = 0. \quad \text{(a)}$$

$$\frac{\mu G}{L} \cdot (L - a) - \frac{\mu G}{L} a = 0 \quad \text{(b)}$$

$$(R_1 - P_1)B = 2\int_0^{L-a} \frac{\mu G}{2L} x\,dx + 2\int_0^{a} \frac{\mu G}{2L} x\,dx =$$

$$= \frac{\mu G}{2L}(L-a)^2 + \frac{\mu G}{2L} a^2 \qquad (c)$$

and correspondingly

$$(P_2 - R_2)B = \frac{\mu G}{2L}(L-a)^2 + \frac{\mu G}{2L} a^2 \qquad (d)$$

The right side of the equations (c) and (d) which had been determined in regard to the resistance forces against the turning of the track is designated as the moment of resistance M_c against the turning of the tank.

From equation (b) we obtain

$$a = \frac{L}{2}.$$

This means that in this case the track center point of turning is located in the center of the track support surface length.

If we substitute the determined value a in the right side of equations (a) and (d), we obtain a new formula for the moment of resistance against turning

$$M_c = \frac{\mu G L}{4} \qquad (6)$$

The equations of moments (c) and (d) can be written in the following form:

$$(R_1 - P_1)B = \frac{\mu G L}{4}$$

$$(P_2 - R_2)B = \frac{\mu G L}{4}.$$

From these equations we will obtain final values of tractive efforts P_2 and P_1 of the outer and inner track by substituting $f G/2$ for R_1 and R_2:

$$P_2 = \frac{fG}{2} + \frac{\mu G L}{4B} \qquad (7)$$

and

$$P_1 = \frac{fG}{2} - \frac{\mu G L}{4B}. \qquad (7a)$$

After giving careful consideration to formulas (7) and (7a) we can reach the following conclusions:

1. The tractive efforts P_2 and P_1 necessary for turning are dependent on the resistances against forward movement of the tank as well as the moment of resistance against turning of the tank and on track width h. (In this case "track" means the indentation left by the track in the ground -(Tr.))

2. In the case of the relationships between tank track length and track width (the track on the tank is meant here -(Tr.)) (4.3 and 4.6) and in the case of the large resistance coefficient against turn μ in comparison to f (Table 1), the second member of the right side of the equation is much larger than the first.

Consequently, the tractive effort of the tracks P_2 and P_1 are actually stipulated by the resistance force against turn.

Table 1. Resistance Coefficients against Turn

Soft Ground	0.7
Hard Ground	0.4
Snow Cover	0.45

3. The tractive effort of the outer track P_2 is a positive value; consequently it must be effected by engine power.

4. Since the second member on the right side of the formula (7a) is larger than the first, the tractive effort of the inner track has a negative effect. This means that the direction has been assumed incorrectly.

Under our assumptions it is really directed against the turning of the track and consequently can be regarded as a braking force.

5. The tractive effort P_2 which must be generated by the engine is greater than the tractive effort required for the tank to travel straight ahead so that for this reason more engine power is required for a turn than for straight ahead travel.

6. An improvement of tank maneuverability can be achieved by a reduction of the required tractive effort P_2 and braking force P_1 which essentially involves reducing moment M_c as we see in formula (6); at the same weight G, this is stipulated by the coefficient of travel resistance μ and of track length L.

Strictly speaking, the usually rectangular course of pressure (Fig. 3) to be determined from M_c does not completely correspond to reality. Actually the line of load is usually represented by a surface which is terminated below by a wavy curve as shown in Fig. 4 where the pressure peaks are located under the road wheels.

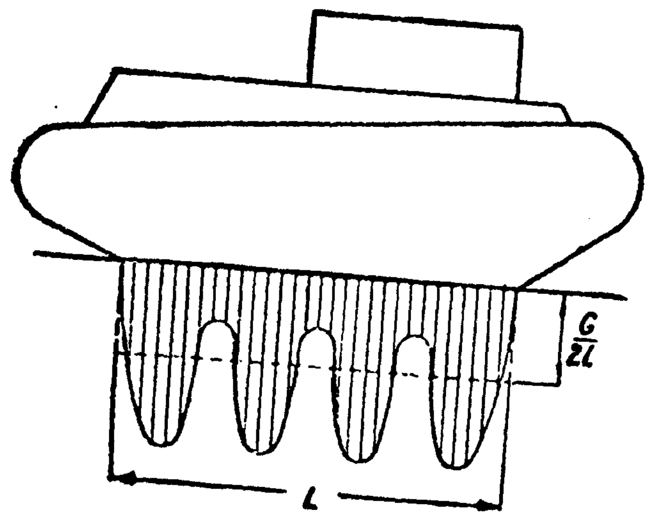


Figure 4.

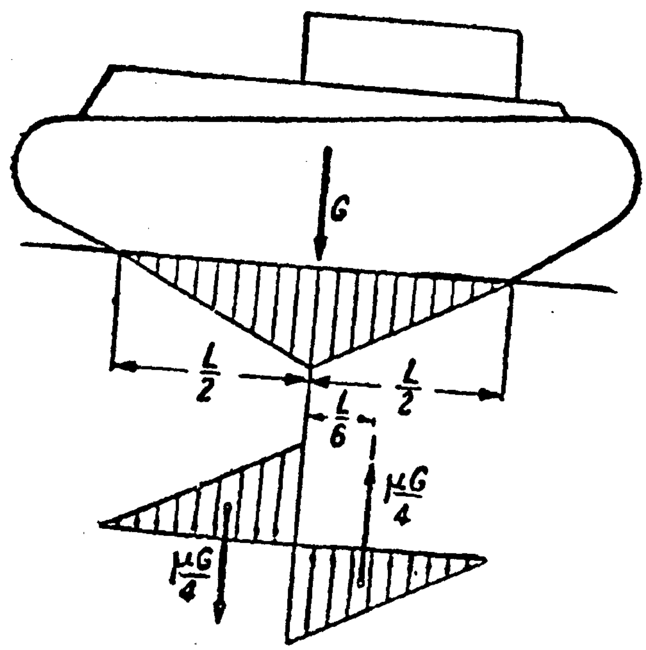


Figure 5.

The pressure curve which we selected by derivation from the formula, as given in Fig. 4 by a dotted line, essentially represents the average pressure.

It is quite understandable that it makes some difference which curve is selected to determine the resistance moment against turn.

But we did not commit a large error because the resistance coefficient against turn μ is likewise determined from the formulas which we derived. Consequently it contains the improvement which is necessary in order that the moments of resistance be equal to each other.

In order to reduce the moment of resistance, we sometimes must resort to the measure of artificially distributing pressure in a non-uniform manner: the pressure on the ends of the track support surface is reduced and is increased in the center of the track.

In order to clarify the value of this measure, we wish to consider how the value of the moment of resistance changes independent of moment M according to formula (6) when mean pressure is represented in the form of an equal-sided triangle (Fig. 7) and all other above-mentioned preconditions remain equal.

The result of each triangle of specific resistances on the track then is μ G/4 while a fourth of the weight is imposed on half of the track. The point at which this resultant force is applied lies at a distance of L/6 from the base of the triangle.

Thus, the following expression contains the moment of resistance against turn with the load triangle:

$$M_c = 2 \cdot 2 \frac{\mu G}{4} \cdot \frac{L}{6} = \frac{\mu G L}{6}. \qquad (8)$$

If we compare the moment of resistance of this formula with the moment of resistance which we obtained with the pressure rectangle, we find that it is one and one-half times smaller than the last mentioned one.

Since it is very difficult to set up the pressure in the form of a triangle, and because specific pressure will sharply increase in the center of the track, which causes a reduction of tank maneuverability, in reality such a load is imposed on the track and suspension for the purpose of reducing M, that we obtain the result shown in Fig. 6.

In this case, the numerical value of the coefficient will lie in the numerator of the resistance moment formula between 4 and 6.

If we assume that the coefficient has the value 5, then we obtain the following equations for the moment of resistance M_c and the tractive efforts P_2 and P_1:

$$M_c = \frac{\mu G L}{5} \tag{8a}$$

$$P_2 = \frac{fG}{2} + \frac{\mu G L}{5B} \tag{9}$$

$$P_1 = \frac{fG}{2} - \frac{\mu G L}{5B} \tag{9a}$$

As we see, tank mobility improves in the formation of trapezoids, and for this reason it seems very desirable to us to use this kind of pressure distribution.

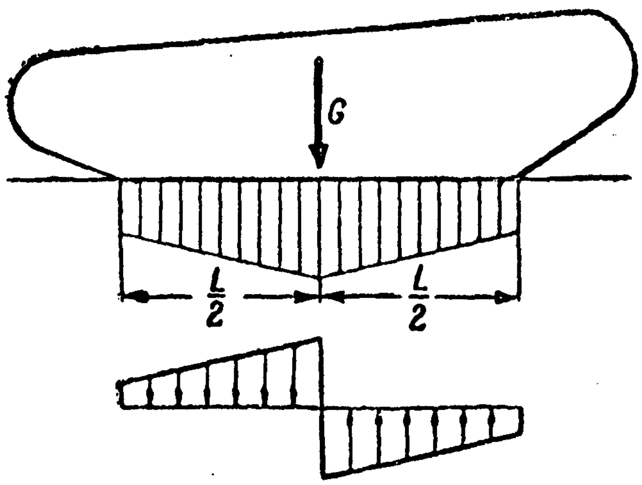


Figure 6. Sketch of Mean Pressure in the Form of Two Trapezoids

We wish to give the proper direction to the tractive effort of the inner track P_1 (Fig. 7). Then after performing the same transformations as we described in the case of formula (7a), we will arrive at the following equation:

$$P_1 = \frac{\mu G L}{4B} - \frac{fG}{2} \tag{10}$$

Later on we will also utilize this expression for P_1.

If we write the moment equation with regard to the symmetrical center point

$$(P_2 + P_1)\frac{B}{2} = M_c$$

or, since

$$M_c = \frac{\mu G L}{4}$$

then

$$(P_2 + P_1)\frac{B}{2} = \frac{\mu G L}{4} \tag{11}$$

The expression in the left part of the formula (11) (P_2+P_1) $B/2$ will be designated as the moment of turn (yaw).

Therefore, such an expression for the relationship between the forces being exerted is convenient because it immediately shows the relationship between the tractive effort of the outer track and the inner track on the one hand and the moment of resistance against turn on the other hand. In addition, we can see from Eq. (11) with consideration of the following relation

$$P_2 - P_1 - R_2 - R_1 = 0$$

that P_2 and P_1 must be constant values with a uniform turn.

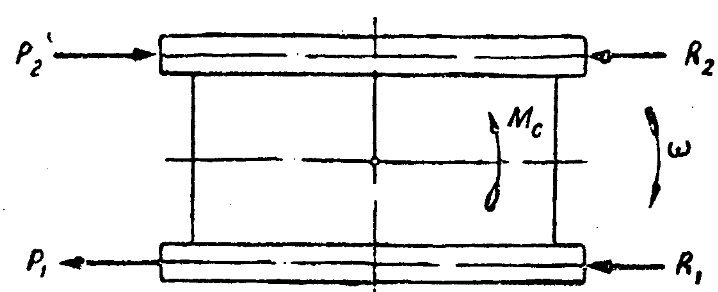


Figure 7. Actual Direction of the Effects of Force P_1 During the Turn of a Tank

If one of these forces is increased, we increase the turning moment M_c (yawing moment) as opposed to the moment of resistance M_c. Consequently, an angular acceleration is imposed on the tank; it will turn in a non-uniform manner and changes will occur in the turning radius.

b) Influence of Centrifugal Force on the Preconditions for a Uniform Tank Turn on Level Ground

When the tank turns at high speed and with a small radius, centrifugal force exerts considerable influence on turning force.

We wish to take up in more detail the effect of centrifugal force on the tank when the tank performs a uniform turn with a definite radius. As in the previous case we will assume that the projection of the center of gravity intersects with the symmetrical axis of the tracks' support surface.

Figure 8 depicts those forces being exerted on the tank; centrifugal force C_o is exerted at the center of gravity and proceeds outward from the center point of turn. The coefficients of centrifugal force -- centrifugal force C and the force of inertia C' -- can be determined from the following equations:

$$\left.\begin{aligned} C &= \frac{m v_c^2}{3.6^2\left(R - \frac{B}{2}\right)} \\ C' &= \frac{NC}{R - \frac{B}{2}} \end{aligned}\right| \qquad (12)$$

m	=	the mass of the tank in kgs.^{2} m.
v_c	=	the speed of the center of gravity in km/h.
R	=	the turning radius of the outer track in m.
B	=	the track width of the tank.

It is clear from Fig. 8 that the vertical projections of specific resistances which lie to the right are larger than those on the left side and that the sum of their components must equal force C, if we wish to achieve an equilibrium of forces which act on the tank in transverse direction.

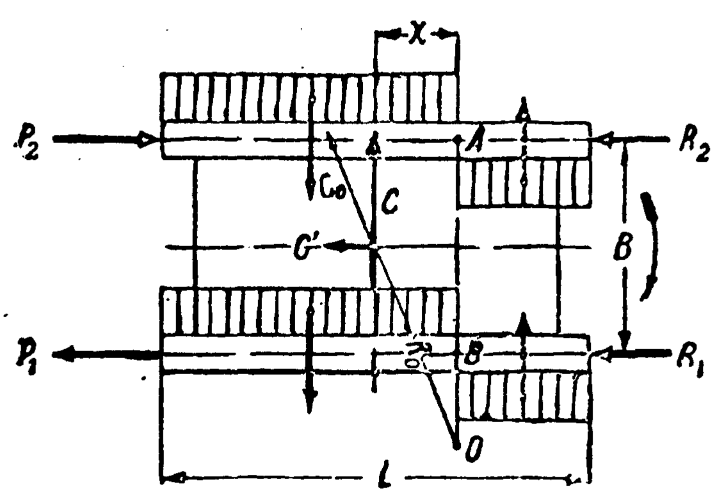


Figure 8. Actual Direction of the Effects of Force P_1 when the Tank turns.

The result is again that the track center point of turn lies grouped to the right of the transverse axis of symmetry at a certain distance X.

In addition to the shift of the center point of turn, centrifugal force will effect an additional regrouping of loads on the tracks: the total load G_2 of the outer track will be larger than the total load G_1 of the inner track. Accordingly, the specific loads will have the following values:

$$\frac{\mu G_2}{L} \text{ and } \frac{\mu G_1}{L};$$

In addition, the forces of resistance against forward movement will undergo change and will have the following values:

$$R_2 = f G_2 \text{ and}$$
$$R_1 = f G_1.$$

We wish to refer to the fact that the moment of resistance M_c found after integration in the right side of the equation is in no way related to the changed distribution of loads on the tracks.

We will set up an equation in regard to the equilibrium of forces and apply the equation with reference to the track center point of turn B:

$$P_2 - P_1 - R_2 - R_1 - C' = 0; \tag{a}$$

$$\left(\frac{\mu G_2}{L} + \frac{\mu G_1}{L}\right)\left(\frac{L}{2} + X\right) - \left(\frac{\mu G_2}{L} + \frac{\mu G_1}{L}\right)\left(\frac{L}{2} - X\right) - C = 0; \tag{b}$$

$$(P_2 - R_2) B + C X - \frac{C' B}{2} = \int_0^{\frac{L}{2}+X} \left(\frac{\mu G_2}{L} + \frac{\mu G_1}{L}\right) x\,dx +$$

$$+ \int_0^{X-\frac{L}{2}} \left(\frac{\mu G_2}{L} + \frac{\mu G_1}{L}\right) x\,dx \quad \frac{\mu G}{2L} \int_0^{\frac{L}{2}+x} x\,dx +$$

$$+ \frac{\mu C}{L} \int_0^{X-\frac{L}{2}} x\,dx = \frac{\mu C}{2L}\left[\left(\frac{L}{2} + X\right)^2 + \left(\frac{L}{2} - X\right)^2\right].$$

or after a transformation:

$$(P_2 - R_2) B + C X - \frac{C' B}{2} = \frac{\mu C L}{4}\left[1 + \left(\frac{2X}{L}\right)^2\right] = M_c \tag{c}$$

We will obtain the same expression for this moment if, with a simple derivation of the mean value of specific resistance, we substitute $\mu G/2L$.

By transforming equation (b) we will obtain the amount of shift in the center point of turning:

$$\frac{\mu G}{L}\left(\frac{L}{2}+X\right)-\frac{\mu G}{L}\left(\frac{L}{2}-X\right)-C=0$$

or

$$\frac{\mu G}{L}\left(\frac{L}{2}+X-\frac{L}{2}+X\right)-C=0.$$

Which results in

$$\frac{2X}{L}=\frac{C}{\mu G} \tag{13}$$

or

$$X=\frac{LC}{2\mu G} \tag{13a}$$

If we substitute (a) into (c), we obtain the formulas for the tractive efforts of the outer and inner track

$$P_2=fG_2+\frac{\mu GL}{4B}\left[1+\left(\frac{2X}{L}\right)^2\right]-\frac{CX}{B}+\frac{C'}{2} \tag{14}$$

and

$$P_1=-fG_1+\frac{\mu GL}{4B}\left[1+\left(\frac{2X}{L}\right)^2\right]-\frac{CX}{B}-\frac{C'}{2} \tag{14a}$$

Now we wish to determine the normal loads of the tracks G_2 and G_1 in order to substitute them into these equations.

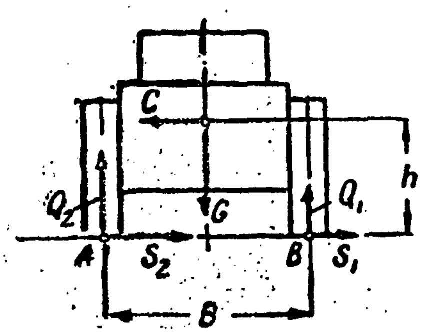


Figure 9. Effect of Centrifugal Force on the Tank When Turning

If we derive the equation for the equilibrium of forces and the equation for the moments with regard to point B from Fig. 9, we obtain the values for the normal reactions of the ground on the tracks Q_2 and Q_1 which are equal to each other in accordance with G_2 and G_1.

We have the following equations:

$$Q_1 + Q_2 = G$$
$$Q_2 B - Ch - G\frac{B}{2} = 0,$$

in which h is the height of the center of gravity.

When we solve these equations, we obtain

$$G_2 = Q_2 = \frac{G}{2} + \frac{Ch}{B} \qquad (15)$$

and

$$G_1 = Q_1 = \frac{G}{2} - \frac{Ch}{B} \qquad (15a)$$

To be able to judge the influence of centrifugal force on M_c, P_2 and P_1 at various speeds and radii of turn, reference must be made to Tables 2 and 3. The following values were taken as a basis in the compilation of the tables: $\mu = 0.5$; $f = 0.06$; L = 3m and B = 2m.

Table 2

Influence of Centrifugal Force on M_c, P_2 and P_1 at $v_2 = 10$ km/h

R (m)	$\frac{C}{G}$	$\frac{C'}{G}$	x (m)	X (m)	$1+\left(\frac{2X}{L}\right)^2$	$\frac{M_c}{G}$	$\frac{P_2}{G}$	$\frac{P_1}{G}$
~	0	0	0	0	1	0,375	0.217	0,157
15	0.048	0.0005	0.00015	0.144	1,0092	0.378	0.217	0,157
10	0,070	0,0016	0.00050	0,210	1,0198	0.382	0,216	0,156
6	0.109	0.0071	0.00230	0,324	1,0480	0,393	0,215	0,148
4	0,146	0,0209	0,00700	0.429	1,0870	0,409	0,218	0,137
2	0,196	0,1051	0,03500	0,537	1,1530	0,432	0,252	0,087

Table 3

Influence of Centrifugal Force on M_c, P_2 and P_1 at $v_2 = 20$ km/h.

R (m)	$\frac{C}{G}$	$\frac{C'}{G}$	x (m)	X (m)	$1+\left(\frac{2X}{L}\right)^2$	$\frac{M_c}{G}$	$\frac{P_2}{G}$	$\frac{P_1}{G}$
—	0	0	0	0	1	0.375	0,217	0,157
40	0.078	0,0005	0	0,234	1	0,375	0,210	0,150
30	0,102	0.0011	0	0,306	1	0,375	0,205	0,145
20	0.150	0,0036	0.0012	0.450	1.090	0,408	0,203	0,139
15	0.192	0.0078	0.0027	0,570	1,144	0,429	0,200	0,132
10	0.282	0,0263	0.0087	0.834	1,318	0,494	0,181	0,095
6	0.435	0,1092	0.0360	1.254	1,740	0,652	0,151	−0,019
4.8	0.518	0.2000	0.0660	1,470	2,050	0,768	0,145	−0,112

If we consider the tables more closely, we observe the following:

a) If the turn is executed at a low speed, the influence of centrifugal force is expressed by an increase of tractive effort P_2 and a decrease of tractive effort P_1.

This change is particularly noticeable at small turning radii.

b) During a turn at high speed, centrifugal force causes tractive efforts P_2 and P_1 to decline, whereby tractive effort P_1 ceases from a certain point in time to act as a braking force and again becomes tractive effort generated by the engine.

If we substitute values X from Eq. (13a) into Eqs. (14) and (14a), we obtain the following equations:

$$P_2 = fG_2 + \frac{\mu G L}{4B} - \frac{C^2 L}{2\mu G}\left(\frac{1}{2B} - \frac{1}{2R-B}\right) \tag{16}$$

$$P_1 = -fG_1 + \frac{\mu G L}{4B} - \frac{C^2 L}{2\mu G}\left(\frac{1}{2B} + \frac{1}{2R-B}\right) \tag{16a}$$

From these equations it is clear that P_1 is also reduced with a reduction of the radius of turn. On the other hand, P_2 declines only at certain values of R. In the case of radii approaching the value of B, P_2 begins to increase again.

c) Uniform Turn of the Tank when Traveling on Sloping Terrain

In order to obtain a clearer picture of the tank during turn on sloping terrain, we will break down the operation into time segments:

1. First we wish to consider those forces being exerted in the point in time when the center line of the tank forms the greatest angle with the horizontal plane (actual climbing);
2. Then, we wish to consider the case when the transverse axis forms the largest angle with the horizontal plane (traveling on an incline in a sloping position).

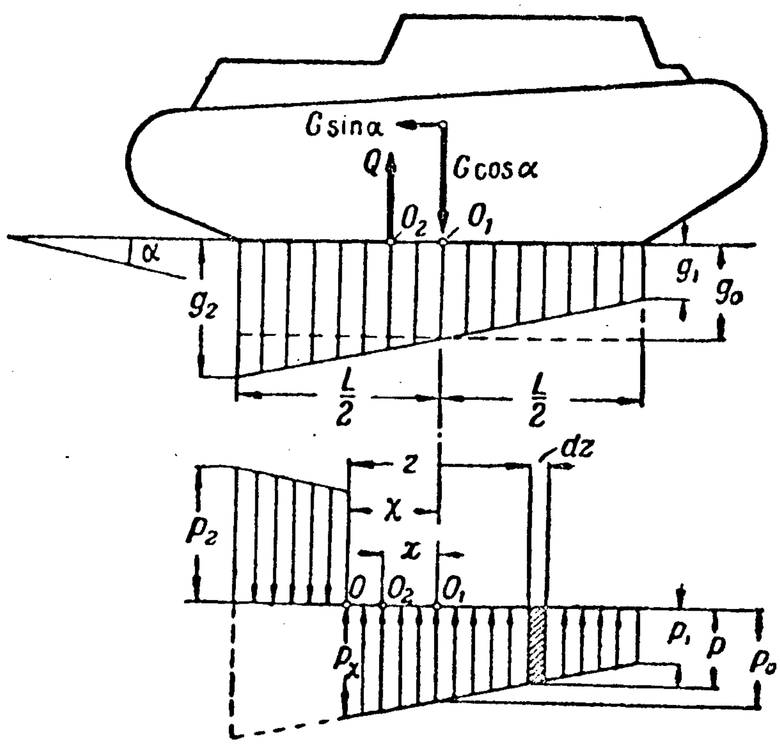


Figure 10. Illustration of the Specific Mean Pressures During a Turn on Sloping Terrain.

3. Finally, we wish to take up the preconditions in the intermediate position when the longitudinal and transverse axis are inclined to the horizontal plane.

In all cases, however, we will neglect the influence of centrifugal force, because we know that traveling speed on a sloping position usually is not very great.

When traveling on sloping terrain, as shown in Section II "Stability", the pressure center point shifts by the amount of

$$x = h \operatorname{tg} \alpha$$

to the rear, and the total pressure on the ground becomes G cos α.

As a result of the shift of the pressure center point, we obtain specific ground pressure in the form of trapezoids as shown in Fig. 10.

The surfaces of the trapezoids, which lie on both sides of the transverse axis through the center of the tank, do not correspond according to size.

The surfaces of the trapezoids which lie to the right and left of the track passing through the pressure center point 0_2 are likewise unequal.

Consequently, the center point of turn of the track will be located neither at point 0_1 or 0_2.

Now we wish to determine the shift of the center point of turn x according to the equilibrium of forces p (the resistance against turn), in which these forces are located to the right and left of the track turning center point 0_1.

We have the following equation:

$$\frac{1}{2}(p_1 + p_x)\left(\frac{L}{2} + X\right) = \frac{1}{2}(p_2 + p_x)\left(\frac{L}{2} - X\right) \qquad \text{(a)}$$

If we express p_x by p_1 and p_2 by p_0, we obtain the following equation:

$$\frac{p_x - p_0}{X} = \frac{p_2 - p_0}{\frac{L}{2}}.$$

From this we obtain:

$$p_x = \frac{2X p_2 + p_0(L - 2X)}{L} \qquad \text{(b)}$$

or in another form:

$$\frac{p_x - p_0}{X} = \frac{p_0 - p_1}{\frac{L}{2}}.$$

From which we obtain:

$$p_x = \frac{(2X + L)p_0 - 2X p_1}{L} \qquad \text{(c)}$$

If we now substitute the value of p_x according to formulas (b) and (c) into this formula (a) we obtain, after transformation, the following equation:

$$[(L - 2X)p_1 + (L + 2X)p_0](L + 2X) = \\ = [(L + 2X)p_2 + (L - 2x)p_0](L - 2X) \qquad \text{(d)}$$

If we replace the values p_1 and p_2 by p_o and substitute p_o into Eq. (e), we obtain

$$p_0 = \frac{p_1 + p_2}{2} \qquad \text{(e)}$$

On the other hand, we obtain the following equation according to the law of the equation of static moments of surfaces (see Fig. 11):

$$M_{trapezoid} = M_{rectangle} + M_{triangle}$$

or, expressed in a different manner:

$$\left(\frac{L}{2} - x\right) L\, p_0 = \frac{L}{2} L\, p_1 + \frac{L}{3} \cdot \frac{L}{2} (p_2 - p_1) \qquad \text{(f)}$$

If we solve Eqs. (e) and (f) together, we are able to determine the following equations for p_1 and p_2:

$$p_1 = p_0 \left(1 - \frac{6x}{L}\right) \qquad (17)$$

$$p_2 = p_0 \left(1 + \frac{6x}{L}\right) \qquad (17a)$$

After we have substituted the determined expressions for p_1 and p_2 into the formula (d) and have undertaken the necessary transformations, we obtain finally:

$$12\, x\, X^2 + 2\, L^2\, X - 3\, x\, L^2 = 0 \qquad (18)$$

or:

$$X^2 + \frac{L^2}{6x} X - \frac{1}{4} L^2 = 0 \qquad (18a)$$

According to this quadratic equation we can determine the extent of the shift of the turning center point x, if we substitute the value of the shift of the pressure center point x into the equation beforehand.

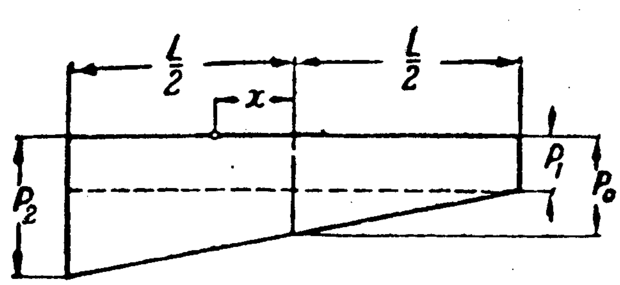


Figure 11. Arrangement of the Trapezoidal Surface into a Rectangle and a Triangle.

As we see from Eqs. (18) and (18a), the shift of the turning center point is conditioned by the shift of the pressure center point.

It should be mentioned here that these equations are only suitable if the pressure center point is shifted up to 1/6 L; that is to say it has such a position in which the trapezoid is converted into a triangle with a base line L. With a further shift, another equation will have to be considered for the determination of X, and other conclusions will need to be reached.

We now wish to study in which position the turning center point will be located in contrast to the pressure center point. For this purpose we wish to use Eq. (18) in the following form:

$$\frac{x}{X} = \frac{2}{3} \frac{1}{1 - \left(\frac{2X}{L}\right)^2} \tag{18b}$$

If we substitute the values X equals 1/6L and less into this equation, we will find that the turning center point within the framework of the trapezoid will be further removed from the various values of specific pressure than the pressure center point.

Now we wish to determine the moment M_c of resistance against turn. The original force of resistance at a distance of z from the turning center point (Fig. 10) will be

$$A p_t = p \, d z.$$

From this the moment of resistance will be expressed in the following form:

$$M_c = 2 \int_0^{\frac{L}{2}+x} p z \, dz + 2 \int_0^{x-\frac{L}{2}} p z \, dz \tag{a}$$

We now wish to express p by the specific average resistance force p_o

$$\frac{p_2 - p_1}{L} = \frac{p_2 - p}{\frac{L}{2} - X + z},$$

From which we obtain

$$L_p = \left(\frac{L}{2} - X + z\right) p_1 + \left(\frac{L}{2} + X - z\right) p_2.$$

If we substitute the expressions from formulas (17) and (17a) for p_1 and p_2, we obtain the following equation of p

$$p = p_0\left(\frac{1 + 12\, x X}{L^2} - \frac{12\, x}{L^2} z\right) \tag{b}$$

The specific resistance forces p_1, p_2, p and p_o stand in direct relation to the specific pressure values and are equal in accordance with μg_1, μg_2, μg and μg_o, but since

$$g_0 = \frac{g_1 + g_2}{2} = \frac{(g_1 + g_2)\, L}{2\, L} = \frac{G \cos\alpha}{L}$$

and consequently

$$p_0 = \frac{\mu G \cos\alpha}{2\, L} \tag{c}$$

If we substitute the expressions for p_o and p according to the formulas (b) and (c) into formula (a), we obtain the following equation:

$$M_c = 2\int_0^{\frac{L}{2}+x} \frac{\mu G \cos\alpha}{2\, L}\left(1 + \frac{12\, x X}{L^2} - \frac{12\, x}{L2} z\right) z\, dz +$$

$$+\, 2\int_0^{x-\frac{L}{2}} \frac{\mu G \cos\alpha}{2\, L}\left(1 + \frac{12\, x X}{L^2} - \frac{12\, x}{L^2} z\right) z\, dz\,. \tag{d}$$

After integrating and simplifying the equation, we will determine the moment of resistance in its final form:

$$M_c = \frac{\mu G L \cos\alpha}{4}\left\{\left[1 + \left(\frac{2\, X}{L}\right)^2\right]\left(1 + \frac{4\, x X}{L^2}\right) - \frac{16\, x X}{L^2}\right\} \quad \ldots \tag{19}$$

This equation is valid only for the case in which the shift of the pressure center point is not greater than 1/6 L, i.e. only until the trapezoid is changed into a pressure triangle.

If we assume that h -- the height of the center of gravity -- equals 1/3 L, we will determine which angle of gradient this corresponds to.

At a uniform speed, we have the following equation:

$$x = h \operatorname{tg} \alpha,$$

from which results:

$$\frac{1}{6}L = \frac{1}{3}L\,\mathrm{tg}\,\alpha$$

or

$$\mathrm{tg}\,\alpha = 0{,}5,$$
$$\alpha \approx 27^\circ.$$

Consequently, when determining the turn on slopes we can apply formula (19) if these are not greater than 27-30°.

Sometimes we obtain the described trapezoid of specific ground load in the plane as a result of the shift of the center of gravity.

In this case, the formula for the moment of resistance, as is clear from Fig. 12, will differ from formula (19) only by the fact that the multiplier cos α is missing. Then the formula becomes:

$$M_c = \frac{\mu G L}{4} K \tag{20}$$

(in which we wish to calculate the member in parentheses by K).

If the shift of the pressure center point and the shift of the turning center point is zero, i.e. if both center points lie on the axis of symmetry, K = 1 and formula (20) will have the following form:

$$M_c = \frac{\mu G L}{4}.$$

We achieved the same result for the rectangle of pressure (Formula 6).

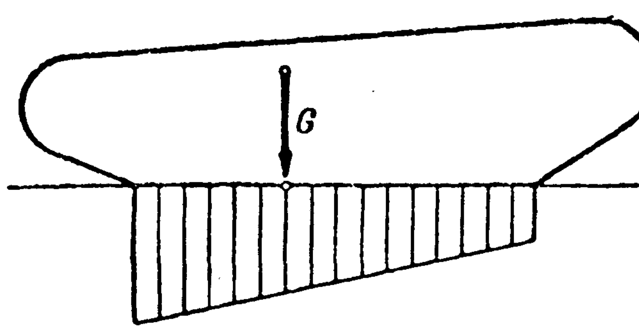


Figure 12. Trapezoidal Form of Specific Average Pressure when Turning on Level Ground.

In order to determine tractive efforts P_2 and P_1, during a turn on sloping terrain according to Fig. 13 we wish to select equation 13 for the equilibrium of the longitudinal forces and moments in regard to point B of the turning center point on the inner track:

$$P_2 - P_1 - R_2 - R_1\, G \sin\alpha = 0 \qquad \text{(a)}$$

$$(P_2 - R_2)\,B - G \sin\alpha \frac{B}{2} = \frac{\mu\, G\, L \cos\alpha}{4} K \qquad \text{(b)}$$

in which

$$R_1 = R_2 = \frac{f G \cos\alpha}{2}.$$

If we solve Eqs. (a) and (b), we will obtain the following equations:

$$P_2 = \frac{f G \cos\alpha}{2} + \frac{\mu\, G\, L \cos\alpha}{4B} K + \frac{G \sin\alpha}{2} \qquad (21)$$

$$P_1 = -\frac{f G \cos\alpha}{2} + \frac{\mu\, G\, L \cos\alpha}{4B} K - \frac{G \sin\alpha}{2} \qquad (21a)$$

In order now to determine the change of the moment of resistance in relation to the angle of inclination and to be able to judge the influence of the individual coefficients on this change, the moments of resistance at various angles of inclination are presented in Table 4.

This table has been compiled as follows: A certain shift of the pressure center point was assumed, an angle of inclination determined which corresponded to the assumed shift of the pressure center point and then the shift of the turning center point X was determined according to formula (18a).

The determined values were now substituted in formula (19); h was assumed to have a value of 1/3 L.

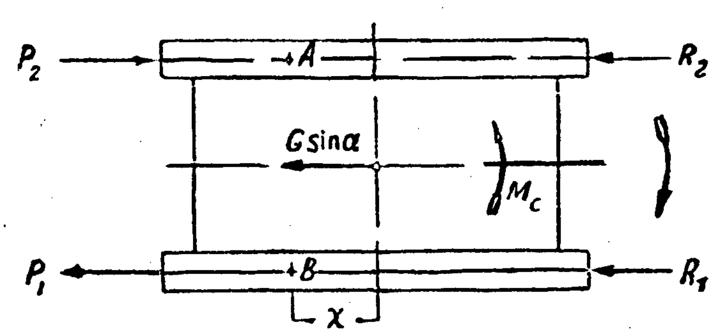


Figure 13. Longitudinally directed forces being exerted on the tank during a turn on inclined terrain.

From Table 4 and Fig. 14 it is clear that the moment of resistance against turn declines in relation to the reduction of the coefficient K and of cos α with an increase of the angles of inclination and logically also with the shift of the pressure center point.

Table 4
Moments of Resistance at Various Angles of Inclination
(first time segment)

$\frac{x}{L}$	α	$\frac{X}{L}$	$\cos\alpha$	K	M_c
0,04	7°	0,06	0,993	0,986	$0{,}978\,\mu\frac{GL}{4}$
0,08	13° 30′	0,11	0,972	0,944	$0{,}917\,\mu\frac{GL}{4}$
0,12	19° 45′	0,16	0,942	0,882	$0{,}831\,\mu\frac{GL}{4}$
0,16	25° 30′	0,20	0,903	0,760	$0{,}686\,\mu\frac{GL}{4}$

With consideration of the circumstance that the coefficient K is only related to the shift of the pressure center point and with the turning center point and for this reason was determined according to the type of the pressure sketch, we refer to the fact that the change of coefficient K in connection with x and X agrees with the data in the table even with a turn on level terrain. The moment of resistance against turn in this case will also decline with an increase of x and X related to the reduction of K.

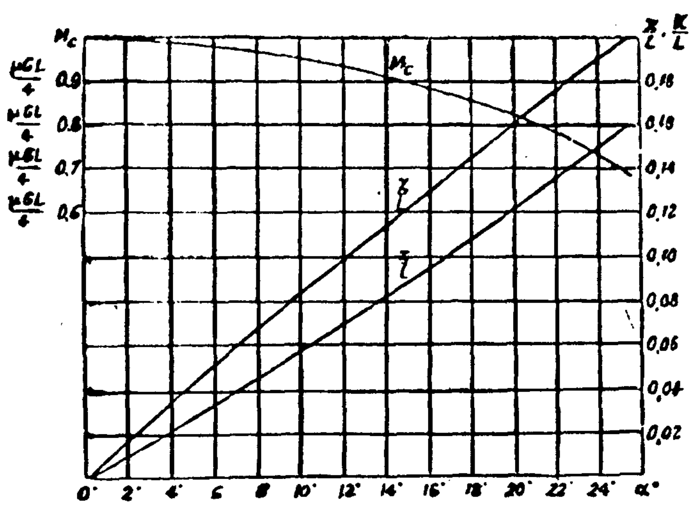


Figure 14. Dependence of the Moment of Resistance M_c on the Shift of the Pressure Center Point x and the Turning Center Point X as well as the Angle of Inclination α.

The change of the tractive efforts P_2 and P_1 in connection with the angle of inclination is presented in Table 5 and Fig. 15.

For the purpose of setting up the table, the following values of the individual members of the formula were taken as a basis:

$$f = 0.06; \quad \mu = 0.5\frac{L}{B} = 1.45.$$

The angles are taken as a basis in the same size as in Table 4.

It is clear from Table 5 and Fig. 15 that both the first members of formulas (21) and (21a) decline according to their absolute size while the last member of these formulas becomes larger.

In the result, the tractive effort of the outer track P_z increases with an increase of the angle of inclination, but less rapidly than the angle increases.

On the other hand, the tractive effort of the inner track increases in this case and at a certain angle (between 13° 30' and 19° 45') becomes a negative value.

Table 5.

Changes of the Tractive Efforts P_2 and P_1 in Relation to the Angle of Inclination α (in the first time segment)

α	$\frac{f\cos\alpha}{2}$	$\frac{\mu L\cos\alpha}{5B}$	$\frac{\sin\alpha}{2}$	$\frac{P_2}{G}$	$\frac{P_1}{G}$
7°	0,0299	0,177	0,0610	+ 0,268	+ 0,086
13° 30′	0,0292	0,166	0,1165	+ 0,312	+ 0,020
19° 45′	0,0282	0,150	0,1690	+ 0,347	— 0,047
25° 30′	0,0271	0,125	0,1250	+ 0,367	— 0,117

With consideration of the assumed force P_1 (against movement), we conclude that at slight angles of inclination, just as on level ground, the inner track must be braked.

At greater angles of inclination tractive effort P_1 becomes a negative value, consequently during a uniform turn it would be superfluous to brake the inner track. On the contrary, driving force must be directed to it from the engine in order to direct its tractive effort in the direction of travel.

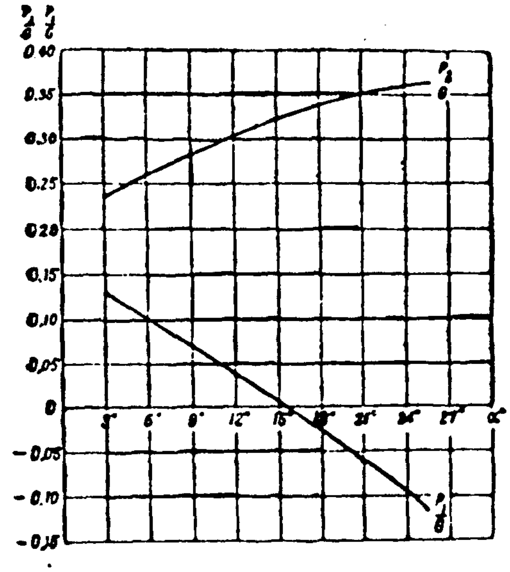


Figure 15. Change of Forces P_2 and P_1, independent of Inclination α

Now we wish to consider the forces being exerted during turn on a given slope if travel is to be continued on the slope.

From Fig. 16 it is clear that in this case an additional transverse force appears besides those forces already known to us, namely the weight components G sin β which causes a backward shift of the turning center point.

The total load of the ground in this case is G cos β . Thus the load caused by each track is different.

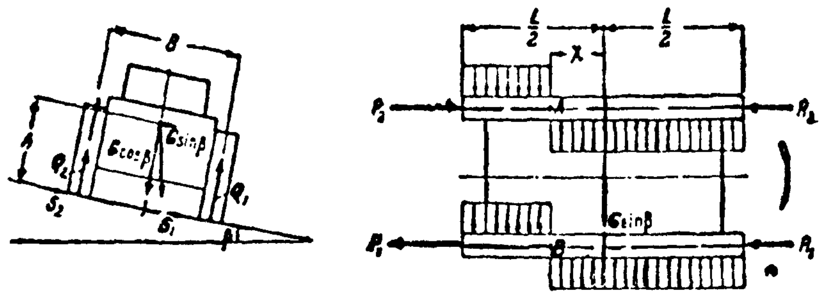


Figure 16. Forces being exerted on the tank and an illustration of the specific ground pressure during turn on a slope.

From the precondition for the equilibrium of forces and moments according to Fig. 16, we will determine the load on the side of the inner track which turns out to be

$$G_1 = \frac{1}{2} G \cos\beta + \frac{h}{B} G \sin\beta \qquad (22)$$

and on the outer track

$$G_2 = \frac{1}{2} G \cos\beta - \frac{h}{B} G \sin\beta \qquad (22a)$$

After determining the equilibrium of the transverse forces, we will determine the shift of the turning center point:

$$\frac{2X}{L} = \frac{G \sin\beta}{\mu G \cos\beta} = \frac{\operatorname{tg}\beta}{\mu} \qquad (23)$$

and

$$X = \frac{L}{2\mu} \operatorname{tg}\beta \qquad (23a)$$

By comparing formulas (13) and (23) we conclude that the ratio 2x/L of each arbitrary transverse force will always equal this transverse force, divided by the product of the coefficient μ and of the normal components of ground load.

The case which we are considering which concerns the effect of forces reminds us of a previously described case in which we gave closer consideration to the influence of centrifugal force. On this occasion we determined that a regrouping of the loads on the tracks had no influence on the size of the resistance moment.

If we take this into account, we can write the following formula for the moment of resistance:

$$M_C = \frac{\mu G L \cos\beta}{4}\left[1 + \left(\frac{2X}{L}\right)^2\right]. \qquad (24)$$

According to Fig. 16, we now turn to the equation of the equilibrium of longitudinally directed forces and the equation of the moments around point B:

$$P_2 - P_1 - R_2 - R_1 = 0 \qquad (a)$$

$$(P_2 - R_2) B + X G \sin\beta = \frac{\mu G L \cos\beta}{4}\left[1 + \left(\frac{2X}{L}\right)^2\right] \qquad (b).$$

When we solve these equations, after transforming the following formulas for the tractive effort of the outer and inner track, we obtain:

$$P_2 = fG_2 + \frac{\mu G L \cos\beta}{4B}\left[1 + \left(\frac{2X}{L}\right)^2\right] - G\sin\beta\frac{X}{B} \quad (25)$$

$$P_1 = -fG_1 + \frac{\mu G L \cos\beta}{4B}\left[1 + \left(\frac{2X}{L}\right)^2\right] - G\sin\beta\frac{X}{B} \quad (25a)$$

If we substitute the values for X, G_1 and G_2 according to formulas (23a), (22) and (22a) for the moment of resistance and the tractive effort into formulas (24), (25) and (25a), we obtain other equations:

$$M_c = \frac{G L \cos\beta}{4}\left[1 + \left(\frac{\operatorname{tg}\beta}{\mu}\right)^2\right] \quad (26)$$

$$P_2 = fG\left(\frac{1}{2}\cos\beta - \frac{h}{B}\sin\beta\right) + \frac{\mu G L \cos\beta}{4B}\left[1 - \left(\frac{\operatorname{tg}\beta}{\mu}\right)^2\right] \quad (27)$$

$$P_1 = -fG\left(\frac{1}{2}\cos\beta + \frac{h}{B}\sin\beta\right) + \frac{\mu G L \cos\beta}{4B}\left[1 - \left(\frac{\operatorname{tg}\beta}{\mu}\right)^2\right] \quad (27a)$$

From these formulas it is clear that the moment of resistance M_c is increased with an increase of the angle of bank β. But in addition there is an increase in the turning moment by the power G sin β.

Since the last-named moment increases more rapidly than the moment of resistance M_c, it is clear that the forces P_2 and P_1 become smaller in regard to their absolute value (with an increase of the angle of gradient), so that the preconditions for the turn improve for this reason.

Table 6 and Fig. 17 present the data concerning the change of the moment of resistance M_c and of the tractive efforts P_2 and P_1 in connection with the angle of bank. (The angle of bank or inclination and other data for the compilation of the table were based on those values used in Tables 4 and 5, i.e.

$$h = \frac{1}{3}L; \quad \frac{L}{B} = 1{,}45; \quad f = 0{,}06; \quad \mu = 0{,}5.$$

In the third case, when the tank is located in an intermediate position between the two named positions, the schematic representation of the forces being exerted is given in Fig. 18 (turn in the direction of the increased sloping position).

Table 6
M_c, P_2 and P_1 at various angle of inclination (2nd time segment)

β	M_c	$\frac{P_2}{G}$	$\frac{P_1}{G}$
7	$1{,}052\,\mu\frac{GL}{4}$	0,195	0.136
13° 30′	$1{,}195\,\mu\frac{GL}{4}$	0,157	0.099
19° 35′	$1{,}417\,\mu\frac{GL}{4}$	0,104	0.047
25° 30′	$1{,}822\,\mu\frac{GL}{4}$	0,030	−0.010

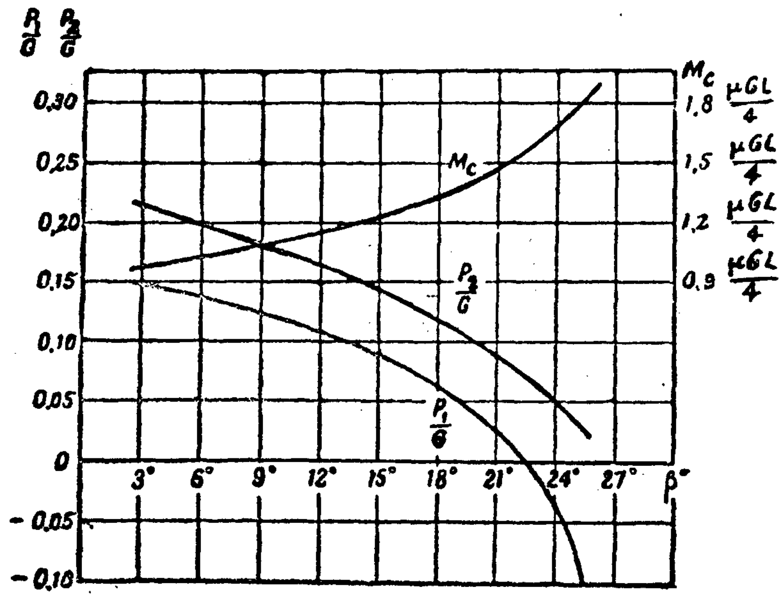


Figure 17. Relation between the moment M_c and the tractive efforts P_2 and P_1 on the one hand, and the angle of bank (angle of inclination β) on the other hand.

In this case, the tank is going uphill and simultaneously is in an inclined position. The transverse force G sin α , the weight components occur because of the tank's sloping position, while the longitudinally directed force G sin β occurs because the tank is climbing.

There exists the following relation between the angle of gradient and the respective position, i.e. the angle α_1, and the angle of maximum gradient α and the angle of inclination β :

$$\sin^2\alpha = \sin^2\alpha_1 + \sin^2\beta.$$

The angles α_1 and β will change in the plane of travel in relation to the angle ψ of tank turn. This relation is expressed in the following formula:

$$\sin\alpha_1 = \cos\psi\sin\alpha.$$
$$\sin\beta = \sin\psi\sin\alpha.$$

The longitudinal components of weight G sin α_1 will cause a shift of the pressure center point and thus the center point of turn as well. The shift of the pressure center point in this case will be

$$x = h\frac{\sin\alpha_1}{\cos\alpha}$$

The transverse components will cause a regrouping of pressure on the track and in addition, as in an exclusive inclination (bank) will exert an influence on the shift of the center point of turn.

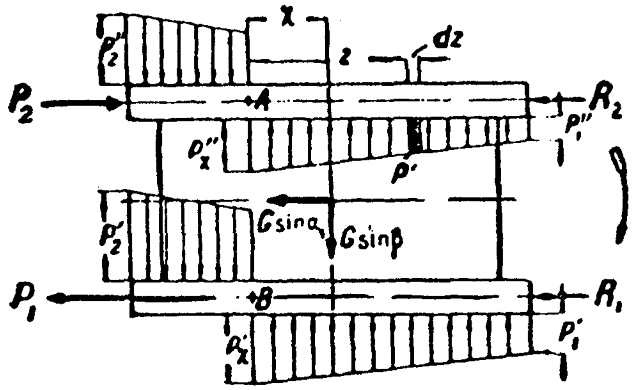


Figure 18.

The pressure will be exerted on the tracks in the following manner:

on the outer track:

$$G_2 = G\left(\frac{1}{2}\cos\alpha - \frac{h}{B}\sin\beta\right) \tag{28}$$

on the inner track:

$$G_1 = G\left(\frac{1}{2}\cos\alpha + \frac{h}{B}\sin\beta\right) \tag{28a}$$

We will determine the shift of the center point of turn X according to the equilibrium of the transversely directed forces (Fig. 18):

$$\frac{1}{2}(p_1' + p_x')\left(\frac{L}{2} + X\right) + \frac{1}{2}(p_1'' + p_x'')\left(\frac{L}{2} + X\right) =$$

$$= \frac{1}{2}(p_2' + p_x')\left(\frac{L}{2} - X\right) + \frac{1}{2}(p_2'' + p_x'')\left(\frac{L}{2} - X\right) + G\sin\beta$$

or

$$\left(p_1' + p_1'' + p_x' + p_x''\right)\left(\frac{L}{2} + X\right) = \left(p_2' + p_2'' + p_x' + p_x''\right)\left(\frac{L}{2} - X\right) + 2G\sin\beta \qquad \text{(a)}$$

In which:

p_1' and p_2' are the specific forces of resistance against turn on the ends of the inner track,

p_1'' and p_2'' are the specific forces of resistance against turn on the ends of the outer track,

p_x' and p_x'' are the specific forces of resistance in the resistance center points of both tracks.

Here the following equations are valid according to the formulas which we derived in the first case of turn on upward sloping terrain:

$$p_1' = p_0'\left(1 - \frac{6x}{L}\right)$$

$$p_1'' = p_0''\left(1 - \frac{6x}{L}\right)$$

$$p_2' = p_0'\left(1 + \frac{6x}{L}\right)$$

$$p_2'' = p_0''\left(1 + \frac{6x}{L}\right)$$

$$p_x' = \frac{(2X + L)p_0' - 2Xp_1'}{L}$$

$$p_x'' = \frac{(2X + L)p_0'' - 2Xp_1''}{L}.$$

If we substitute these values into formula (a), we obtain the following equation:

$$\left(p_0' + p_0''\right)\left(\frac{L}{2} + X\right)\left(2 - \frac{6x}{L} + \frac{12xX}{L^2}\right) =$$
$$= \left(p_0' + p_0''\right)\left(\frac{L}{2} - X\right)\left(2 + \frac{6x}{L} + \frac{12xX}{L^2}\right) + 2G\sin\beta \qquad \text{(b)}$$

The specific mean resistance of the inner and outer track will be expressed in the following equations:

$$p_0' = \mu G_0' = \mu\frac{G_1}{L}$$

$$p_0'' = \mu G_0'' = \mu\frac{G_2}{L}.$$

When we combine these equations, we obtain the following equation:

$$p_o' + p_o'' = \mu \frac{G_1 + G_2}{L} = \mu \frac{G \cos \alpha}{A} \tag{c}$$

When we substitute this expression into the formula (b) and undertake a transformation, then we finally obtain the following equation:

$$12 x X^2 + 2 L^2 X - 3 L^2 x - \frac{L^3 \sin \beta}{\mu \cos \alpha} = 0 \tag{29}$$

According to this equation, assuming that we know the shift of the pressure center point x and the inclination and angle of slope, we can determine the shift of the center point of turn X.

The moment of resistance against turn is calculated in the following manner:

$$M_c = M_1 + M_2,$$

in which:

M_1 represents the resistance moment of the inner track and

M_2 represents the resistance moment of the outer track.

$$M_c = \int_0^{\frac{L}{2}+x} p' z\,dz + \int_0^{x-\frac{L}{2}} p' z\,dz + \int_0^{\frac{L}{2}+x} p'' z\,dz + \int_0^{x-\frac{L}{2}} p'' z\,dz. \tag{a}$$

In accordance with the formulas derived with inclination, the following applies:

$$p' = p_o' \left(1 + \frac{12 x X}{L^3} - \frac{12 x}{L^3} z\right);$$

$$p'' = p_o'' \left(1 + \frac{12 x X}{L^3} - \frac{12 x}{L^3} z\right).$$

If we substitute these expressions into the formula (α) and combine the first integral with the third, the second with the fourth, we obtain:

$$M_c = \int_0^{\frac{L}{2}+x} (p_o' + p_o'') \left(1 + \frac{12 x X}{L^3} - \frac{12 x}{L^3} z\right) z\,dz +$$

$$+ \int_0^{x-\frac{L}{2}} (p_o' + p_o'') \left(1 + \frac{12 x X}{L^3} - \frac{12 x}{L^3} z\right) z\,dz.$$

$$P_\cdot' + P_\cdot'' \quad \frac{\mu G \cos\alpha}{L}:$$

$$M_c = \int_0^{\frac{L}{2}+x} \frac{\mu G \cos\alpha}{L}\left(1 + \frac{12 x X}{L^2} - \frac{12 x}{L^2} z\right) z\,dz +$$

$$+ \int_0^{x-\frac{L}{2}} \frac{\mu G \cos\alpha}{L}\left(1 + \frac{12 x X}{L^2} - \frac{12 x}{L^2} z\right) z\,dz.$$

After integration, we obtain an equation which corresponds to Eq. (19):

$$M_c = \frac{\mu G L \cos\alpha}{4}\left\{\left[1 + \left(\frac{2X}{L}\right)^2\right]\left(1 + \frac{4 x X}{L^2}\right) - 16\frac{x X}{L^2}\right\} \tag{30}$$

But in this case, X and x are determined according to other formulas, which can be considered common for turn in all three cases.

In order to determine tractive efforts P_2 and P_1, we will take as a basis the equation of equilibrium of the longitudinally directed forces and moments with regard to point B:

$$P_2 - P_1 - R_2 - R_1 - G\sin\alpha = 0 \ldots\ldots\ldots\ldots \tag{a}$$

$$(P_2 - R_2)B - G\sin\alpha_1\frac{B}{2} + G\sin\beta X = \frac{\mu G L \sin\alpha}{4}K = M_c \tag{b}$$

Here K is the member in parenthesis (30).

If we solve equations (a) and (b) together, we obtain:

$$P_2 = f G_2 + \frac{\mu G L \cos\alpha}{4B}K + \frac{G\sin\alpha_1}{2} - \frac{X G \sin\beta}{B} \tag{31}$$

$$P_1 = -f G_1 + \frac{\mu G L \cos\alpha}{4B}K - \frac{G\sin\alpha_1}{2} - \frac{X G \sin\beta}{B} \tag{31a}$$

In order to determine the relationship between the moment of resistance M_c as well as the forces P_2 and P_1 on the one hand and the angle of slope α on the other hand, we refer to Tables 7 and 8.

In compiling the tables, the same values were taken as a basis for the individual variables as those in Tables 4, 5 and 6, namely: $h = \frac{1}{3}L$; $\frac{L}{B} = 1{,}45$; $f = 0{,}06$ and $\mu = 0{,}5$.

The angle of turn in the plane of travel ψ was assumed to be 45°.

When we consider these tables and Fig. 19, it is clear that the moment of resistance against turn M_c is increased with each increase of the angle of slope α mainly due to the shift of the turning center point X to the rear.

Table 7.

Moments of Resistance M_c and Various Angles of Inclination α (third time segment)

α	$\frac{x}{L}$	$\frac{X}{L}$	$\cos\alpha$	K	M_c
7°	0,0290	0,120	0,993	1,017	$1,010\mu\frac{GL}{4}$
13° 30′	0,0565	0,235	0,972	1,075	$1,045\mu\frac{GL}{4}$
19° 45′	0,0845	0,340	0,942	1,127	$1,070\mu\frac{GL}{4}$
26° 30′	0,1130	0,400	0,903	1,214	$1,098\mu\frac{GL}{4}$

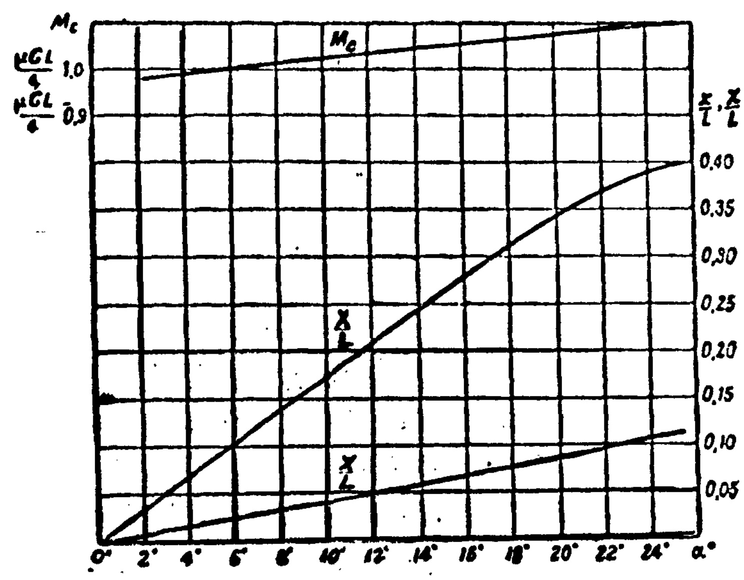


Figure 19. Relationship between the moment of resistance M_c, the value of the shift of the pressure center point x and the center point of turn X on the one hand and the angle of inclination α during a turn in the direction of the increase of the position of slope (bank).

It can be concluded from Fig. 20, and Table 8 that the tractive effort P_2 declines with an increase of the angle of inclination α.

Likewise, but much more rapidly, the force P_1 is reduced and at many angles (between $13^\circ\ 30'$ and $19^\circ\ 45'$) it sinks to zero.

When the angle is increased further, it obtains a negative value. This means that we must also have a propulsive force on the inner track during turn which can be achieved only by the propulsive force of the engine.

At this juncture we wish to observe the tank turning in the opposite direction when traveling upward while the tank is tilted to one side.

Table 8

Tractive Efforts P_2 and P_1 at Various Angles of Inclination α
(third time segment)

α	fG_2	fG_1	$\frac{\mu L \cos\alpha}{4B}K$	$\frac{\sin\alpha_1}{2}$	$\sin\beta\frac{X}{B}$	$\frac{P_2}{G}$	$\frac{P_1}{G}$
7°	0,0273 G	0.0322 G	0.183	0,043	0.015	+ 0,238	+ 0,093
13° 30′	0,0244 G	0,0339 G	0,089	0,082	0.056	+ 0,240	+ 0,017
19° 45′	0,0214 G	0.0352 G	0.194	0.119	0,118	+ 0,216	− 0,078
25° 30′	0,0184 G	0,0359 G	0.199	0,152	0,176	+ 0,193	− 0,165

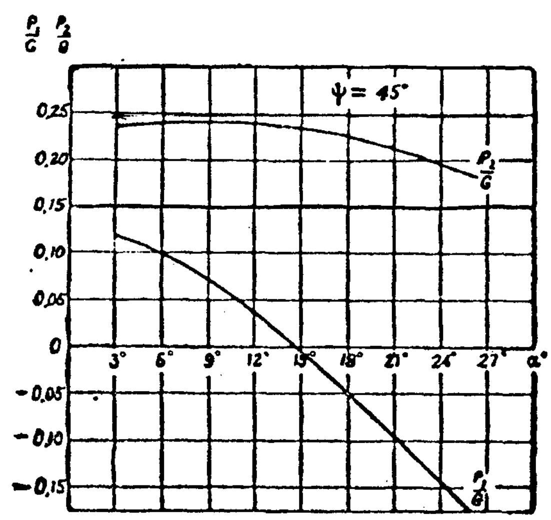


Figure 20. Change of the tractive efforts P_2 and P_1 in relation to the inclination when turning the tank in the direction of increased inclined position (bank).

On the basis of conclusions reached earlier, we can write the following:

$$P_2 = fG_2 + \frac{\mu G L \cos\alpha}{4B} K + \frac{G \sin\alpha_1}{2} - \frac{X G \sin\beta}{B} \tag{32}$$

$$P_1 = -fG_1 + \frac{\mu G L \cos\alpha}{4B} K - \frac{G \sin\alpha_1}{2} - \frac{X G \sin\beta}{B}; \tag{32a}$$

$$K = \left[1 + \left(\frac{2X}{L}\right)^2\right]\left(1 - \frac{4xX}{L^2}\right) + \frac{16xX}{L^2}. \tag{33}$$

$$12xX^2 - 2L^2X - 3L^2x + \frac{L^3 \sin\beta}{\mu \cos\alpha} = 0 \tag{34}$$

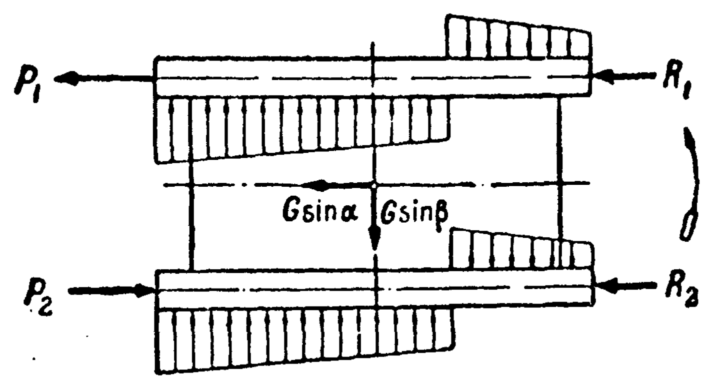


Figure 21. Forces being exerted on the tank and illustration of the specific mean ground pressures during a turn in the direction of inclination.

Table 9 and Fig. 22 give the relationship between M_c, P_2 and P_1 on the one hand and the angle of inclination α on the other hand.

The values of the individual sizes are the same as in the previous cases.

Table 9

Tractive Efforts P_2 and P_1 at Various Angles of Inclination α During a Turn while Traveling Upward.

α	$\frac{x}{L}$	$\frac{X}{L}$	$\frac{\mu L \cos\alpha}{4B}K$	$\frac{P_2}{G}$	$\frac{P_1}{G}$
7°	0,0290	0,043	0,184	0,254	0,109
13° 30'	0,0565	0,088	0,192	0,287	0,091
19° 45'	0,0845	0,137	0,204	0,310	0,016
25° 30'	0,1130	0,195	0,230	0,332	− 0,026

With consideration of Table 9 and Fig. 22, we observe that the moment of resistance as well as tractive effort P_2 are increased in this case when angle α is increased. On the other hand, tractive effort P_1 declines when angle α becomes larger.

In order to determine the changes M_c, P_2 and P_1 in relation to the angle of turn, a graphic representation for the angle α = 19° 45' is given in drawing 23.

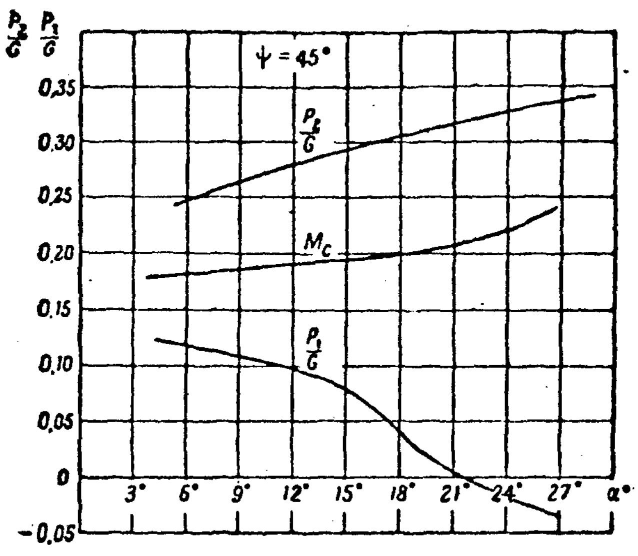


Figure 22. Change of Tractive Efforts P_2 and P_1 dependent on Slope α during a Turn while Traveling Uphill.

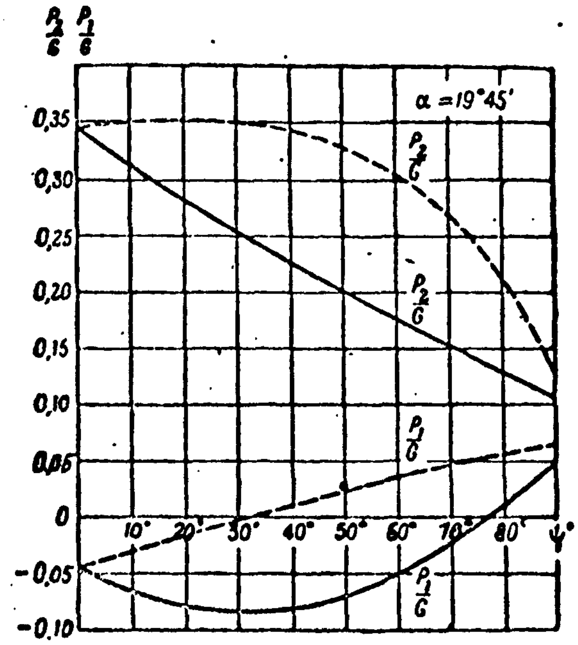


Figure 23. Change of the moment of resistance M_c and tractive efforts P_2 and P_1 depending on the angle of turn φ.

Here P_2 and P_1 are given by a line during the turn of a tank traveling downhill, while the dotted line shows the forces P_2 and P_1 with the tank traveling uphill.

Taking into account the circumstance that the negative force increases the propulsive force required for the turn, we conclude when studying the graphic representation that the most difficult preconditions will prevail for turning while traveling uphill in the first position, i.e. when the longitudinal axis of the tank forms the largest angle with the horizontal plane.

For this case it is necessary to calculate turning when traveling uphill.

d) Turning of a Tracked Caterpillar and of a Trailer

A uniform turn of a non-steerable trailer can be performed only with a definite direction and with a certain tractive effort on the towing hook P_k.

If the angle between the longitudinal axis of the trailer and force P_k is smaller than is necessary in the most extreme case, the trailer will not execute the turn. It will continue straight ahead regardless of the effect of force $P_k \sin\gamma$ which is trying to turn the trailer.

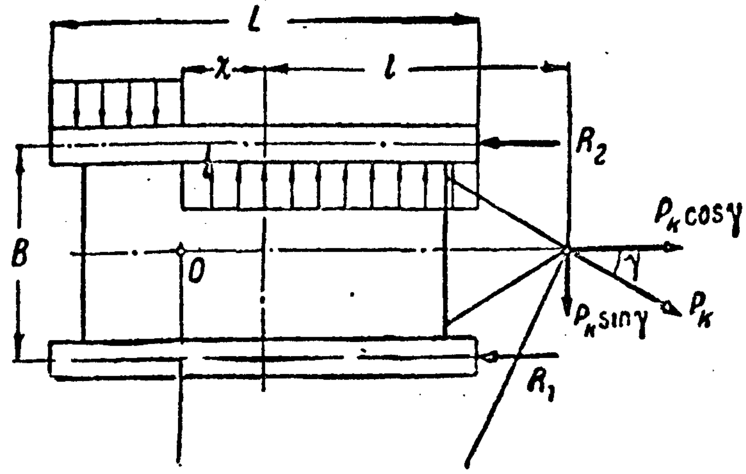


Figure 24. The Turn of a Tracked Trailer

The extent of tractive effort on the hook and the angle γ are determined according to the equality of the moments of force $P_k \sin\gamma$ opposite point 0 (Fig. 24) and of the moment of resistance against turn M_c.

$$(X + l) P_k \sin\gamma = \frac{\mu G_n L}{4}\left[1 + \left(\frac{2X}{L}\right)^2\right] \qquad \text{(a)}$$

The following equation can be written on the basis of formula (13):

$$\frac{2X}{L} = \frac{P_k \sin\gamma}{\mu G_n}.$$

If we substitute this expression into formula (a), we will obtain the following equation after transformation:

$$P_k^2 \sin^2\gamma + 4\mu G_n \frac{l}{L} P_k \sin\gamma - \mu^2 G_n^2 = 0. \qquad \text{(b)}$$

The numerical value of $P_k \sin\gamma$ can be calculated from this equation, it is

$$P_k \sin\gamma = \mu G_n \left[\sqrt{\frac{4l^2}{L^2} + 1} - \frac{2l}{L}\right] \qquad (35)$$

On the other hand, assuming that the trailer moves in a uniform manner with a uniformity of the longitudinally directed forces, we will have to deal with the following equation:

$$P_k \cos\gamma = R_2 + R_1 = fG_n \ . \tag{36}$$

If we divide formula (35) by formula (36) we will obtain the expression for determining the size of angle γ.

$$\mathrm{tg}\,\gamma = \frac{\mu}{f}\left[\sqrt{\frac{4\,l^2}{L^2}+1}-\frac{2\,l}{L}\right]. \tag{37}$$

The amount of force P_k is determined according to the following formula:

$$P_k = \sqrt{(P_k \sin\gamma)^2 + (P_k \cos\gamma)^2} \ . \tag{38}$$

or from the formula (36):

$$P_k = \frac{fG_n}{\cos\gamma} \ . \tag{38a}$$

In order to obtain a concept of the values of the angle γ and of the force P_k, we will now calculate these amounts numerically and take as a basis the following numerical values:

$$L = 2{,}0 \text{ m};\ l = 3{,}0 \text{ m};\ B = 1{,}8 \text{ m};\ f = 0{,}1;\ \mu = 0{,}4.$$

Using these data, we will attempt the following results:

$$\mathrm{tg}\,\gamma = 0{,}647;\ \gamma \approx 33°;\ P_k = 0{,}119\,G_n.$$

If we assume with these data that $\mu = 0.3$, then we obtain:

$$\mathrm{tg}\,\gamma = 0{,}486;$$
$$\gamma \approx 26°;$$
$$P_k = 0{,}111\,G_n.$$

As these examples show, the angle between the direction of motion and the longitudinal axis of the trailer, with which a uniform turning is possible, is rather large.

The direction of motion on the hook P_k, which is necessary for turning, is increased only slightly in comparison to the force required for a uniform and straight ahead travel.

In order to reduce angle γ and force P_k, it is necessary, as is clear from formulas (35) and (37), to move the location of the hook as far as possible from the trailer axis.

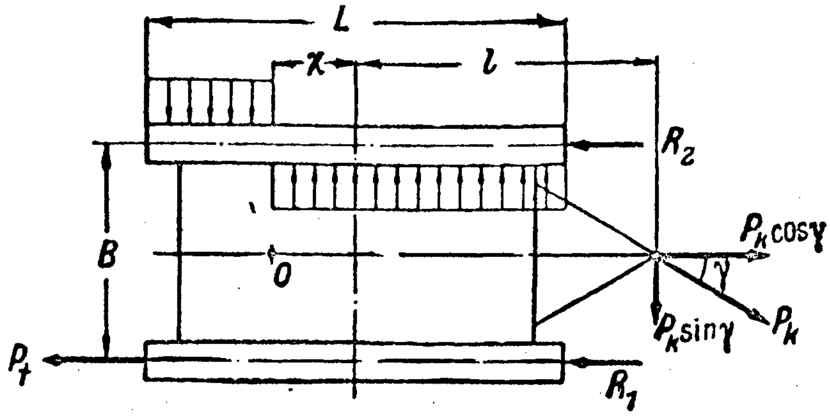


Figure 25. Turn of the Trailer When Braking the Inner Track

If the inner track of the trailer is braked, the angle γ is reduced significantly.

In fact (Fig. 25),

$$(X + l)\, P_k \sin\gamma + \frac{B}{2} P_t = \frac{\mu G_n L}{4}\left[1 + \left(\frac{2X}{L}\right)^2\right]. \tag{a}$$

in which P_t signifies braking force.

After transformation this equation assumes the following appearance:

$$P_k^2 \sin^2\gamma + 4\mu G_n \frac{l}{L} P_k \sin\gamma - \mu^2 G_n^2 + \frac{B}{L} k_t \mu G_n^2 = 0. \tag{b}$$

k_i is the braking coefficient which equals the relative value P_t in comparison to weight $Gn/2$ which is lost on the inner track.

According to this equation we find:

$$P_k \sin\gamma = \mu G_n \left[\sqrt{\frac{4l^2}{L} + 1 - \frac{B}{L}\frac{k_t}{\mu}} - \frac{2l}{L}\right]. \tag{39}$$

In addition,

$$P_k \cos\gamma = \left(f + \frac{k_t}{2}\right) G_n. \tag{40}$$

If we divide the two last-named equations by each other, we obtain:

$$\operatorname{tg}\gamma = \frac{\mu d}{f + \frac{k_t}{2}}. \tag{41}$$

in which d represents the member of formula (39) in brackets.

If we utilize the same numerical values as in the former examples, we will obtain the following results for the angle γ and force P:

At $\mu = 0.4$ and $k_r = 0.15$: $\mathrm{tg}\,\gamma = 0.245$; $\gamma \approx 14°$; $P_k = 0180\,G_n$;

At $\mu = 0.3$ and $k_r = 0.15$: $\mathrm{tg}\,\gamma = 0.184$; $\gamma = 10° 30'$ and $P_k = 0.178\,G_n$.

The result is that angle γ is significantly decreased even at light braking. To be sure the required tractive effort on the hook is increased somewhat by braking the trailer.

The forces which are exerted on the towing machine (tractor) during its turn together with the trailer are represented in Fig. 26 (we will neglect the influence of centrifugal force).

The moment of resistance against turn will be expressed in the same manner as when considering the influence:

$$M_c = \frac{\mu G L}{4}\left[1 + \left(\frac{2X}{L}\right)^2\right]$$

where

$$\frac{2X}{L} = \frac{R_k \sin\gamma_1}{\mu G}$$

R_k is the total resistance of the trailer on the towing hook.

The equation of the longitudinally directed forces and moments opposite point B is written down in the following form:

$$P_2 - P_1 - fG - R_k \cos\gamma_1 = 0 \qquad \text{(a)}$$

$$P_2 B - \frac{fG}{2} B - M_c - R_k \cos\gamma_1 \frac{B}{2} - R_k \sin\gamma_1 (l_1 - X) = 0 \qquad \text{(b)}$$

From this we determine the expressions for tractive efforts:

$$P_2 = \frac{fG}{2} + \frac{\mu G L}{4B}\left[1 + \left(\frac{2X}{L}\right)^2\right] + \frac{R_k \cos\gamma_1}{2} + R_k \sin\gamma_1 \frac{l_1 - X}{B} \;. \qquad (42)$$

$$P_1 = -\frac{fG}{2} + \frac{\mu G L}{4B}\left[1 + \left(\frac{2X}{L}\right)^2\right] - \frac{R_k \cos\gamma_1}{2} + R_k \sin\gamma_1 \left(\frac{l_1 - X}{B}\right). \qquad (42a)$$

If we substitute the values of X into these formulas, we will, after transformation, obtain other expressions for the tractive efforts:

$$P_2 = \frac{fG}{2} + \frac{\mu G L}{4B} - \frac{(R_k \sin\gamma_1)^2 L}{4\mu G B} + \frac{R_k \cos\gamma_1}{2} + \frac{R_k \sin\gamma_1 l_1}{B} \qquad (43)$$

$$P_1 = -\frac{fG}{2} + \frac{\mu G L}{4B} - \frac{(R_k \sin\gamma_1)^2 L}{4\mu G B} - \frac{R_k \cos\gamma_1}{2} + \frac{R_k \sin\gamma_1 l_1}{B} \qquad (43a)$$

If $\gamma_1 = 0$ (at the beginning of turn), the formulas will have the following form:

$$P_2 = \frac{fG}{2} + \frac{\mu G L}{4B} + \frac{R_k}{2} \qquad (44)$$

$$P_1 = -\frac{fG}{2} + \frac{\mu G L}{4B} - \frac{R_k}{2} \qquad (44a)$$

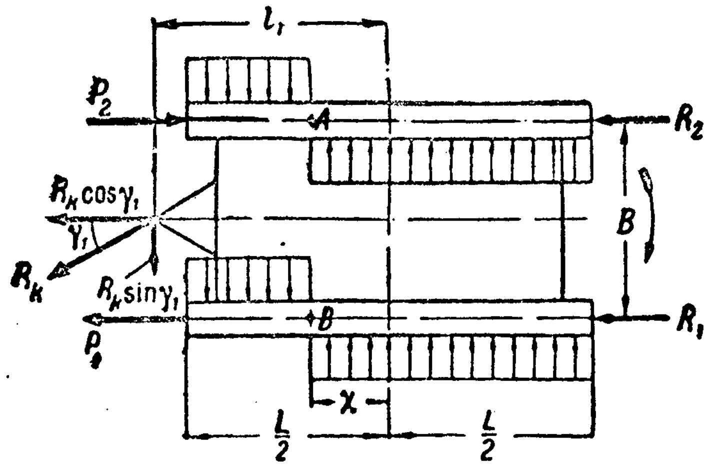


Figure 26.

If $\gamma_1 = 90°$, we will have to deal with the following formulas:

$$P_2 = \frac{fG}{2} + \frac{\mu G L}{4B} - \frac{R_k^2 L}{4\mu G B} + \frac{R_k l_1}{B} \qquad (45)$$

$$P_1 = -\frac{fG}{2} + \frac{\mu G L}{4B} - \frac{R_k^2 L}{4\mu G B} + \frac{R_k l_1}{B} \qquad (45a)$$

If we give closer study to the formulas cited here, we reach the conclusion that tractive effort P_2 is smallest when the angle $\gamma_1 = 0$. If we increase the angle γ_1, the tractive effort will increase up to a certain extreme limit and then decline again when the sizes of the angle reach almost 90^o.

Tractive effort P_1 can turn out to be negative values at small angles γ_1; it can be generated only by the engine, but not by braking.

At a certain angle γ_o its value equals zero while at γ_1 it becomes greater than γ_o already at braking effort and will increase with an increase of γ_1.

e) Non-uniform Turn of the Tank

Up to this point we have dealt only with a uniformly executed turn with a definite uniform radius and uniform angular velocity and from this we have determined the necessary tractive efforts of the outer and inner track for this turn under varying preconditions.

In order to accomplish such a turn, the tank must perform the initial part of the turn in a straight line in which it travels with the required angular velocity to generate turning velocity with a changing radius, where this radius changes from an infinite to an arbitrarily finite value.

In order to generate an angular acceleration it is necessary that

$$M_p > M_c$$

in order that the moment of turn M_p (Formula 11) be larger than the moment of resistance.

The equation of turning motion will assume the following appearance:

$$(P_2 + P_1)\frac{B}{2} - M_c = M_\varphi \qquad (46)$$

$$M_\varphi = J_0 \varphi''.$$

J_o is the polar moment of inertia around the vertical axis which passes through the center point of turn, and φ'' is the angular velocity.

The greater the sum of the tractive efforts P_2 and P_1, the greater will be the angular acceleration φ'' and the more rapidly the tank will adapt to the required angular velocity at the appropriate radius.

If the sum of tractive efforts P_2 and P_1 as well as the moment of resistance M_c reach constant values, the result will be a uniform increase of angular velocity.

When the moment of resistance in relation to the relative value of forces P_2 and P_1 exhibit a constant value, four characteristic cases of a non-uniform turn can occur:

1. Uniform movement of the center of gravity with uniform angular acceleration;
2. Uniformly accelerated movement of the center of gravity and uniformly accelerated turn;
3. Uniformly delayed movement of the center of gravity and uniformly accelerated turn;
4. Neither the angular acceleration of the tank nor acceleration of the center of gravity can be constant values.

The problem of studying the non-uniform turn involves checking the type of the turn in the first time segment and to determine the ratio of forces which are involved.

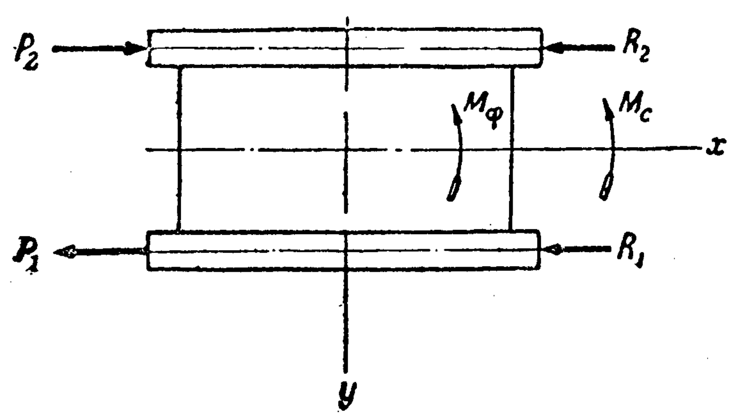


Figure 27.

Studying the non-unform turn analytically will be very difficult because the mathematical derivations are very involved. But to study the type of non-uniform turning it is sufficient to study the above-mentioned four cases with certain allowances so as to check the conclusions as to what influence these allowances exert.

For example, we always wish to neglect the influence of centrifugal force. In four cases of turning this will exert no noticeable influence not even on the final derivations of the study. Consequently, this will also exert no influence on the general conclusions concerning the type of turning.

1. So that the center of gravity might continue its uniform movement (b = 0) and in order that the velocity which prevailed before the turn (v_o) might be continued, the sum of the forces being exerted on the tank must equal zero.

Consequently, it is particularly necessary that

$$P_2 - P_1 - R_2 - R_1 = 0 \tag{a}$$

When we study Eqs. (46) and (a) more closely we will recognize that in this case the difference between the forces P_1 and P_2 must be constant (we are assuming that the resistances R_2 and R_1 are constant and equal to each other). On the other hand, their sum may be different but must definitely be larger than 2 M_c/B, but must not change during the turn. By solving Eqs. (46) and (a) we obtain the following formulas for the tractive effort of the outer and inner track:

$$P_2 = \frac{fG}{2} + \frac{\mu G L}{4B} + \frac{M_\varphi}{B} \tag{47}$$

and

$$P_1 = -\frac{fG}{2} + \frac{\mu G L}{4B} + \frac{M_\varphi}{B} \tag{47a}$$

If we compare these formulas for tractive effort P_2 and braking force P_1 with similar formulas which we have obtained for a uniform turn without considering centrifugal force (Formulas 7 and 10) we see that at a non-uniform turn P_2 as well as P_1 are increased by the same value of $M\varphi/B$.

In order now to study the type of turn, we must investigate the related movements between the individual variables which characterize the turn.

We will deal with the following variables: The turning angle of the tank, the path which the center of gravity follows, the angular velocity of the tank and the radius of turn at the appropriate point in time.

The following equations are available to us for determining these values:

$$\varphi'' = \frac{1}{J_0}\left[(P_2 + P_1)\frac{B}{2} - M_c\right] = E \qquad \text{(a)}$$

$$\left.\begin{aligned} v_x &= v_0 \cos\varphi; \\ v_y &= v_0 \sin\varphi; \end{aligned}\right\} \cdots\cdots\cdot \qquad \text{(b)}$$

In this case, we obtained equation (a) from formula (46); on the other hand, equations (b) represent the projections of uniform speed v_o of the movement of the center of gravity, which changes its direction simultaneously with the turn of the tank. Angle φ in these equations is the tank angle of turn.

If we write equation (a) in differential form and integrate, we obtain the formula for the angular velocity independent of time.

$$\int d\varphi' = \int E\,dt;$$

$$\varphi' = \omega = Et + \text{const.}$$

If we select the initial moment of turn (transition from straight ahead travel to the curve) as starting point for the calculation, we obtain the final formula for the angular velocity of tank turn, i.e.

$$\omega = Et \qquad (48)$$

Likewise by means of integration, we obtain the turning angle as a function of time from this formula, i.e.

$$d\varphi = Et\,dt \qquad \text{(c)}$$

$$\varphi = \frac{E}{2}t^2 \qquad (49)$$

If we substitute this formula for angle φ into the equations (b) and quote them in differential form, we will obtain the following equations:

$$\left.\begin{aligned} dv_x &= v_0 \cos\frac{Et^2}{2}dt; \\ dv_y &= v_0 \sin\frac{Et^2}{2}dt \end{aligned}\right\} \cdots\cdots \qquad \text{(d)}$$

Unfortunately, these equations cannot be integrated so that we might determine the equation for the curve.

Since now v_o = const. we obtain:

$$s = v_0 t \qquad (50)$$

According to this formula we will find the method for determining the distance the center of gravity will cover from the curve in a certain time interval.

The turning radius, dependent on time, may be determined with the aid of formula (c) and formula (50), i.e.:

$$R - \frac{B}{2} = \frac{ds}{d\varphi} = \frac{v_0}{Et} \qquad (51)$$

Eqs. (48), (49), (50) and (51) which are related to each other by a variable size t, characterize the accelerated turn at a uniform speed of the center of gravity.

If the size of the angle is known around which the tank must execute the turn, by using formula (49) we can determine the interval of time t during which the turn is executed:

$$t = \sqrt{\frac{2\varphi}{E}}.$$

When we substitute these determined times into formulas (50), (48) and (51), we obtain:

a) the path traveled over by the center point of inertia during the turn of the tank around angle φ :

$$s = v_0\sqrt{\frac{2\varphi}{E}};$$

b) the angular velocity which is achieved at the end of the turn:

$$\omega = \sqrt{2E\varphi};$$

c) the turning radius during this point of time:

$$R = \frac{v_0}{\sqrt{2E\varphi}} + \frac{B}{2}.$$

Accordingly we can determine the speeds of the outer and inner tracks v_1 and v_2 according to formulas (3) and (4).

2. The center of gravity will shift with uniform acceleration during a uniformly accelerated turn of the tank if the sum of the forces being exerted on the tank equals zero.

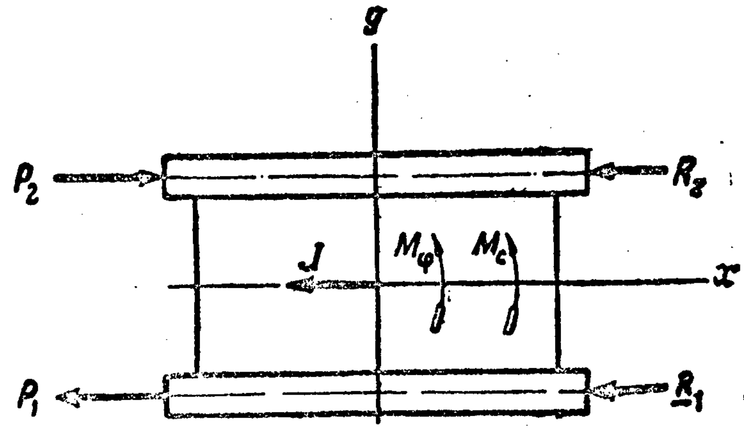


Figure 28.

The equation with regard to the equilibrium of forces (Fig. 28) will have the following appearance:

$$P_2 - P_1 - R_2 - R_1 - m \cdot b = 0, \tag{a}$$

where $m \cdot b$ is the force of inertia.

The equations of equilibrium, of moments and of turn remain the same as in the previous case.

If we solve Eqs. (46) and (a) together, we obtain the formulas for the tractive effort of the outer and inner track:

$$P_2 = \frac{fG}{2} + \frac{\mu G L}{4B} + \frac{M_\varphi}{B} + \frac{mb}{2} \tag{52}$$

$$P_1 = -\frac{fG}{2} + \frac{\mu G L}{4B} + \frac{M_\varphi}{B} - \frac{mb}{2} \tag{52a}$$

From these formulas we will see that in this case the tractive effort P_2 increases still more than tractive effort P_2 in the previous case, while braking force P_1 declines.

When we take this circumstance into account, we can conclude that it can equal zero under certain conditions and then the turn will be executed without braking the inner track.

In accordance with the previous case, we will determine the formulas for the values which give us the type of accelerated turning in the present case.

As in the previous case, we will deal with the same formulas (48) and (49) for ω and φ :

$$\omega = E t \tag{53}$$

and

$$\varphi = \frac{E}{2} t^2$$

The speed of the shift of the center of gravity will be:

$$v = v_0 + b t \tag{53}$$

Accordingly, we will determine the following formula for the path traversed by the center point of inertia:

$$s = v_0 t + \frac{b t^2}{2} \tag{54}$$

From:

$$d s = (v_0 + b t) d t$$
$$d \varphi = E t d t$$

we obtain:

$$R - \frac{B}{2} = \frac{ds}{d\varphi} = \frac{v_0 + b t}{E t} \tag{55}$$

Assuming we know the value of angle φ around which the tank must execute its turn, we can determine the path which the center of gravity must traverse, the angular velocity and the turning radius in the last moment of turn by using formulas (49), (53), (48) and (55).

According to a definite radius R up to which the non-uniform turn is performed, we can also determine the angle of turn, the path traversed by the center of gravity and the angular velocity.

Now we wish to determine under what circumstances a turn of the tank at uniform radius is possible (if the forward movement of the center of gravity and the turn of the tank are subject to the same acceleration).

In this case:

$$\left(R - \frac{B}{2}\right) \varphi'' = b$$

or

$$\frac{v_0 + b t}{E t} \varphi'' = b$$

Since now, however,

$$E = \varphi''$$

$$v_0 + b t = b \cdot t$$

This is impossible, however, because we can assume the velocity in an arbitrary time interval of the earlier one as initial velocity v_o if, depending on conditions, the center of gravity exhibited a velocity which was not equal to zero.

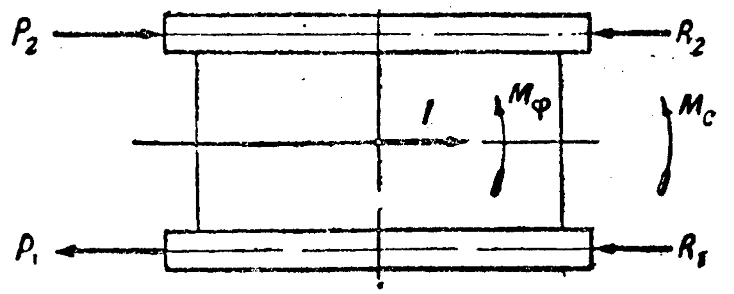


Figure 29.

Consequently, under these preconditions the tank does not move with a uniform radius.

3. In the third case, if the center of gravity moves with a uniform lag (Fig. 29), the equilibrium of forces will appear as follows:

$$P_2 - P_1 - R_2 - R_1 + m \cdot b = 0 \qquad \text{(a)}$$

The equilibrium equation of forces will be the same as in the previous cases; according to Formula (46) it will be as follows:

$$(P_2 + P_1)\frac{B}{2} - M_c = M_\varphi .$$

The tractive efforts of both tracks will be expressed in the following formulas:

$$P_2 = \frac{fG}{2} + \frac{\mu G L}{4B} + \frac{M_\varphi}{B} - \frac{mb}{2} \qquad (56)$$

and:

$$P_1 = \frac{fG}{2} + \frac{\mu G L}{4B} + \frac{M_\varphi}{B} + \frac{mb}{2} \qquad (56a)$$

In this case tractive effort P_2 will be smaller than in the previous case by the amount $m \cdot b$; on the other hand, braking effort will increase by the same value. Force P_1 will be larger than force P_2.

For the sizes which characterize the preconditions for turn, we obtain the following formulas in this case as before, according to the same considerations:

$$\omega = E t,$$

$$\varphi = \frac{E}{2} t^2,$$

$$s = v_0 t - \frac{b t^2}{2} \tag{57}$$

$$R - \frac{B}{2} = \frac{ds}{d\varphi} = \frac{v_0 - b t}{E t} \tag{58}$$

We wish to determine in which case the velocity v_o of the outer track will be uniform and equal to the original velocity v_o.

We have the equation:

$$v_2 = \omega R,$$

where

$$\omega = E t \quad . \; . \; . \; . \; . \tag{53}$$

$$R = \frac{v_0 - b t + \frac{B E}{2} t}{E t},$$

Consequently,

$$v_2 = v_0 - b t + \frac{B E}{2} t.$$

Assuming that $v_2 = v_o$, we obtain the following equation:

$$\left(\frac{B E}{2} - b\right) t = 0.$$

The result is:

$$b = \frac{B E}{2}$$

or

$$b = \frac{B}{2 J_0}\left[(P_2 + P_1)\frac{B}{2} - M_c\right] \tag{59}$$

If we substitute this expression for b in the formula for the equilibrium of forces (a), we obtain the ratio of the forces P_2 and P_1 to each other where it is possible to maintain the speed of the outer track at a constant value equal to 0.

4. The fourth case: when the throttle valve is opened during increasing angular velocity and acceleration of the center of gravity, with variable values of tractive efforts P_2 and P_1 (which always occurs during the start of a turn with the brakes applied).

It is very difficult to set up an exact law for the change of forces P_2 and P_1 of time because the time interval for applying the brakes, opening the throttle valve and the operating efficiency when braking is completely dependent on the tank driver.

In our conclusion, which is necessary only to comprehend the phenomena which occur at the moment of turning, it is sufficient to assume braking force to be in a direct ratio to time.

Up to the point in time in which forces P_2 and P_1 have not reached the following values:

$$P_{02} = \frac{fG}{2} + \frac{\mu G L}{4B}$$

and

$$P_{01} = -\frac{fG}{2} + \frac{\mu G L}{4B}$$

applying the brake and opening the throttle valve will serve no purpose; the tank will execute no turn. The turn does not begin until these forces (or one of them) become larger than the named values).

We will assume that the beginning of readings will be the point of time at which the turn begins, and that the tractive effort of the outer track will not change later on.

Then the equations of forces and moments (Fig. 29) will take on the following form:

$$P_{02} - P_{01} - \frac{P_1 - P_{01}}{t_0} t - R_1 - R_2 + m b = 0$$

and

$$(P_{02} - P_{01})\frac{B}{2} + \frac{P_1 - P_{01}}{t_0}\frac{B}{2} t - M_c = M_\varphi$$

or

$$\frac{P_1 - P_{01}}{t_0} t = \frac{G}{g} b \qquad (a)$$

and

$$\frac{P_1 - P_{01}}{t_0}\frac{B}{2} t = J_0 \varphi'' \qquad (b)$$

where t_0 is the full time interval for applying the brake. From this we obtain:

$$\dot{v} = \frac{g}{G}\frac{\Delta P_1}{t_0} t = A t \tag{60}$$

$$\varphi'' = \frac{B}{2 J_0}\frac{\Delta P_1}{t_0} t = M_1 t \tag{61}$$

If we integrate these equations in succession we obtain in the function of time the formulas for the values which indicate the type of movement:

$$\omega = \frac{M_1 t^2}{2} \quad \dots\dots\dots\dots\dots \tag{62}$$

$$\varphi = \frac{M_1 t^3}{6} \quad \dots\dots\dots\dots\dots \tag{63}$$

$$v = v_0 - \frac{A t^2}{2} \dots\dots\dots\dots\dots \tag{64}$$

$$s = v_0 t - \frac{A t^3}{6} \dots\dots\dots\dots \tag{65}$$

$$R - \frac{B}{2} = \frac{2 v_0 - A t^2}{M_1 t_2} \quad \dots\dots\dots\dots \tag{66}$$

The method of determining these values under any precondition remains the same as in the previous cases.

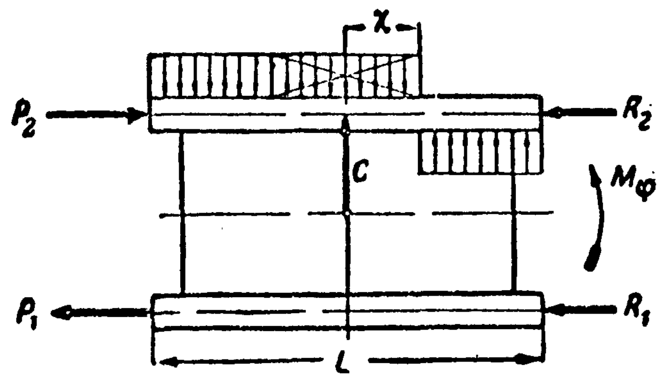


Figure 30.

From these equations it is evident that the tank will execute the turn more slowly at variable braking force than at uniform braking force.

As soon as the brake is applied and force P_1 achieves a uniform value, the type of turn will change and become of the same type as was described in the third case.

Now we wish to consider the influence of centrifugal force on accelerated turning.

Fig. 30 gives a schematic representation of the forces being exerted. The shaded area represents the sum of reactions from the ground by means of which centrifugal force C is again equalized. This surface will also change, dependent on centrifugal force. The formula for this opposing moment will take the following form:

$$M_c = \frac{\mu G L}{4} - \frac{C^2 L}{4 \mu G}.$$

When we select the moments of forces around the center of gravity, we obtain the following formula:

$$(P_2 + P_1)\frac{B}{2} - \frac{\mu G L}{4} + \frac{C^2 L}{4 \mu G} = J_0 \varphi'' . \qquad (67)$$

Comparing this equation with the equation for the turning motion in the previous cases (46) it is clear that the left part of the equation is increased in view of the invariability of P_2 and P_1, the constant value of v_o, a shortening of the turning radius (i.e. an increase of angular velocity) because of the increase of $C^2L/4\mu G$.

This illustrates that in addition angular velocity is increased and becomes increasingly larger than the accelerations to which we referred when considering all characteristic cases of non-uniform turns.

This means that the turn will really be performed somewhat more rapidly under the influence of centrifugal force than was indicated above.

3. Power Efficiency of the Engine during the Turn

a) Power equilibrium of the tank with various steering gears.

The engine power efficiency is used for turning the tank without considering the circumstance; the steering gear is installed in the tank to overcome external resistances against forward movement of the tank, as well as against the braking of the inner track.

The total force of external resistances, at the same size ratios of the tank and at the same turning radius and turning velocity, depends only on the properties of the ground on which the turn is executed.

Another part of this power efficiency, the so-called braking force, at the same preconditions is related not only to the characteristics of the ground but also to the make of the vehicle.

We now wish to derive the formulas for engine load when turning the tank using various types of steering gears.

As already mentioned, when deriving the exterior forces and moments which exert their influence on the tank, it proved true in most cases that the tractive effort of the outer track was directed in the direction of travel.

On the contrary, the tractive effort of the inner track was most frequently directed opposite the direction of travel; thus it was a braking force.

Consequently, a tank with a steering coupling will almost always execute the turn with the steering clutch disengaged and the inner drum braked; this drum is coupled to the drive sprocket over the final drive.

If we consider this circumstance, we reach the conclusion that the total power efficiency of the engine is exerted on the outer track.

The tractive effort of the outer track which is necessary for the turn is P_2 according to the previous derivations. Consequently, the full power required to turn a tank with a steering coupling will be:

$$N_D = \frac{P_2 v_2}{270\,\eta}\ \text{PS} \tag{68}$$

v_2 is the speed of the outer track in km/h.

The power N_D should be used to overcome the exterior forces and for braking, according to our above considerations.

$$N_D = N_e + N_T \tag{69}$$

It is completely understandable that the power efficiency which is used to brake the inner track (braking effort) amounts to

$$N_T = \frac{P_1 v_1}{270\,\eta}\ \text{PS} \tag{70}$$

with a tank having a steering coupling in which v_1 will be the speed of the inner track in km/h.

The difference of the drive and braking force will of course be the power used against the exterior resistance

$$N_e = N_D - N_T$$

or

$$N_e = \frac{P_2 v_2 - P_1 v_1}{270\,\eta}\ \mathrm{PS} \tag{71}$$

This division can be derived directly from the sum of resistances being exerted against forward movement and the moment of resistance against turn.

It is clear that the power efficiency of the exterior resistances in the case of such a derivation, the turn being executed uniformly on level ground without the influence of centrifugal force, will be expressed in the following form:

$$N_0 = \frac{\frac{fG}{2} v_2 + \frac{fG}{2} v_1 + \frac{\mu G L}{4} \omega}{270\,\eta} \tag{a}$$

According to formula (2) we have,

$$\frac{v_2 - v_1}{B} = \frac{v_2}{R}$$

or

$$\frac{v_2 - v_1}{B} = \omega$$

When we substitute this expression for angular velocity into the formula (a) we obtain an equation of the following form:

$$N_e \frac{\left(\frac{fG}{2} + \frac{\mu G L}{4B}\right) v_2 - \left(-\frac{fG}{2} + \frac{G L}{4B}\right) v_1}{270\,\eta}.$$

This expression corresponds to the expression for the power of the exterior resistances according to formula (71) because the members in parenthesis according to formulas (7) and (7a) represent nothing more than the values of forces P_2 and P_1.

If the tractive effort of the inner track exhibits a direction which corresponds to the direction which the track is traveling, the full power which is required to execute a turn of the tank will be furnished by exterior resistances and the power used to operate the steering clutch.

The force of exterior resistances in this case will also be expressed by formula (71).

In view of the negative value of P_1, the second member in the formula is not subtracted but added.

The power used to operate the steering clutch is expressed in the following equation:

$$N_b = -\frac{P_1 (v_2 - v_1)}{270\,\eta} \text{ PS} \tag{72}$$

If we compile this power efficiency with that of exterior resistances (Formula 71), we obtain the total power efficiency utilized during the turn, if P_1 is a positive value; it amounts to:

$$N'_D = N_e + N_b = \frac{(P_2 - P_1)\,v_2}{270\,\eta} \tag{73}$$

The propulsive output of the engine N_D used to turn the tank, if the tank has a rotating steering gear, becomes just as great as the sum of the power conducted to the rear axle shaft wheels:

$$N_D = \frac{M_2 \omega_2}{75\,\eta'} + \frac{M_1 \omega_1}{75\,\eta'} \tag{a}$$

M_2 is the turning moment on the left rear axle shaft wheel (Fig. 31) in mkg which is directed back to the outer track.

M_1 is the turning moment on the right rear axle shaft wheel, in mkg, which is connected to the inner track.

ω_2 and ω_1 are the corresponding angular velocities of these gear wheels in 1/s.

η' is the coefficient of resistance which indicates the power loss of the power plant parts which are mounted between the engine and the rear axle shaft wheels.

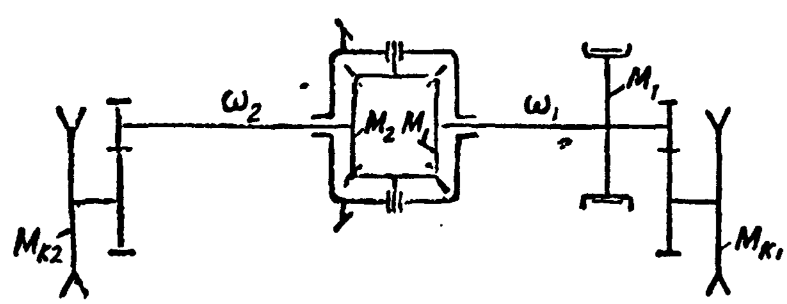


Figure 31. Turning moments and rotational velocities when turning the tank equipped with a differential.

The moments are manifest on the rear axle shaft wheels during a uniform turn of the tank equipped with a rotating steering gear:

$$M_2 = M_1$$

For this reason Formula (a) can be written in the following form:

$$N_D = \frac{M_2}{75\eta'} \cdot (\omega_2 + \omega_1) \qquad \text{(b)}$$

We wish to express turning moment M_2 by tractive effort P_2 and angular velocities ω_1 and ω_2 by the speeds of the tracks v_2 and v_1. Then we obtain the following equation:

$$M_2 = \frac{P_2 r}{i\eta''},$$

$$\omega_2 = \omega_{k_2} i = \frac{v_2 i}{3{,}6\, r}$$

$$\omega_1 = \omega_{k_1} i = \frac{v_1 i}{3{,}6\, r}.$$

In these formulas:

r = the radius of the drive sprocket in m.
i = the gear ratio of the final drive.
η'' = the coefficient indicating the power loss in the final drive and in the track.

If we substitute these values for M_2, ω_2 and ω_1 into formula (b), and perform the necessary transformations, we obtain finally:

$$N_D = \frac{P_2(v_2 + v_1)}{270\,\eta}.$$

Since now the product $\eta' \eta''$ indicates all losses, it is thus equal to η.

Formula (74) can also be written in the following form:

$$N_D = \frac{2 P_2 v_0}{270\,\eta} \qquad \text{(74a)}$$

in which $v_0 = \frac{v_1 + v_2}{2}$ is the speed of the longitudinal axis.

In this case, braking force is determined in the following manner:

$$N_T = \frac{M_T \omega_1}{75\eta'} = \frac{\omega_1}{75\eta'}\left(M_1 + \frac{M_{k_1}}{i\eta''}\right) \qquad \text{(c)}$$

M_r = the moment on the brake drum in mkg,

M_{kt} = the moment on the drive sprocket in mkg.

For M_1, M_{k1} and ω_1 we have the following formulas:

$$M_1 = M_2 = \frac{P_k r}{i \eta''}$$

$$M_{k1} = P_1 r$$

$$\omega_1 = \frac{v_1 i}{3{,}6\, r}.$$

If we substitute these expressions into formula (c), we obtain after transformations:

$$N_T = \frac{(P_2 + P_1) v_1}{270\, \eta} \tag{75}$$

The force of exterior resistance in the case of a tank equipped with a rotating steering gear will be just as great as if the tank were equipped with a steering coupling (71):

$$N_0 = N_D - N_T = \frac{P_2 v_2 - P_1 v_1}{270\, \eta}.$$

Formulas (74), (75) and (71) apply to the case that force P_1 is directed in the direction the track is traveling. In this case, it will only exhibit a negative value.

With a double differential we are dealing with the following equations for movement:

$$\omega_1 = (1 - i_0)\,\omega_0 + i_0\,\omega_3;$$

$$\omega_2 = (1 + i_0)\,\omega_0 - i_0\,\omega_3;$$

which results in:

$$\omega_1 + \omega_2 = 2\,\omega_0.$$

In these equations:

ω_1 = the angular velocity of the right rear axle shaft which is coupled to the inner track.

ω_2 = angular velocity of the left rear axle shaft, coupled with the outer track.

ω_0 = angular velocity of the differential housing.

$$i_0 = \frac{A_2 B_1}{B_0 A_1},$$

where A_3 and A_1 are the radii of the right brake and rear axle shaft wheel.

B_2 and B_1 are the radii of the large and small transmission wheels.

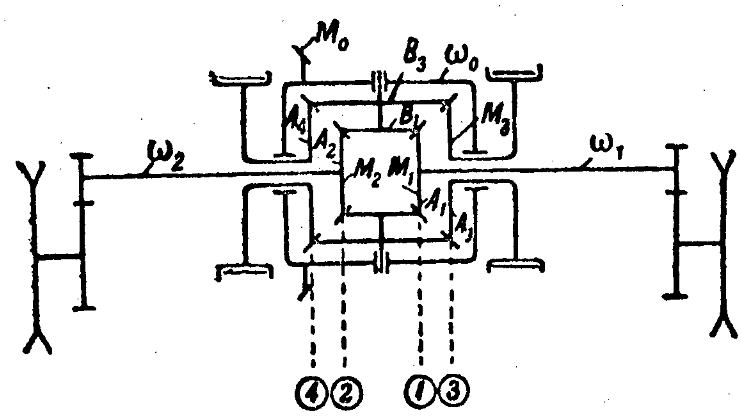


Figure 32. Schematic representation of the double differential gear (Clectrac).

The engine power required to execute a turn of the tank having a double differential gear is:

$$N_D = \frac{M_0 \omega_0}{75 \eta'} = \frac{M_0 (\omega_1 + \omega_2)}{2 \cdot 75 \eta'} \tag{a}$$

M_o is the turning moment in mkg directed to the differential gear.

According to the conditions of equilibrium in the differential gear we have the following equation:

$$M_0 = M_2 - M_1 + M_3 \tag{b}$$

In which:

M_2 = the moment on the left rear axle shaft,

M_1 = the moment on the right rear axle shaft,

M_3 = braking moment.

In addition, we obtain the following equation according to the conditions of equilibrium:

$$\frac{M_0 B_1}{A_3} = \frac{M_2 B_2}{A_2} + \frac{M_1 B_1}{A_1}.$$

The result is:

$$M_3 = M_2 \frac{A_3 B_2}{B_3 A_2} + M_1 \frac{A_3 B_1}{B_3 \, l_1} = M_2 i_0 + M_1 i_0. \qquad (c)$$

If we replace M_3 in formula (b) by the expression obtained from formula (c), we will obtain the following equation:

$$M_0 = M_2 (1 + i_0) - M_1 (1 - i_0) \; . \; . \qquad (76)$$

If we substitute the expression obtained for M_0 into formula (a) and substitute their values according to the above equations instead of M_2, M_1, ω_2, ω_1; after transformations we obtain the formula for propulsive output in the final form:

$$N_D = \frac{v_1 + v_2}{540 \eta} [P_2 (1 + i_0) - P_1 (1 - i_0)] \; . \qquad (77)$$

We can also write this equation in another form:

$$N_D = \frac{v_0}{270 \eta} [P_2 (1 + i_0) - P_1 (1 - i_0)] \; . \; . \qquad (77a)$$

The power efficiency of the exterior resistances, just as in the case of other steering gears, will be expressed in the following equation:

$$N_0 = \frac{P_2 v_2 - P_1 v_1}{270 \eta}.$$

Braking force in the case of adouble differential gear will be expressed in the following equation:

$$N_T = N_D - N_0 = \frac{P_2 - P_1}{540 \eta} [(v_1 + v_2) i_0 + v_1 - v_2] \qquad (78)$$

The tank with the double differential gear will then be able to execute the turn with the smallest possible radius, if the brake drum is firmly applied. Since in this case $\omega_3 = 0$, we obtain the following equation for the movement of the double differential gear:

$$\frac{\omega_2}{\omega_1} = \frac{1 + i_o}{1 - i_o}.$$

On the other hand, we have the following equation:

$$\frac{\omega_2}{\omega_1} = \frac{v_2}{v_1} = \frac{R_{min}}{R_{min} - B}.$$

Accordingly, we obtain:

$$\frac{1 + i_o}{1 - i_o} = \frac{R_{min}}{R_{min} - B}. \tag{79}$$

If we study formula (77a) more closely, we will come to the conclusion that the full power efficiency when turning with a double differential gear will be smaller, the smaller the gear ratio i_o.

On the other hand, it is clear from formula (79) that the smallest possible turning radius R_{min} will be larger with the reduction of i_o.

We wish to refer to the fact that in the case of $i_o = 1$, the formula (77) of power efficiency N_D will have the same appearance as formula (74) which was derived for the determination of the full power efficiency for a turn with a simple differential gear. The smallest radius in this case will be as large as the track width.

In the case of tracked vehicles, the smallest turning diameter R_{min} is usually 2 B. This value of R_{min} is determined from the same idea that when R_{min} is smaller than 2 B, the power required for the turn is much greater, whereas when R_{min} is larger than 2 B, maneuverability already declines significantly.

If we assume that $R_{min} = 2$ B, and substitute this in formula (79) we will find that $i_o = 1/3$.

If we substitute this value i_o into formula (79) we will thus obtain the value of N_D for the appropriate R_{min}; from now on this value will be used in further derivations:

$$N_D = \frac{v_1 + v_2}{3 \cdot 270\,\eta}\,(2\,P_2 - P_1) \tag{80}$$

or

$$N_D = \frac{2 + v_0}{3 \cdot 270\,\eta}\,(2\,P_2 - P_1). \tag{80a}$$

Since the speed of the outer track v_2 at the same engine rotational speed for tanks with steering couplings equals v_o the average speed for single and double differential gears, according to the formulas for the propulsion force (68) and (74a), we can conclude immediately that with the same preconditions only half the power efficiency is needed for tanks with steering couplings than for tanks with single differential gears.

Also according to formulas (68), (74a) and (80), we notice by a comparison that, considering P_2 almost equals P_1, the necessary power efficiency for turning will be smaller for double differential gears as well as for steering couplings than will be the case for a tank equipped with a simple differential gear.

In order to be able to compare the power required with a uniform turn at various radii on level ground without considering centrifugal force, Tables 10, 11 and 12 present data for tanks with various steering gears.

In compiling these tables, the rotational speeds of the engine were equal in all cases; the rotational speeds were uniform and correspondingly the maximum power efficiency was assumed:

Data for the tank and ground:

Weight of the tank	G = 10,000 kg
Speed in straight ahead travel	v = 10 km/h
Track length	L = 2.8 m
Track width	B = 2.0 m

Centrifugal force is assumed to be 0. $A = 0{,}07$; $\mu = 0{,}05$; $\eta = 0{,}75$.

In the first columns of these tables the power is calculated which is required for a tank traveling straight ahead.

Table 10.

Power Distribution in Tank Turning Equipped with a Steering Coupling.

R m	v_2 km/h	v_1 km/h	P_2 kg	P_1 kg	N_D PS	N_0 PS	N_T PS
∞	10	10	350	350	34,5	34,5	—
[illegible]	10	10	2100	1400	103,7	34,5	69,2
50	10	9,6	2100	1400	103,7	37,4	66,3
25	10	9.2	2100	1400	103,7	40,1	63,6
15	10	8,66	2100	1400	103,7	43,9	59,8
10	10	8,0	2100	1400	103,7	48,4	55,3
4	10	5	2100	1400	103,7	69,1	34,6
2	10	0	2100	1400	103,7	103,7	0

Table 11.

Power Distribution when Turning a Tank with Simple Differential Gear.

Rm	V_2 km/h	V_1 km/h	P_2 Kg	P_1 Kg	N_D PS	N_0 PS	N_T PS
∞	10	10	350	350	34,5	34,5	—
∞	10	10	2100	1400	207,4	34,5	172,9
50	10,2	9,8	2100	1400	207,4	38,0	169,4
25	10,4	9,6	2100	1400	207,4	41,4	166,0
15	10,7	9,3	2100	1400	207,4	46,6	160,8
10	11,11	8,89	2100	1400	207,4	53,7	153,7
4	13,34	6,67	2100	1400	207,4	92,2	115,2
2	20	0	2100	1400	207,4	207,4	0

Table 12.

Power Distribution when Turning a Tank with a Double Differential Gear

Rm	V_2 km/h	V_1 km/h	P_2 Kg	P_1 Kg	N_D PS	N_0 PS	N_T PS
∞	10	10	350	350	34,5	34,5	—
∞	10	10	2100	1400	92,2	34,5	57,7
50	10,2	9,8	2100	1400	92,2	38,0	54,2
25	10,4	9,6	2100	1400	92,2	41,4	50,8
15	10,7	9,3	2100	1400	92,2	46,6	45,6
10	11,11	8,89	2100	1400	92,2	53,7	38,5
4	13,34	6,67	2100	1400	92,2	92,2	0
2	—	—	—	—	—	—	—

From these tables it is clear that the propulsive power in the case of all steering gears which is needed to turn the tank (without considering centrifugal force and at a constant value of μ) is not related to the turning radius but remains constant.

In order to turn a tank equipped with a simple differential gear we need greater propulsive power than for tanks with a double differential gear and steering coupling. A tank equipped with a double differential gear requires the least propulsive power in order to turn.

The force of exterior resistances and braking force change with the change of the turning radius.

The force of exterior resistances is increased with a decrease of the turning diameter in the case of all steering gears. In contrast to this, braking force is decreased with a decrease of the radius and will have a value of zero at the possible minimum value of the turn.

From the power diagram (Fig. 33) which was drawn according to the data in the tables, it is clear that the most rapid change in the force of exterior resistances and braking power occurs at small turning radii. In addition, the force of exterior resistance is somewhat smaller when a tank equipped with steering gears is turned than when a tank with differential gears is turned. This is due to the fact that a tank with a differential gear executes the turn more rapidly than a tank with a steering coupling.

To increase the required power N_o in the case of a tank with a simple differential gear, braking force makes a great contribution in comparison with the other steering gears.

All these data make it clear that the simple differential gear used as a steering gear is almost unsuitable. Because of its relative simplicity it can perhaps be used in exceptional cases on particularly light tanks if these tanks have only a slight value ratio L/B and a large power reserve in the engine.

Even if the double planetary gear showed somewhat better results in regard to power losses than the steering coupling, it is still much more complex and much more expensive than steering clutches. Therefore it cannot be used in tanks because it reduces the maneuverability of the tank in comparison with the steering clutches (R_{min} greater than B).

For this reason in recent times steering clutches have found preferential use in tanks and towing vehicles.

b) Relationship between power during a uniform turn, coefficients f and μ and the turning radius R (without considering the influence of centrifugal force).

In the formulas for power loss with various steering gears, we wish to replace tractive efforts P_2 and P_1 by their values according to formulas (7) and (10) and to replace their velocities v_2 and v_1 according to these formulas by constant speeds v_o in the case of rotating steering gears and v_2 in the case of steering couplings.

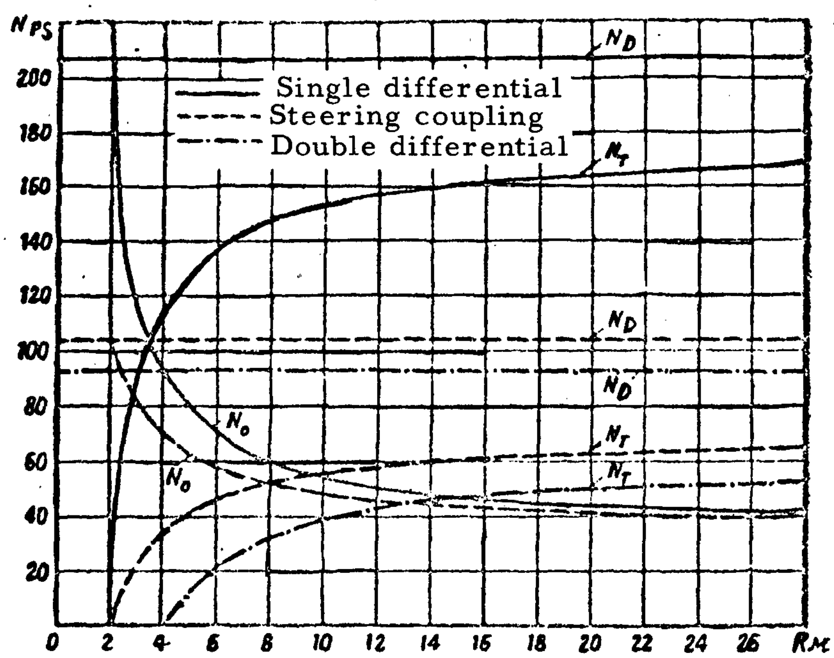


Figure 33. Power Diagram for Various Steering Gear Designs.

Then, after the appropriate transformations, we obtain the following equations:

a) For steering clutches:

$$N_D = \frac{G v_2}{270\,\eta}\left(\frac{1}{2}+\frac{\mu L}{4 B}\right) \qquad (81)$$

$$N_o = \frac{G v_2}{270\,\eta}\left[\left(1-\frac{B}{2 R}\right)+\frac{\mu L}{4 B}\right] \qquad (82)$$

$$N_T = \frac{G v_2}{270\,\eta}\left(\frac{R-B}{R}\right)\left(-\frac{1}{2}+\frac{\mu L}{4 B}\right) \qquad (83)$$

b) For the simple differential gear:

$$N_D = \frac{2 G v_0}{270\,\eta}\left(\frac{1}{2}+\frac{\mu L}{4 B}\right) \qquad (84)$$

$$N_o = \frac{G v_0}{270\,\eta}\left(1-\frac{\mu L}{2(2 R-B)}\right) \qquad (85)$$

$$N_T = \frac{G v_0}{270\,\eta}\left(\frac{R-2 B}{2 R-B}\right)\frac{\mu L}{3 B} \qquad (86)$$

c) For the double differential gear at $R_{min} = 2\ B$:

$$N_D = \frac{G v_0}{270\,\eta}\left(f+\frac{\mu L}{6 B}\right) \qquad (87)$$

$$N_o = \frac{G v_0}{270\,\eta}\left(1+\frac{\mu L}{2(2 R-B)}\right) \qquad (88)$$

$$N_T = \frac{G v_0}{270\,\eta}\left(\frac{R-2 B}{2 R-B}\right)\frac{\mu L}{3 B} \qquad (89)$$

From these equations it is clear that the propulsion force and the force of exterior resistances will increase in all cases with an increase of coefficients f and μ and that in the case of a tank with a differential gear, braking force is in no way related to coefficient f.

On the other hand, braking efficiency declines in the case of a tank with steering clutches if the value of f increases.

The braking efficiency of all steering gears will increase with an increase of coefficient μ.

From these formulas it is also possible to detect the relationship between power loss and turning radius R. This relationship was already dealt with earlier and is presented in the diagrams (Fig. 33).

We now wish to write down formula (82) in the following form:

$$N_0 - \frac{fG_2}{270\eta} = \frac{Gv_2}{270\eta} \frac{\mu L - 2B}{4} \frac{1}{R} \tag{90}$$

With a closer consideration we recognize that the curve of the relationship between the force of exterior resistances and the turning radius (Fig. 33) represents a hyperbola, the asymptotes of which stand at a distance of $fGv_2/270\eta$ (power in straight ahead travel) from the ordinate axis and the abscissa axis.

We now wish to transform this formula (83) in the following manner:

$$N_T - \frac{Gv_2}{270\eta}\left(-\frac{1}{2} + \frac{\mu L}{4B}\right) = \frac{Gv_2}{270}\left(\mu L - \frac{f}{2}\right)\frac{B}{R} \tag{91}$$

We also see from this formula that the curve of dependence on braking efficiency and turning radius is likewise a hyperbola. Its asymptotes are formed by the ordinate axis (in the direction of the negative signs) and by the line which goes straight to the abscissa axis (in the positive direction) at a distance of $P_1 v_2/270\eta$.

We will likewise determine that the curves of dependence on the force of exterior resistances and braking force on the one hand, and R on the other hand is a hyperbola in the case of tanks with differential gears.

c) Influence of Centrifugal Force on Power Consumption during a Uniform Turn.

In order to gain a concept of the influence of centrifugal force on power consumption when a tank equipped with steering clutches is turned, Tables 13 and 14 present a compilation of the results of calculations presented in formulas (68), (70) and (71); they give the power consumption of tanks with steering clutches.

For the purpose of considering centrifugal force, the following values of P_2 and P_1 were substituted into these formulas (according to formulas (16) and (16a) and C' = 0.)

$$P_2 = fG_2 + \frac{\mu G L}{4B} - \frac{C^2 L}{4\mu G B}$$

$$P_1 = -fG_1 + \frac{\mu G L}{4B} - \frac{C^2 L}{4\mu G B}.$$

where the following equations exist according to formulas (15), (15a) and (12):

$$G_2 = \frac{G}{2} + \frac{C h}{B}$$

$$G_1 = \frac{G}{2} - \frac{C h}{B}$$

$$C = \frac{m v_c^2}{3{,}6^2\left(R - \frac{B}{2}\right)} = \frac{m v_2^2\left(R - \frac{B}{2}\right)}{3{,}6^2 R^2}.$$

The following basic values have been selected for the tank and for the ground:

Weight of the tank	G	=	10,000 kg
Speed	v	=	10-20 kg/h
Length of the track support surfaces	L	=	3.0 m
Track width	B	=	2.0 m
Height of the center of gravity	h	=	1.00 m

$f = 0.06$; $\mu = 0.5$; $\eta = 0.75$.

Table 13.

Compensation of power efficiency during a uniform turn of the tank with steering clutches and with consideration of centrifugal force (v = 10 km/h).

Rm	V_a km/h	V_i km/h	P_a kg	P_i kg	N_D PS	N_a PS	N_T PS
∞	10	10	300	300	29,6	29,6	0
∞	10	10	2175	1575	107,4	29,6	77,8
15	10	8,66	2175	1575	107,4	40,1	67,3
10	10	8,0	2160	1560	106,7	45,1	61,6
6	10	6,67	2150	1480	106,2	57,5	48,7
4	10	5,0	2180	1370	107,7	73,9	33,8
2	10	0	2520	870	124,5	124,5	0

Table 14.

Compensation of power efficiency during a uniform turn of the tank with steering clutches and with consideration of centrifugal force (v = 20 km/h).

Rm	V_a km/h	V_i km/h	P_a kg	P_i kg	N_D PS	N_a PS	N_T PS
∞	20	20	300	300	- 59,2	59,2	0
∞	20	20	2175	1575	214,8	59,2	155,6
40	20	19	2100	1500	207,4	66,6	140,8
30	20	18,67	2050	1450	201,4	67,7	133,7
20	20	18,0	2030	1390	200,5	77,0	123,5
15	20	17,33	2000	1320	197,6	84,8	112,8
10	20	16,0	1810	930	178,8	103,8	75,0
6	20	13,34	1514	— 190	168,0	161,8	6,2[1]
4,8	20	11,67	1480	— 1120	157,0	210,9	46,1[1]

1) Power efficiency during slip.

From Tables 13 and 14 as well as Fig. 34, it is clear that centrifugal force exerts no influence whatever on power consumption if traveling speed is low (up to 4 km/h) and the turning radius is more than 4 m.

As soon as traveling speed is increased to above 10 km/h, the influence of centrifugal force on power consumption becomes noticeable. Initially it is noticeable by a reduction of power efficiency of 20% as well as in the rapid increase of turning radii.

If we compare Figs. 33 and 34, it becomes clear that the power efficiency of the exterior forces is somewhat greater (if centrifugal force is taken into account) while on the other hand, braking force is somewhat less than the power efficiency achieved without taking centrifugal force into account.

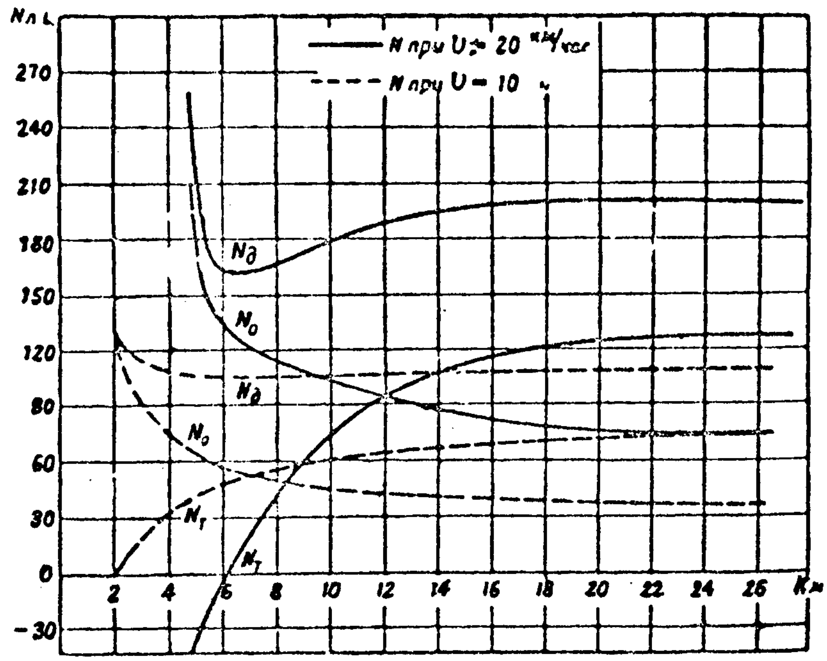


Figure 34. Equalization of Force with Consideration of Centrifugal Force.

Since the change of power efficiency as a result of centrifugal force is relatively insignificant, it can be neglected in approximate calculations, all the more as the power loss amounting to 20% is equalized completely.

d) <u>Force requirement during a uniform turn when climbing uphill.</u>

The power required to execute a turn on inclined terrain is given by formulas with which we are already familiar.

In the case of tanks with steering clutches, it is given in formulas (68), (70) and (71).

The values of P_2 and P_1 must be selected at various positions of the tank on sloping terrain according to formulas (21), (21a), (25), (25a), (31) and (31a).

In order to be able to make a comparison of the power consumption for turning a tank with steering clutches on upward sloping terrain at various positions of the tank, Table 15 has been set up which is a compilation of the data given in Tables 5, 6 and 8.

The following basic data were assumed for the tank and the ground when the table was compiled:

Weight	G = 10,000 kg
Speed	v = 10 km/h
The relationship between the support surface of the track and track width	L/B = 1.45

$f = 0.06; \quad \mu = 0.5; \quad \eta = 0.05$

Table 15.

Power consumption during the uniform turn of a tank on upward sloping terrain.

α	$\psi = 0$			$\psi = 45°$			$\psi = 90°$		
	N_D	N_0	N_T	N_D	N_0	N_T	N_D	N_0	N_T
7°	132,4	98,3	34,1	117,5	80.8	36,7	96,4	42,7	53,7
13° 30'	154,0	146.1	7,9	118.4	111,7	6,7	77,6	38,5	39,1
19° 45'	194.5	189.9	4,6*	145,1	137,4	7,7*	51,4	32,6	18,6
25° 30'	239,0	227,5	11,5*	176,7	160,4	16,3*	19,8	9,9	9,9*

Table 15 gives the power efficiency values designated with asterisks in the braking force column; these values are required during "slip" of the steering clutches.

These values correspond to the values of P_1 which are provided with a minus sign in Tables 5, 6 and 8. The power efficiencies during "slip" which are designated by asterisks were calculated according to formula (72). The corresponding propulsive output N_D in these lines were calculated according to formula (73).

When we consider Table 15 and Fig. 35, we see that the propulsive output N_D during a turn of the tank on the appropriate upward sloping terrain, insofar as is necessary to turn the tank, declines on the inclined plane with an increase of the angle of turn ψ and is at its minimum value in a purely inclined position ($\psi = 90°$).

This decline of power N_D occurs due to a decline of exterior resistances.

If, on the other hand, the angle of inclination α becomes larger at the first and second position of the tank (at ψ = 0 and ψ = 45°), power will become greater due to the significant increase of exterior forces.

The opposite phenomenon is observed at a purely sloped position, i.e. power N_D declines with a decrease of the angle of inclination. This occurs due to the reversed effect of weight transverse components.

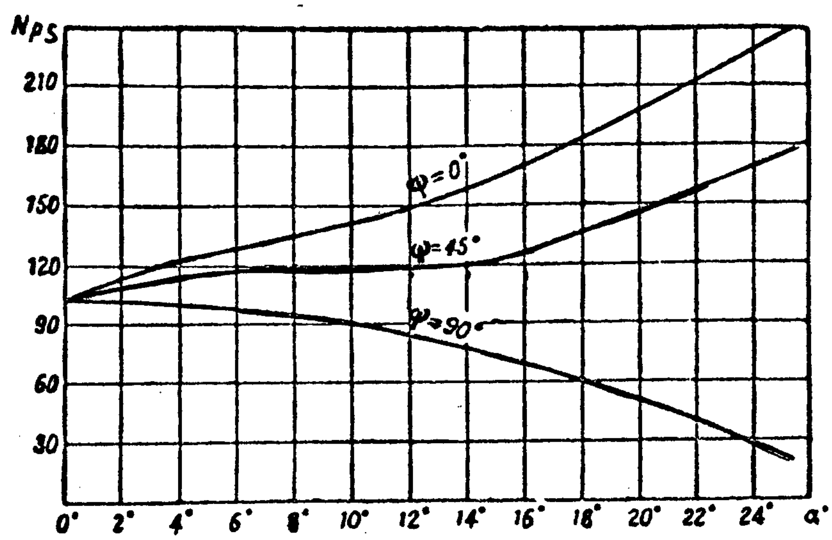


Figure 35. Power Consumption during a Uniform Turn of the Tank on Upward Sloping Terrain.

It will also be observed here that in both the first instances braking force is decreased with an increase of the angle of inclination and finally reaches the value 0, then becomes stronger again due to "slip" of the steering clutch and thus exerts a significant influence on power consumption.

4. Preconditions for Tank Mobility

a) Tank mobility without applying the brakes.

We have already referred to the fact that in many cases a tank in uniform travel on upward sloping terrain can execute a turn without braking the inner track.

This phenomenon can also be observed when turning a tracked vehicle on level terrain, if the tracked vehicle is being towed. The turn is accomplished merely by disengaging the steering clutches.

This occurs because in the first case the weight components G sin α and in the second case tractive effort on the hook which is directed opposite the direction of travel (movement), together with tractive effort P_2 of the outer track, generate a sufficient turning force to overcome the resistance moment against turning.

A tank can execute a uniform turn on level ground without braking the inner track, only under the following condition:

$$P_1 = -f\frac{G}{2} + \frac{\mu G L}{4B} \leq 0,$$

which results in the following formula:

$$\frac{B}{L} \geq \frac{\mu}{2f} \tag{92}$$

Formula (92) gives us the mutual ratio of the dimensions of B and L on the one hand and the coefficients ψ and μ on the other hand, at which the tank can execute a uniform turn on level ground without braking the inner track.

When we assume that $f = 0.07$ and $\mu = 0.5$, we will obtain the following equation:

$$\frac{B}{L} \geq 3{,}57.$$

In the case of $f = 0.1$ and $\mu = 0.6$, we will obtain the following:

$$\frac{B}{L} \geq 3{,}0.$$

This means that track width B must be 3-3.5 times larger than the length of the track support surface L in order to execute a turn without braking.

But such a power ratio can never be achieved in actual fact. In reality L is larger than B and the ratio L/B lies between 1 and 1-1/2.

For this reason the tank is always turned by braking the inner track.

If only the steering clutches are disengaged, the tank will describe a barely noticeable turn at an extraordinarily large radius.

This will occur on hard ground because the resistances against turning cause slip of the track on the ground and a slight deformation of the ground by the edges of the track links. On soft ground the coefficient μ will not reach its maximum value at once but will increase only in measure as the ground is deformed.

With the existing mutual relationships of L and B, we now wish to determine the value of the coefficient μ at which a tank can execute a turn without braking the track.

We will assume that $L/B = 1$ and $f = 0.07$, which means that we will obtain the following equation:

$$\mu \leq 0,14.$$

If $L/B = 1$ and $f = 0.1$, then

$$\mu \leq 0,20.$$

If $L/B = 1.4$ and $f = 0.07$, then

$$\mu \leq 0,1, \text{ and}$$

If $L/B = 1.4$ and $f = 0.1$, then

$$\mu \leq 0,14.$$

We must assume that when an insignificant bend in the path which the tank is following occurs, the value of the resistance coefficient against turn will fluctuate within the limits of the cited numerical values.

b) <u>Preconditions for the mobility of the tank on level ground.</u>

In order for a tank to be able to turn uniformly on level ground at an arbitrarily large radius, it is necessary above all that the power of its engine be greater than the power consumption or is equal to that value calculated according to formulas (68), (74) and (77) in which it was assumed that the values of P_2 and P_1 correspond to the following equations:

$$P_2 = \frac{fG}{2} + \frac{\mu G L}{4B}$$

and

$$P_1 = -\frac{fG}{2} + \frac{\mu G L}{4B}.$$

Consequently, it is necessary that

$$N_e \geq N_D.$$

N_c is the given power rating of the engine.

For a tank with steering clutches, this precondition can be written in the following form:

$$P_D \geq \frac{fG}{2} + \frac{\mu G L}{4B},$$

that means that the tractive effort which the engine can generate on the outer track must be equal to or greater than that required to perform a uniform turn.

If we divide the tractive effort P_2 of the outer track at turning by the tractive effort P, which is required for straight ahead travel, we will obtain the following equation:

$$\frac{P_2}{P} = \frac{\frac{fG}{2} + \frac{\mu G L}{4B}}{fG} = \frac{1}{2} + \frac{\mu L}{4fB} \tag{93}$$

If we assume that $L/B = 1.4$, $\mu = 0.5$ and $f = 0.07$, we will obtain the following result:

$$\frac{P_2}{P} = 3.0.$$

From this we can see that the resistance against turn on level ground is approximately three times as great as the resistance against straight-ahead forward travel. This means that the execution of a uniform turn at an arbitrary diameter is impossible by shifting to a higher gear.

Disregarding formula (93), it is necessary for the performance of a turn for care to be taken that the outer and inner tracks have ground adhesion. This second precondition is expressed as follows:

$$\frac{\varphi G_2}{2} \geq P_2 \tag{94}$$

$$\frac{\varphi G_1}{2} \geq P_1 \tag{94a}$$

Since the tractive effort P_2 during a uniform turn will always be greater than P_1, while G_1 declines more slowly than P_1 when centrifugal force is exerting an influence, both the aforementioned preconditions can be replaced by one, in which the beginning of the turn is observed when the preconditions are the most difficult:

$$\frac{\varphi G}{2} \geq \frac{fG}{2} + \frac{\mu G L}{4B}.$$

From this we will finally obtain the formula for mobility with regard to ground adhesion:

$$\varphi \geq f + \frac{\mu L}{2B} \tag{95}$$

If we assume that L/B = 1, μ = 0.7 and f = 0.1 (soft ground), we will obtain the necessary value of the coefficient for the adhesion of the tracks on the ground:

$$\varphi \geq 0{,}45.$$

If L/B = 1.4, we obtain the following formula for the turn on the same ground (μ = 0.7 and f = 0.1):

$$\varphi \geq 0{,}59.$$

Since the adhesion coefficient φ for tanks on soft ground fluctuates between 0.5 and 0.6, we can conclude from the cited examples that adhesion is present in the first case; here the required check of the condition for mobility with reference to adhesion will be:

$$\varphi \geq 0{,}31$$

If L/B = 1.4, f = 0.06 and μ = 0.5, we will arrive at the following result:

$$\varphi \geq 0{,}41.$$

On hard ground the adhesion coefficient will fluctuate between 0.6 and 0.8. Consequently, we can assume that the tracks will adhere sufficiently to the ground when a turn is being executed on hard ground.

c) Preconditions for the mobility of a tank on sloped terrain.

In order for the tank to be able to execute a uniform turn on upward sloping terrain as well as on level terrain, it is necessary that

$$N_e \geq N_D$$

In the case of steering clutches this precondition can be expressed as follows, if P_1 is a positive value:

$$P_D \geq \frac{fG\cos\alpha}{2} + \frac{\mu G L \cos\alpha}{4B} K + \frac{G \sin\alpha}{2}.$$

If P_1 is a negative value, the form of mobility with reference to the engine will appear as follows:

$$P \geq P_2 - P_1 \tag{96}$$

$$P_d \geq fG\cos\alpha + G\sin\alpha, \tag{96a}$$

i.e. it will be identical with the formula for forward movement. This means that in this case a tank will have enough power for a turn if the engine has some excess power when climbing uphill.

We wish to divide tractive effort P_2, which is necessary for a turn on upward sloping terrain (Formula 21), by the tractive effort required for uniform travel on upward sloping terrain:

$$\frac{P_2}{P} = \frac{\frac{fG\cos\alpha}{2} + \frac{GL\cos\alpha}{4B}K + G\frac{\sin\alpha}{2}}{fG\cos\alpha + G\sin\alpha}$$

or

$$\frac{P_2}{P} = \frac{1}{2} + \frac{\mu L}{4B}\frac{K}{f + \operatorname{tg}\alpha} \tag{97}$$

From this formula we can see that the ratio P_2/P declines with an increase of the angle of inclination. This means that the probability of being able to execute the turn without shifting back will increase with the power efficiency of the engine.

If we assume $L/B = 1.4$; $f = 0.07$; $\mu = 0.5$; we will obtain for the angle of inclination $\alpha = 13^{\circ}$, $K = 0.94$ (Formula 33) and

$$\frac{P_2}{P} = 0.5 + \frac{0.5 \cdot 1.4}{4} \cdot \frac{0.94}{0.07 + 0.23} = 1.05$$

The result is that a tank which is traveling uphill at an angle of inclination of 13° with a small power reserve, can execute a turn on this uphill travel without shifting gears.

If the angle of inclination is greater than 13°, the possibility for the tank to be able to execute the turn without changing gears will persist.

When the angle of inclination declines below 13°, the value of P_2 will increase; that is to say, the possibility of turning uniformly without shifting will be reduced.

At $\alpha = 0$ and $K = 1$, Formula (97) will have the same appearance as Formula (93) which under the same preconditions on our last cited example results in a triple increase of the force P_2 in comparison with P.

By comparing P_2 according to Tables 5, 6 and 8, it is clear that the reduction of tractive effort in the outer track during transition of the tank from the first into the third position occurs much more rapidly than the reduction of adhesion weight being lost by the outer track.

We see from the same formulas (21), (21a), (27), (31) and (31a) that in all positions of the tank during a turn on upward sloping terrain, P_2 is larger than P_1.

In addition, we know that P_2 achieves its maximum limit during the first time segment of the turn, when the longitudinal axis of the tank forms the maximum angle with the horizontal plane.

We conclude from all these data that the formula for the mobility of the tank with reference to adhesion during uphill travel is

$$\varphi \frac{G\cos\alpha}{2} \geq \frac{G\cos\alpha}{2} + \frac{\mu G L \cos\alpha}{4B} K + \frac{G\sin\alpha}{2}$$

or

$$\varphi \geq f + \frac{\mu L}{2B} K + \operatorname{tg}\alpha \tag{98}$$

In the case of $\alpha = 0$ and $K = 1$, we obtain Formula (95) according to Formula (98) which we have derived as a precondition for mobility with reference to ground adhesion on level ground.

According to Formula (98) we recognize that the adhesion coefficient φ is larger, the larger the angle of inclination α.

With $K = 1$ and appropriate transformation of Formula (98), we obtain:

$$\operatorname{tg}\alpha < \varphi - f - \frac{\mu L}{2B} \tag{99}$$

Formula (99) can serve as an equation for determining the maximum angle of inclination at which the tank can execute a turn on the appropriate base.

If we assume that $L/B = 1.4$, $\mu = 0.5$, $f = 0.06$ and $\varphi = 0.7$, we obtain:

$$\operatorname{tg} \alpha \leq 0{,}29,$$
$$\alpha \leq 16°.$$

If we assume at the same preconditions that $L/B = 1$, we obtain:

$$\operatorname{tg} \alpha \leq 0{,}39$$
$$\alpha \leq 21°.$$

The maximum angle of rectilinear climb is expressed on this ground in the following formula:

$$\operatorname{tg} \alpha \leq \varphi - f.$$

If we now substitute the same values for K and f, we obtain:

$$\operatorname{tg} \alpha \leq 0{,}64,$$
$$\alpha \leq 33°.$$

From this we conclude that the maximum angle of inclination at which a turn of the tank can be executed at appropriate ground traction is almost only half as large as the maximum angle of inclination at which the tank can travel in a straight direction.

d) Minimum turning radius

Fig. 9 gives a schematic representation of the tank cross section showing the forces being exerted on it during a uniform turn.

Admittedly centrifugal force C has an effect on the transverse forces of the ground S_2 and S_1 and change the usual reactions of the ground against the tracks Q_2 and Q_1.

The equations for the equilibrium of forces and of resistance moment with reference to point 4 is written down in the following form:

$$Q_1 + Q_2 = G \qquad \text{(a)}$$
$$S_2 + S_1 = C \qquad \text{(b)}$$
$$G\frac{B}{2} - Ch - Q_1 B = 0 \qquad \text{(c)}$$

The greater the centrifugal force, the greater the counteraction Q_2. In constrast to this, the reaction Q_1 will decrease with an increase of centrifugal force.

Of course the tank will begin to tip around the outer track as soon as reaction Q_1 reaches the value of zero.

For $Q_1 = 0$ in equation (c), we obtain the precondition for the tipping of the tank; it is

$$C = G\frac{B}{2h} \tag{100}$$

or, if we substitute the appropriate expression for C according to formula (12)

$$\frac{m\,v}{3{,}6^2\left(R-\frac{B}{2}\right)} = G\frac{B}{2h} \tag{101}$$

v_c is the velocity for straight ahead travel. If we are dealing with a tank with steering clutches, this formula is more practically written in the following form:

$$\frac{m\,v_2^2\left(R-\frac{B}{2}\right)}{3{,}6^2 R^2} = G\frac{B}{2h} \tag{101a}$$

According to Formulas (101) and (101a) we find according to the given velocity v_c or v_2 the maximum radius at which the tank begins to return to its proper position.

During the turn, the tank will begin to rectify itself again when centrifugal force is greater than the force of ground transverse adhesion on the tracks, where the adhesion force can be expressed in the following equation:

$$\mu' Q_2 + \mu' Q_1 = \mu' G$$

$$C = \mu' G \tag{102}$$

or

$$\frac{m\,v_c^2}{3{,}6^2 R^2\left(R-\frac{B}{2}\right)} = \mu' G. \tag{103}$$

In the case of a tank with steering clutches, it is better to write the formula in the following form:

$$\frac{m\,v_2^2\left(R-\frac{B}{2}\right)}{3{,}6^2 R^2} = \mu' G. \tag{103a}$$

According to formulas (103) and (103a) and at the given velocity v_c (or v_2) we obtain the maximum turning radius at which the tank begins to straighten out again.

If we substitute the value C from formula (102) into formula (13a) and take into account that in this case M is equal to M', we obtain the following equation:

$$X = \frac{L}{2}.$$

This means that the turning center point is shifted beyond the track support surface the moment the tank begins to tip.

With consideration of the circumstance that B/2h is usually somewhat larger than 1, while μ' is not larger than 1, we can write:

$$\mu' G < G \frac{B}{2h}. \quad \ldots \qquad (104)$$

Using formulas (101) and (103) we determine the following:

$$\frac{m v_c^2}{3.6^2\left(R_3 - \frac{B}{2}\right)} < \frac{m v_c^2}{3.6^2\left(R_0 - \frac{B}{2}\right)}$$

or

$$R_0 < R_3.$$

This means that the smallest turning radius R_3, after straightening out, is larger than the largest radius during the tip R_0. Consequently, there is no danger that a tank can tip over when executing a turn on level ground because the tank will straighten out again before the tipping motion can take effect.

e) Preconditions for a non-uniform turn.

If the power efficiency of the engine is less than that required to turn the tank uninterruptedly, the tank cannot turn continuously around an arbitrary angle at an arbitrary radius.

Under these circumstances the tank will be able to turn by using its weight. However, this turn can only be made by sufficiently braking the inner track. Thus the speed of the center of gravity, as well as the rotational speed of the engine will be smaller.

The brake must not be applied for too long a time, since the engine will choke out. The brake must be released before engine rotational velocity reaches its minimum value.

During braking the tank will turn at some definite, not too large angle.

Thereupon a certain speed must be reached again and, if necessary, the brake applied again. This turning phenomenon will be repeated.

The path of motion of the center of gravity will be a broken line which is composed of individual curves and straight lines in a certain succession.

In order that such a graduated turn be executed, it is necessary to calculate according to the following preconditions (formula):

$$\left(P + \frac{\varphi G}{2}\right)\frac{B}{2} > \frac{\mu G L}{4}$$

in which P is the tractive effort of the engine.

We now wish to determine around which angle the tank will turn in one time segment of turn.

The preconditions for a non-uniform turn in this case will agree with point "e" described in Section 2.

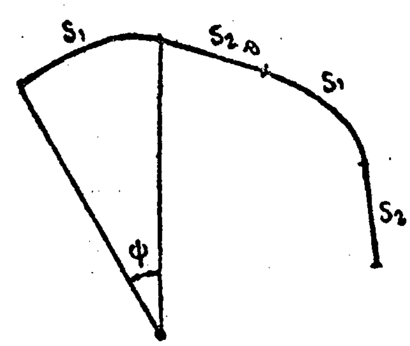


Figure 36. Path of the Tank during a Non-uniform Turn

Consequently, on the basis of formulas (a) and (46) we can write the following:

$$P - \frac{\varphi G}{2} - fG + J = 0 \qquad \text{(a)}$$

$$\left(P + \frac{\varphi G}{2}\right)\frac{B}{2} - \frac{\mu G L}{4B} = J_\varphi \psi'' \qquad \text{(b)}$$

Using these equations we can determine the angle ψ around which the tank will turn during one time segment. In the same manner as we derived formula (49), we obtain:

$$\psi = \frac{E t^2}{2}, \tag{106}$$

where $$E = \frac{1}{J_\varphi}\left[\left(P + \frac{\varphi G}{2}\right)\frac{B}{2} - \frac{\mu G L}{4B}\right]$$

The time interval t in equation (106) can be determined by the following formula:

$$v = v_0 - b\,t,$$

in which v is the minimum rotational velocity considered permissible with normal engine operation. Its value can be assumed as 1/4 v_0. Then formula (106) becomes

$$t = \frac{3}{4}\frac{v_0}{b} \tag{107}$$

The acceleration of the center point of inertia b is determined according to the following formula:

$$b = \frac{g}{\delta}\left(\frac{P}{G} - \frac{\varphi}{2} - f\right) \tag{108}$$

If we substitute b into formula (107) and then substitue t into formula (106), we will have determined the value of the angle sought ψ.

The path over which the center of gravity moves during this time will be determined by the following formula:

$$S_1 = v_0 t - b\frac{t^2}{2} \tag{109}$$

We will determine the path of the later tank movement according to the formula:

$$s_2 = \frac{\left[v_0^2 - \left(\frac{1}{4}v_0\right)^2\right]\delta m}{P - fG} \tag{110}$$

The average turning radius of the tank can be determined approximately in this case according to the formula:

$$R - \frac{B}{2} = \frac{s_1 + s_2}{\psi} \tag{111}$$

If the brake is not applied after disengaging the steering clutches, the precondition for a non-uniform turn will have the following form:

$$P_1 \frac{B}{2} > \frac{\mu G L}{4} \qquad (112)$$

If we assume that $N_c = 300$ HP, $G = 10{,}000$ kg, $L = 2.7$ m, $B = 2$, $\mu = 0.4$, $\eta = 0.75$ and $v = 15$ km/h, we will obtain the following results:

$$P_1 \frac{B}{2} = 4050 \text{ kg}; \quad \frac{\mu G L}{4} = 2800 \text{ kg}.$$

Consequently, in this case precondition (112) will be maintained.

This means that in many cases, if the engine has sufficient power reserve, a turn during travel around a certain small angle is possible without braking the inner track for this purpose (thus merely by disengaging the steering couplings).

It is only necessary here that tank speed before the turn be less than the maximum speed possible in whichever gear the tank happens to be.

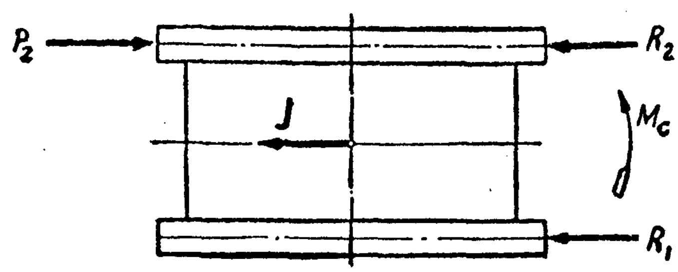


Figure 37.

In this case, the force of inertia $J = m \cdot b$ will play the role of the braking force P_1, where the force of inertia is directed backward due to the accelerated travel (Fig. 37).

The angle of turn and the path of the center of gravity in this case can be determined according to formulas (46), (47a), (48), (49), (53) and (54).

5. Coefficient of Resistance Against Turn

The coefficient of resistance μ against turn with various types of soil has been insufficiently studied as yet. Nevertheless, it can be stated that the value of the coefficient depends upon the following circumstances: on the friction of the tracks rubbing on the ground, on resistance when the grips go into the ground and on the resistance when the ground is deformed by the track edges.

Each of these circumstances will change in relation to the type of soil.

It can be assumed that the turning radius exerts an influence on the value of the coefficient μ with the same soil (base), whenever the angular velocity remains constant during the turn. A small wall of earth is built up along the tracks; it becomes higher the smaller the turning radius, which makes turning more difficult.

Determining the value of coefficient μ must be accomplished in such a way that its value is found with various soil types in relation to the tank speed and turning radius.

Today there are three main methods for determining the coefficient: the first method involves towing the tank with a winch or with a towing vehicle vertically to the longitudinal axis (Fig. 38) and reading the power efficiency on a gauge.

The number P read on the gauge is divided by the weight $\frac{P}{G} = \mu'$.

The coefficient μ' determined in this manner differs from the actual coefficient by the fact that the resistance of the grips against the ground, which plays a role in turning, is not taken into account.

It would be more correct to designate the coefficient μ' as the coefficient at lateral shift.

The coefficients of resistance quoted earlier (Table 1) are actually the coefficients μ' which were determined by making additional calculations for the resistance of the grips.

The second method involves turning the tank by force while rolling the towing hawser onto the winch drum and fastening the other end of the hawser on a derrick specially attached on the tank.

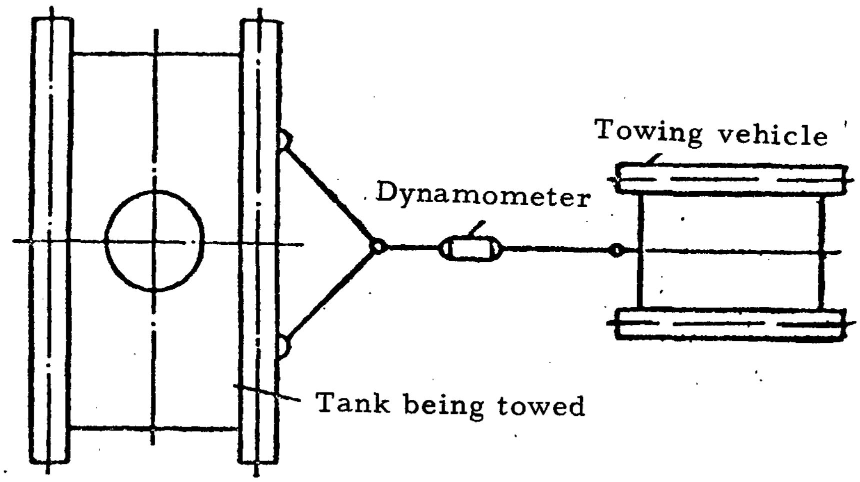


Figure 38. Schematic representation of the tank being towed by a towing vehicle in a direction vertical to the longitudinal axis for the purpose of determining the coefficient μ.

The tractive effort shown on the gauge is read at that point in time when the hawser is stretched vertically to the longitudinal axis of the tank. In addition, at the same time we measure the shift of the center point of turn X according to the resultant track left by the sliding tank.

We then calculate the value of the coefficient according to basic values P and X determined by the measurement:

$$\mu = \frac{4P(1 + X) - 2fGB}{GL\left[1 + \left(\frac{2X}{L}\right)^2\right]}.$$

The coefficient μ determined in this manner likewise differs from the correct one, but this difference is not so large as in the case of μ'.

That this μ is also imprecise is attributable to the fact that first of all the tank is turned in a non-uniform manner the entire time (it is difficult to achieve a uniform turn) and secondly to the fact that the force being exerted in transverse direction distributes the load on the tracks; it can occur that the right track bores deeply into the ground, which causes an increase of the resistance against turn.

Thirdly, the lack of precision of μ is attributable to the fact that the tracks are not stretched as tightly so that the pressure distribution pattern, which exerts a direct influence on turning resistance, is changed.

The third method in which the tow rope leads from the winch to the upper turret of the unbraked track and is joined there produced results more in conformity with reality (the second track was braked during towing) (Fig. 40).

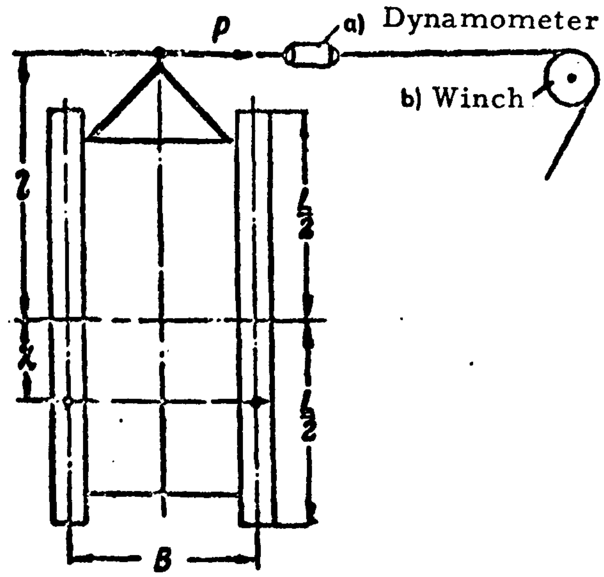


Figure 39. Forceful Turning of the Tank in Position to Determine the Coefficient μ.

The tractive effort indicated by the gauge is read in the moment the direction that the rope is being pulled corresponds to the longitudinal axis of the track.

At this moment the shift of the turning center point X is also measured.

In this case, the coefficient μ is calculated according to the following formula:

$$\mu = \frac{4P(2B + \alpha) - 2fGB}{GL\left[1 + \left(\frac{2X}{L}\right)^2\right]}.$$

It must be noted here that with all these methods it is possible to determine neither the relationships between the resistances during turn nor the radius.

The most correct method for the determination of μ would be to measure the tractive efforts on the track during turn or the turning moments on the drive sprockets.

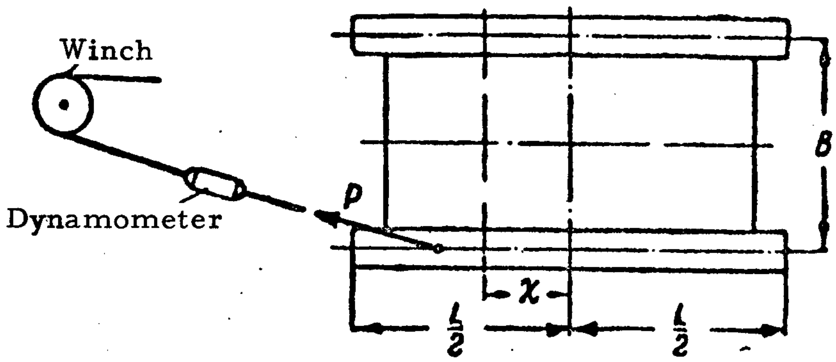


Figure 40.

Then we will calculate the proper value of μ, or in any case an approximate value according to the following formula:

$$(P_2 + P_1)\frac{B}{2} = \frac{\mu G L}{4}\left[1 + \left(\frac{2X}{L}\right)^2\right].$$

One disadvantage of this method is due to the fact that it is difficult to execute a uniform turn with a definite radius, although this disadvantage can be nullified with sufficient practice by the driver.

Section IV.

TANK TRACK AND SUSPENSION

1. Basic Types of Tank Track and Suspension

By track and suspension of the tank we mean the totality of all the individual parts of its track and suspension, by means of which the road wheels are connected with the tank hull.

The first English tanks exhibited a rigid track and suspension, i.e. a support by means of which the road wheel axes were rigidly connected to the tank without the aid of springs.

This rigid track and suspension was considered satisfactory only up to certain low speeds (5-6 km/h). On the other hand, at high speeds the tank, especially on uneven terrain, was subjected to such sharp impacts and bumps that the tank crew suffered fatigue and the working life of the tank was shortened.

For this reason, all recent tanks have already been significantly improved by equipping them with springs.

Recent tanks exist in many track and suspension designs, but almost all can be classified into three groups according to the manner in which the load is distributed on the road wheels:

1. Road wheels with individual springs,
2. Swing arm supports,
3. Bogie wheel track and suspension.

In the case of the first track and suspension, the road wheels are fastened independently on the tank hull.

Usually they are supported with coil springs.

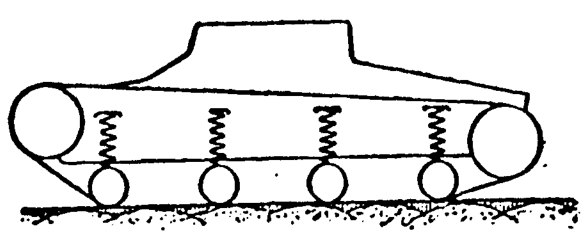
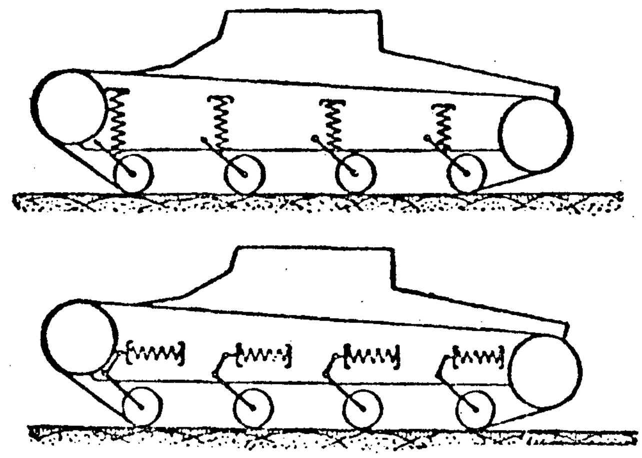
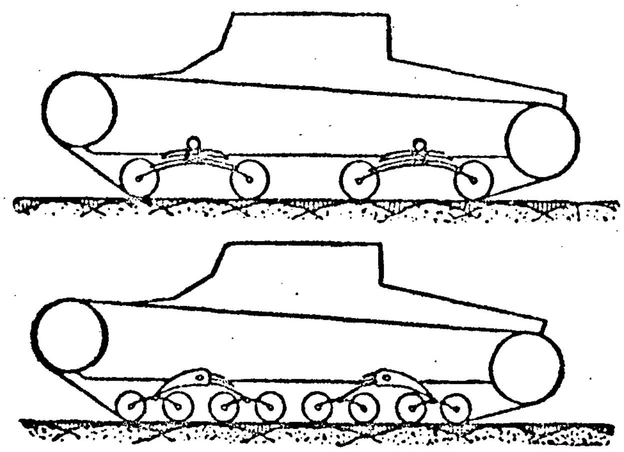


Figure 1. Road Wheels on the Tank with Individual Springs.



Figures 2 and 3. Swing Arm Track and Suspensions



Figures 4 and 5. Bogie Wheel Suspension Frame

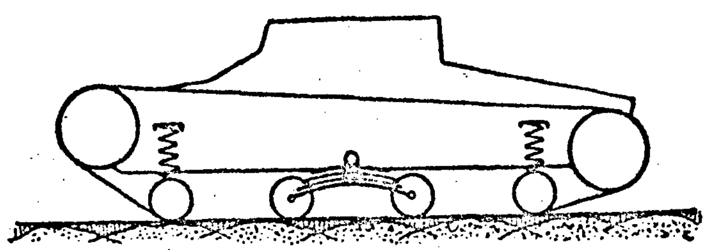


Figure 6. Combination Track and Suspension

Two intermediate levers and pull rods of various designs are fastened between the road wheels and the springs of the independent suspension (see Figs. 1, 2 and 3).

The wheels are independent of each other on this swing arm track and suspension. If every two or more road wheels have common springs, it becomes a bogie wheel suspension frame.

A suspension is designated as a combination track and suspension if some of the road wheels are independent of each other on the tank hull while others are joined by means of springs and swing arms (see Fig. 6).

2. Exterior Forces being Exerted on the Road Wheels of a Tank

When designing a tank track and suspension system it is very important to distribute the loads on the road wheels in a proper manner.

To comprehend these loads better, we wish to consider the exterior forces which exert an influence on the road wheels.

In view of the circumstance that these forces are equal on the left and right side of the tank, we wish to limit ourselves to considering those forces which are exerted on the right side.

If it is necessary to distinguish between the road wheels on the right and left side, we will designate all forces related to the right side with letters and an apostrophe, the forces exerted on the left side will be designated with letters and quote marks. Where no marks are used, this means that the force is exerted on both sides.

Any road wheel (we will designate them with the letter i), with the exception of those furthest to the rear, will be influenced by the following forces (Fig. 7).

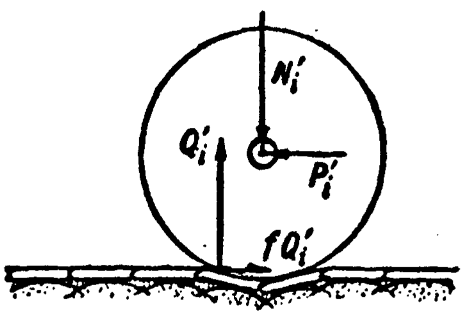


Figure 7.. Forces Acting on the Road Wheel

Normal load N' and shearing force P' are exerted from the side; the normal reaction of the ground Q' is exerted by it, while force R' is exerted over the track onto the road wheel. Force R' is rolling resistance.

Taking into account all forces being exerted on the road wheel axis, we obtain the following equations:

$$N_i' = Q_i'$$
$$P_i' = fQ_i'.$$

The rear n^{th} road wheel as well as all other road wheels is subjected to the effect of forces N_n, P_n, Q_n' and fQ_n'. In addition the stress of the lower and rear part of the track -P', which with sufficient approximation can be considered of the same value, exerts an effect on this wheel (Fig. 8).

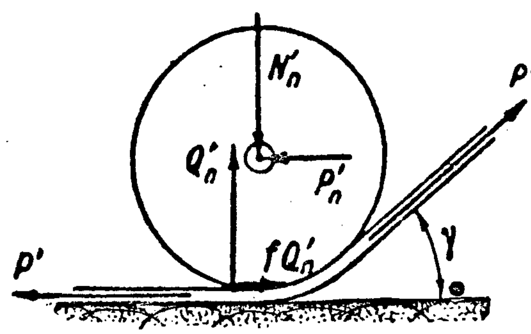


Figure 8. Forces being Exerted on the Rear Road Wheel

The operating stress of the track is generated by the propulsion force of the rotating drive sprocket and corresponds to the tractive effort on this track P''.

In straight ahead tank travel, this tractive effort amounts to

$$P' = fQ_1 + \frac{mb + G\sin\alpha}{2} \qquad \text{(a)}$$

in which Q' represents the total reaction which is equal to the adhesion weight of the tank lost by the track; this means that

$$Q' = \Sigma Q_i'$$

If we set up equations for the equilibrium of forces which are exerted on the rear road wheel, they will have the following appearance:

$$Q_n' = N_n' - P'\sin\gamma$$
$$P_n' = fQ_n' - P'(1 - \cos\gamma) \qquad (2)$$

Equations (2) prove that the normal load Q'_n of the ground by the track under the rear road wheel is smaller by $P' \sin \gamma$ than the normal load N'_n of the track through the road wheel due to the relieving effect of the slanted part of the track. Likewise, shearing force P'_n on the last road wheel is smaller than the resistance against the horizontal components $P' (1 - \cos \gamma)$ of the road wheel load $2 P' \sin \gamma/2$ which is stipulated by track tension.

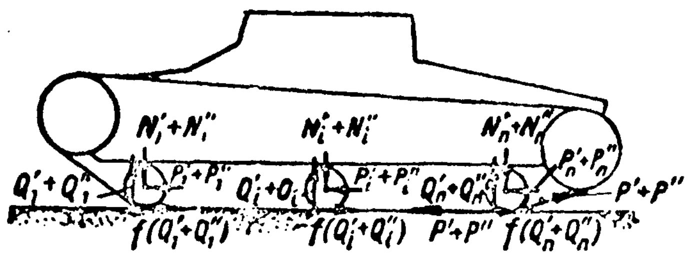


Figure 9. Track and Suspension of a Tank

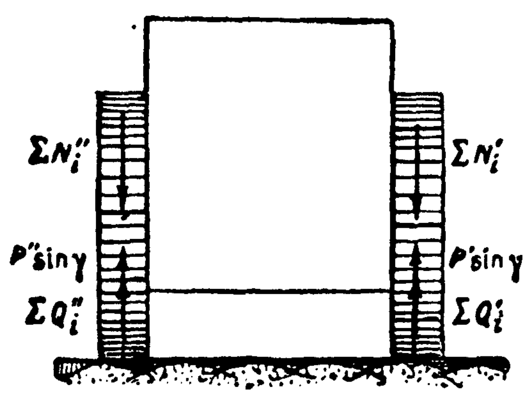


Figure 10. Force Diagram on the Track and Suspension of a Tank

The exertion of all exterior forces on the tank road wheels is shown on Figs. 9 and 10. This diagram can be simplified, however.

From mechanics we know that the end result Q of all vertical reactions of the ground on the road wheels is as great as the adhesion weight of the tank G:

$$\Sigma Q_i = \Sigma (Q'_i + Q''_i) = Q \qquad \text{(b)}$$

It is applied at the so-called pressure center point, the position of which is determined by the longitudinally-directed x and the transversely directed y-shift of the pressure center point, as opposed to the projection of the tank's center of gravity on the plane of travel.

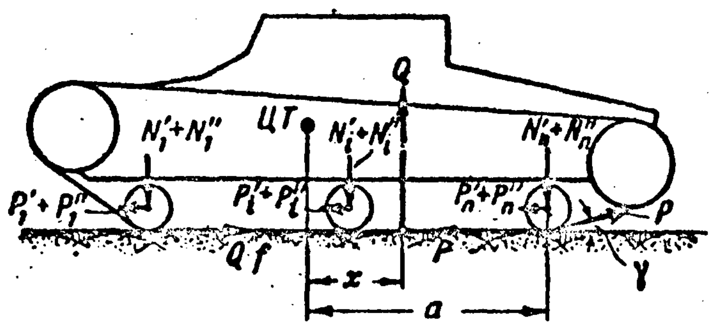


Figure 11. Forces Being Exerted on the Tank's Track and Suspension.

Where:

and

$$\left.\begin{aligned} x &= \frac{mb + G\sin\alpha}{Q}h \\ y &= \frac{G\sin\beta}{Q}h. \end{aligned}\right\} \qquad (c)$$

We wish to consider the shift of the pressure center point to the rear and the left side as positive, and the shift to the front and to the right side as negative.

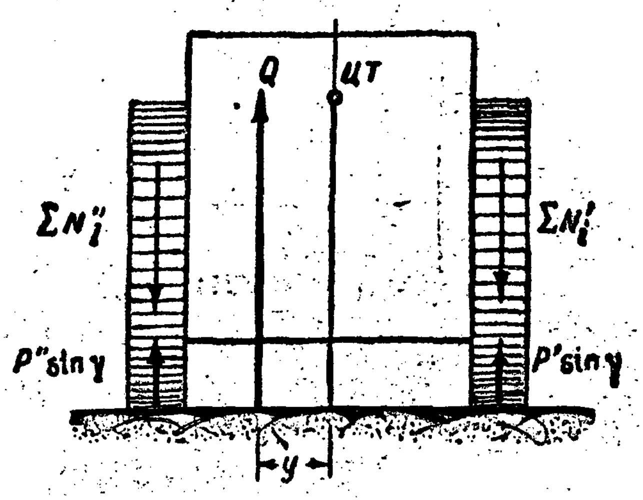


Figure 12. Forces Being Exerted on the Tank's Track and Suspension.

In this manner we can replace all forces Q_i' and Q_i'' by one force Q, which is exerted on the pressure center point of the tank.

The rolling resistance is:

$$\Sigma\left(fQ_i' + fQ_i''\right) = f\Sigma Q_i = fQ \qquad \text{(d)}$$

In this manner we can replace all forces fQ_i' and fQ_i'' by one and the same force fQ.

In addition we wish to refer to the fact that the sum of the stresses being exerted on the right and left side of the tank track is as large as the tractive effort P of the tank.

$$P' + P'' = P.$$

The simplified schematic of the effects of all exterior forces on the road wheels is shown in Figs. 11 and 12.

3. Determination of Road Wheel Loads with Independent Suspension of the Tank.

We wish to determine the normal loads N_i' and N_i'' on the road wheels when they are individually equipped with springs.

We cite the following designations:

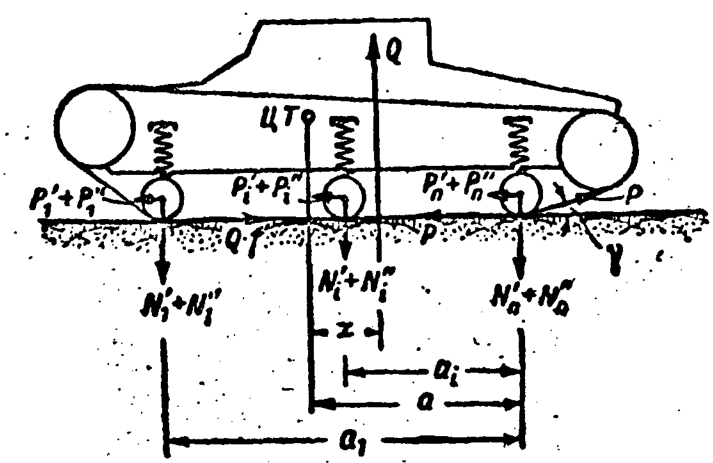


Figure 13. Forces Being Exerted on Individually Sprung Tank Road Wheels.

a_i -- the distance between the axis of any road wheel and the axis of the rear road wheel. The index i can include all values from 1 to n, where we assume for the rear n^{th} road wheel that $a_u = 0$.

a -- distance between the axis of the rear road wheel and the transverse surface of the tank which passes through the center of gravity.

B -- the tank track width (See Fig. 14).

r -- radius of the road wheel.

To determine the vertical loads N_i' and N_i'' we wish to set up equations of the forces vertical to the plane of travel and one equation of the force moments relative to the longitudinal axis through the tank center of gravity (Figs. 13 and 14).

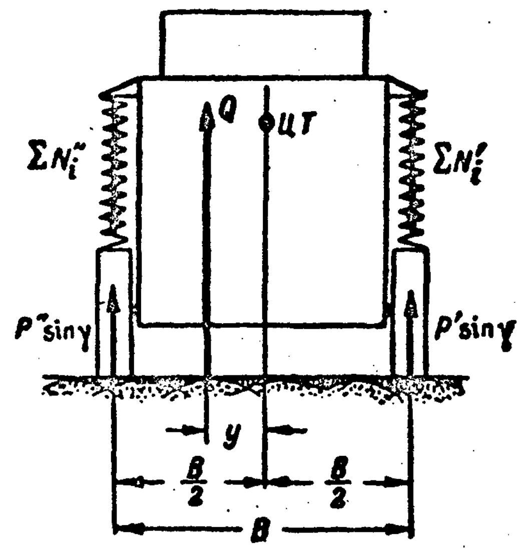


Figure 14. Forces Being Exerted on an Individually Sprung Track and Suspension.

$$Q + P\sin\gamma - \Sigma(N_i' + N_i'') = 0 \tag{3}$$

$$\Sigma(N_i' + N_i'')a_i + Q(fr - a + x) = 0 \tag{4}$$

$$\frac{B}{2}\sum(N_i' - N_i'') + Qy - \frac{B}{2}(P' - P'')\sin\gamma = 0 \tag{5}$$

Eqs. (3), (4) and (5) form a system of three equations with 2 n unknown quantities.

$$N_1', N_2', N_3' \cdots\cdots N_n'$$
$$N_1'', N_2'', N_3'' \cdots\cdots N_n''.$$

To obtain the missing equations, we wish to replace the vertical loads N_i' and N_i'' of the road wheels by the deflection of the springs f_i' and f_i''.

$$N_i' = h f_i' \quad \ldots . \qquad \text{(a)}$$

$$N_i'' = h f_i'' \quad \ldots . \qquad \text{(a)}$$

where c represents the spring constant, i.e. the relation between the spring load and its deflection.

The deflection of the springs can be expressed by the bank of the tank hull, k_x lengthwise and k_y laterally, and by the shift of the tank center of gravity downward z_o.

The height differences in the longitudinal and transverse direction are designated as angles k_x and k_y, which are formed by the longitudinal or transverse axis of the tank with the tank's plane of travel (see Figs. 15 and 16).

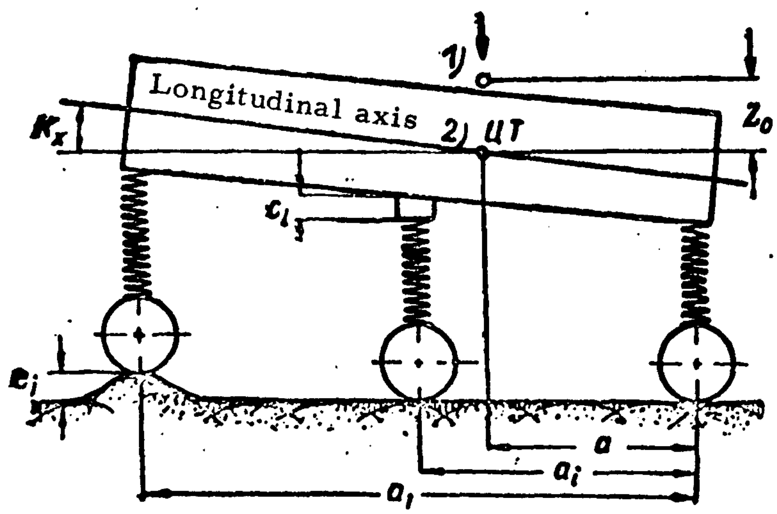


Figure 15. Longitudinal Bank of the Tank Hull

1) Position of the center of gravity with the springs unloaded.
2) Longitudinal axis (points 1 and 2) also apply to Fig. 16.

The shift of the center of gravity downward is designated by the amount of weight settling as a result of the elastic deformation of the track and suspension springs.

We now wish to select certain springs, one on the right and the other on the left side of the suspension and to express their deflection by the values k_x, k_y and k_o; we are assuming that the spring deflections are positive on the rear and on the left side:

$$\left.\begin{aligned} f_i' &= z_0 - (\alpha_i - \alpha)\sin k_x - \frac{B}{2}\sin k_y + e_i' \\ f_i'' &= z_0 - (\alpha_i - \alpha)\sin k_x + \frac{B}{2}\sin k_y + e_i'' \end{aligned}\right\} \qquad \text{(b)}$$

e_i' and e_i'' are the additional spring deflections due to the spring supports or because the adjustment plates were unscrewed or because of unevenness of the ground, (see Fig. 15).

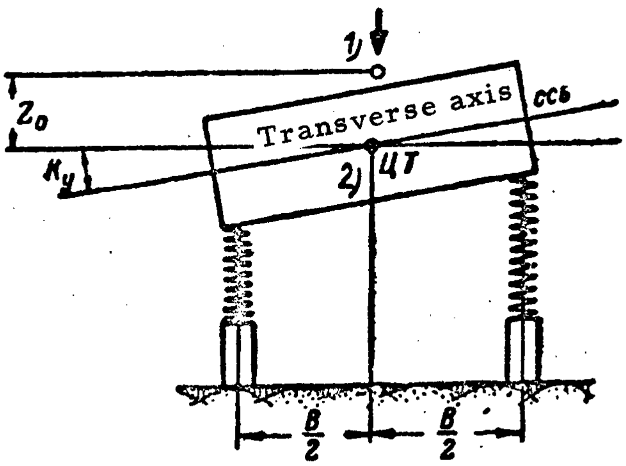


Figure 16. Lateral Bank of the Tank Hull

Usually these height differences k_x and k_y represent small values in themselves; thus we can calculate that

$$\sin k_x = k_x$$

and

$$\sin k_y = k_y$$

then the following equations will be justified:

$$\left.\begin{aligned} f_i' &= z_0 - (\alpha_i - \alpha) k_x - \frac{B}{2} k_y + e_i' \\ f_i'' &= z_0 - (\alpha_i - \alpha) k_x + \frac{B}{2} k_y + e_i'' \end{aligned}\right\} \qquad \text{(c)}$$

If we write expressions in the manner of (c) for the deflection of all springs and substitute them into equation (a), we obtain:

$$\left.\begin{aligned} N_i' &= a\left[z_0 - (\alpha_i - \alpha) k_x - \frac{B}{2} k_y + e_i'\right] \\ N_i'' &= a\left[z_0 - (\alpha_i - \alpha) k_x + \frac{B}{2} k_y + e_i''\right] \end{aligned}\right\} \qquad \text{(6)}$$

Now Eqs. (3), (4) and (5) with Eq. (6) form an equation system which is made up of $2n + 3$ independent equations with $2n + 3$ unknown quantities:

$$N_1', \; N_2', \; N_3' \cdots\cdots\cdots N_n'$$

$$N_1'', \; N_2'', \; N_3'' \cdots\cdots\cdots N_n''$$

consequently, we can calculate all unknown quantities from them.

We now wish to substitute the expressions for vertical loads of the road wheels from Eq. (6) into Eq. (5). After we have substituted these expressions and similar members into the numerator, Eq. (3) will have the following appearance:

$$-\frac{n h B^2}{2} k_y + \frac{h B}{2} \sum (e_i' - e_i'') + Q_y - \frac{B}{2}(P' - P'') \sin \gamma = 0.$$

The result is that:

$$k_y = \frac{h B \sum (e_i' - e_i'') + 2 Q_y - B(P' - P'') \sin \gamma}{h n B^2} \qquad \text{(d)}$$

We wish to change the obtained equation for k_y in such a way that we replace the value $P' - P''$ by the adhesion weight Q of the tank and the transverse shift of the pressure center point y. Then we obtain the following equation:

$$P' - P'' = \left(Q' f + \frac{m b + G \sin \alpha}{2} - \left(Q'' f + \frac{m b + G \sin \alpha}{2}\right) = (f Q' - Q'').\right.$$

But since

$$Q' = \frac{Q}{2} - \frac{y}{B} Q \qquad \text{and}$$

$$Q'' = \frac{Q}{2} - \frac{y}{B} Q \qquad \text{consequently}$$

$$P' - P'' f(Q' - Q'') = -2 f Q \frac{y}{B}.$$

When we substitute the obtained expression for $P' - P''$ into Eq. (d), we obtain finally:

$$k_y = \frac{h B \sum (e_i' - e_i'') + 2 Q_y (1 + f \sin \gamma)}{h n B^2} \qquad (7)$$

We wish to refer to the fact that the lateral tilt is in no way related to longitudinal tilt k_x; it is caused exclusively by the following circumstances:

a) by the difference of the additional deflections of the springs on the left and right sides of the tank $\sum(e_i' - e_i'')$ as well as

b) the transverse shift of the pressure center point y.

To determine the longitudinal tilt k_x of the tank and the downward shift of the center of gravity we will substitute expression (6) for normal road wheel loads into Eqs. (3) and (4). After we have done this and have brought similar members of the equation to the numerator, we obtain the following equations:

$$2 k n z_0 - 2 k k_x \Sigma (a_i - a) + k \Sigma (e_i' + e_i'') - Q - P \sin = 0$$

and

$$2 k z_0 \Sigma a_i - 2 k k_x \Sigma (a_i - a) a_i + k \Sigma (e_i' + e_i'') a_i + Q (f r - a + x) = 0 \qquad (8)$$

Both these equations (8) represent a system of equations of the first degree with the unknown quantities k_x and z_o. The solution of this equation system presents no difficulties after substituting the numerical values into the equations.

After we have determined the unknown quantities k_x, k_y and z_o in this manner, we now wish to determine the values of normal road wheel loads N_i' and N_i'' with the aid of Eqs. (6).

If we know the vertical loads on the road wheels, we can determine the values of the vertical reactions of the ground under the road wheels Q_i' amd Q_i'' with the aid of formulas (1) and (2) and in addition the horizontal loads on the road wheels P_i' and P_i''.

EXAMPLE

The vertical loads on the tank road wheels during travel at a uniform speed on level terrain will be determined with the coefficient of travel resistance $f = 0.1$:

The basic values for the tank are: G = 5600 kg.

The number of road wheels on one side is n = 4.

$r = 20$ cm	$a_1 = 240$ cm
$\gamma = 30°$	$a_2 = 160$ cm
$m = 100$ kg/cm	$a_3 = 80$ cm
$a = 130$ cm	$a_4 = 0$

The second and third springs on the right and left side of the tank are additionally stressed by supports of 5 cm each, i.e.:

$$e_1' = e_1'' = e_3' = e_3'' = 5 \text{ cm}$$
$$e_1' = e_1'' = e_4' = e_4'' = 0.$$

SOLUTION:

1. The lateral tilt k_y equals 0, since the additional spring deflections on the right and left sides are equal and the lateral shift of the pressure center point is $y = 0$.

2. We shall determine the longitudinal tilt k_x and the shift of the tank center of gravity z_o downward. Since when traveling on level ground the adhesion weight corresponds with the actual tank weight G, in this case Eq. (8) will have the following appearance:

$$2hnz_0 - 2hk_x\Sigma(a_i - a) + h\Sigma(e_i' + e_i'') - G - P\sin\gamma = 0,$$
$$2hz_0\Sigma a_i - 2hk_x\Sigma(a_i - a)a_i + h\Sigma(e_i' + e_i'')a_i + G(fr - a) = 0.$$

We shall calculate the coefficients of both these equations:

$$2hn = 2 \cdot 100 \cdot 4 = 800;$$
$$2h\Sigma(a_i - a) = 2h(\Sigma a_i - na) = 2 \cdot 100(240 + 160 + 80 - 4 \cdot 130) = -8000;$$
$$h\Sigma(e_i' + e_i'') = 100(5 + 5 + 5 + 5) = 2000;$$
$$P\sin\gamma = fG\sin\gamma = 5600 \cdot 0{,}1 \sin 30^\circ = 280;$$
$$2h\Sigma a_i = 2 \cdot 100(240 + 160 + 80) = 96{,}000;$$
$$2h\Sigma(a_i - a)a_i = 2h(\Sigma a_i^2 - a\Sigma a_i) = 2000(240^2 + 160^2 + 80^2 - 130(240 + 160 + 80) = 5{,}440{,}000;$$
$$h\Sigma(e' + e'')a_i = 100[(5 + 5)160 + (5 + 5)80] = 240{,}000$$
$$G(fr - a) = 5600(0{,}1 \cdot 20 - 130) = -716{,}800.$$

Consequently, we have:

$$800z_0 + 8\,000k_x + 2\,000 - 5600 - 280 = 0$$
$$96{,}000z_0 - 5{,}440{,}000k_x + 240{,}000 = 716{,}800 = 0.$$

When we solve these equations, we obtain:

$$z_0 = 4{,}87 \text{ cm}$$
$$k_x = 0{,}00175$$

3. We determine the normal road wheel loads:

$$N_i' = N_i'' = h\,[z_0 - (a_i - a)\,k_x + e_i];$$
$$N_1' = N_1'' = 100\,[4{,}87 - (240 - 130)\cdot 0{,}00175] = 506 \text{ kg};$$
$$N_2' = N_2'' = 100\,[4{,}87 - (160 - 130)\cdot 0{,}00175 + 5] = 992 \text{ kg};$$
$$N_3' = N_3'' = 100\,[4{,}87 - (80 - 130)\cdot 0{,}00175 + 5] = 978 \text{ kg};$$
$$N_4' = N_4'' = 100\,[4{,}87 - (0 - 130)\cdot 0{,}00175] = 464 \text{ kg}.$$

4. We check the solution by substituting the determined values of N_i' and N_i'' into Eq. (3).

$$\Sigma\,(N_i' + N_i'') - G - P\sin\gamma = 0,$$
$$2\,(506 + 992 + 978 + 464) - 5600 - 280 = 0.$$

Thus the solution is correct.

Without further deliberations which the reader can make for himself, we present comparisons of the results for the case that the tank which we selected as an example was in rest position P = 0:

$$P' = 280,\ P' = 0,\ P' = 280,\ P' = 0,\ P' = 280,\ P' = 0$$
$$N_1' = N_1'' = 506,\ 555 \quad Q_1' = Q_1'' = 506,\ 555 \quad P_1' = P_1'' = 55{,}5\ \ 0$$
$$N_2' = N_2'' = 992,\ 985 \quad Q_2' = Q_2'' = 992,\ 985 \quad P_2' = P_2'' = 93{,}5\ \ 0$$
$$N_3' = N_3'' = 978,\ 915 \quad Q_3' = Q_3'' = 978,\ 915 \quad P_3' = P_3'' = 91{,}5\ \ 0$$
$$N_4' = N_4'' = 464,\ 345 \quad Q_4' = Q_4'' = 324,\ 345 \quad P_4' = P_4'' = -17\ \ 0.$$

If we compare the corresponding normal road wheel loads of the stationary and moving tank, we see that significant load changes occur during travel (movement).

In the example which we considered, these changes of normal loads reach 34% on the rear road wheel.

The negative value P_4 refers to the fact that the rear road wheels shove the tank hull forward and not conversely, as is the case with all other road wheels. Thus, rolling resistance $R = fQ = 560$ kg is not the sum of the shearing force exerted by the hull against the road wheels $\Sigma P_i = 457$ kg.

4. Calculation of Independent Suspensions

In the preceding section the equation was determined according to which the vertical road wheel loads with individual springs were calculated; see Fig. 1.

We can use these same equations for determining the vertical loads of road wheels with individual springs in the design depicted in Figs. 2 and 3.

If we use these equations from the preceding section for determining the vertical loads of road wheels with individual springs as depicted in Figs. 2 and 3, we must clothe these supports in a simple calculation scheme.

This method involves replacing the actual tank suspension in the sense of that depicted in Figure 1. In this change we must maintain the position of the road wheels in comparison to the tank hull and replace the temper of all springs of the swing arm track and suspension, which serves as a substitute for the actual one in the calculation, by the temper of the corresponding actual springs.

The springs of the ideal substitute track and suspension are a replacement for the actual ones. Their temper is designated as apparent temper.

The precondition under which the value of the apparent temper was determined involves the same normal reactions of the road wheel on its own axis corresponding to the same vertical shifts of the road wheel in comparison to the tank hull.

This precondition is met if the spring deflection of the substitute spring in the vertical direction is equal to the shift of the road wheel in comparison to the tank hull.

We wish to determine the expression for apparent spring temper of the suspension which is depicted in Fig. 2.

We wish to designate the angle which is formed by lever O B and the road wheel rolling plane as α.

We will designate the value of the angle α of completely unloaded springs as α_0.

The angle formed by the spring axis and the lever O B is designated as β and the value of this angle with unloaded springs as β_0. Then we can assume that $\beta \approx \beta_0 + \alpha$ if α is not equal to 0.

We now wish to set up the formula for the spring tension of springs during vertical shift of the road wheel relative to the hull by the value of z. This expression appears in the following formula (a):

$$z = c(\sin\alpha - \sin\alpha_0) \quad \text{(a)}$$

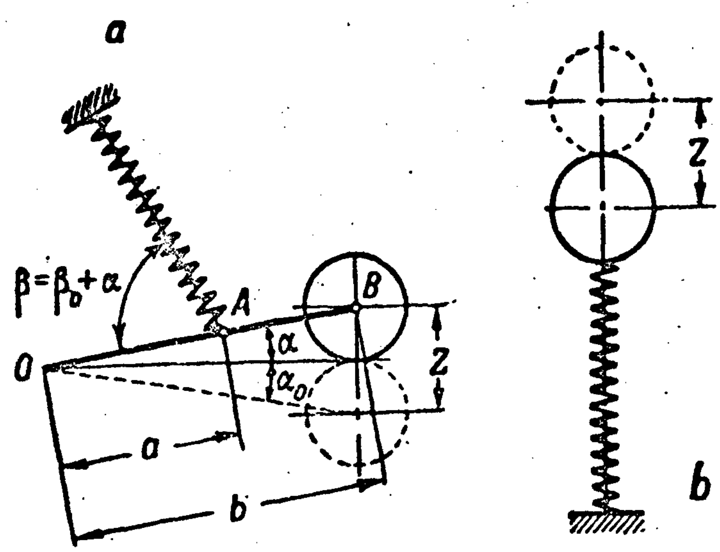


Figure 17. Road Wheel on the Tank Hull

The full deflection of the spring f_p will be expressed in the following formula:

$$f_p = a\,[\cos(\beta_0 + \alpha_0) - \cos(\beta_0 + \alpha)] \qquad \text{(b)}$$

The spring tension is determined by the following equation:

$$V_p = \frac{h_p f_p^2}{2} = \frac{1}{2} h_p a^2 [\cos(\beta_0 + \alpha_0) - \cos(\beta_0 + \alpha)]^2.$$

The derivative of spring tension of the spring according to the vertical shift of the road wheel will be expressed in the following equation:

$$\frac{dV_p}{dz} = \frac{dV_p\, d\alpha}{d\alpha\, dz} =$$
$$= h_p a^2 [\cos(\beta_0 + \alpha_0) - \cos(\beta_0 + \alpha)] \sin(\beta_0 + \alpha) \frac{d\alpha}{dz} \qquad \text{(c)}$$

We now wish to remove the lever O B and to connect the road wheel by a spring without a lever (see Fig. 17b) to the tank hull.

The power efficiency of these substitute springs which corresponds to the vertical shift of the road wheel by the value z, will be as follows:

$$V = \frac{hf^2}{2} = \frac{hz^2}{2} = \frac{1}{2} h b^2 (\sin\alpha - \sin\alpha_0)^2.$$

The temper of the substitute spring will be expressed in the following formula:

$$\frac{dV}{dz} = \frac{dV\,d\alpha}{d\alpha\,dz} = h\,b^2(\sin\alpha - \sin\alpha_0)\cos\alpha\,\frac{d\alpha}{dz}. \tag{d}$$

If we equate the right halves of Eqs. (c) and (d), we obtain:

$$h_p\,a^2[\cos(\beta_0+\alpha_0) - \cos(\beta_0+\alpha)],\sin(\beta_0+\alpha) = h\,b^2(\sin\alpha - \sin\alpha_0)\cos\alpha \tag{9}$$

$$h = h_p\frac{a^2}{b^2} \tag{10}$$

However, if β_o does not equal 90°, we must conclude from Eq. (9) that the value of the assumed temper is stipulated by the variable angle α which in turn is stipulated by the deflection of the spring f_p according to Eq. (b).

Consequently, if β_o does not equal 90°, the assumed temper forms a function of spring deflection f_p.

In order to make the calculation of road wheel loads less involved, and in order to be in a position to make use of these calculations of the preceding section, we will not assume that β_o equals 90° or that the value of the assumed temper is a constant value so that an error in determining the wheel loads will be as slight as possible.

We will assume that the spring deflection equals approximately f_p and proceed on the assumption that the static spring deflection of the sprung tank weight usually makes up one-half to one-third of the maximum deflection $f_{p\,max}$ allowed by the suspension design, i.e.

$$\frac{f_{p\,max}}{2} \geq f_p \geq \frac{f_{p\,max}}{3}.$$

Consequently it is reasonable to assume that $f_p = f_{p\,max}/2.5$ if the calculation angle α required to determine the assumed temper h, is determined according to equation (b).

When we have determined angle α we will determine the approximate expression for the assumed modulus according to Eq. (9) which in this case we can write in the following form:

$$h = h_p\frac{a^2}{b^2}\,\frac{f_{p\,max}\sin(\beta_0+\alpha)}{2,5(\sin\alpha - \sin\alpha_0)\cos\alpha}. \tag{11}$$

We now wish to evaluate the error which can occur in such an approximate determination of the value of the assumed modulus.

For example, we wish to assume that

$$f_{p\,max} = 0{,}66\,a$$

and

$$\beta_0 = 75°;\ \alpha_0 = -10°$$

We determine the mathematical value of the angle α in which we take the following relationship as a basis:

$$\frac{f_{p\,max}}{f_p} = 2{,}5;\quad f_p = \frac{f_{p\,max}}{2{,}5} = a[\cos(\beta_0 + \alpha_0) - \cos(\beta_0 + \alpha)].$$

When we replace their values for $f_{p\,max}$, α_o and β_o, we obtain:

$$0{,}264 = \cos(75° - 10°) - \cos(75° + \alpha),$$

$$\cos(75° + \alpha) = 0{,}1586,\ \alpha = 5°\,50'.$$

The value of the apparent spring temper corresponding to this angle α will be the following:

$$h = h_p \frac{a^2}{b^2}\,\frac{0{,}66 \sin(75° + 5°\,50')}{2{,}5(\sin 5°\,50' + \sin 10°)\cos 5°\,50'} = 0{,}95\,h\,\frac{a^2}{b^2}.$$

In such a determination of the value of apparent spring temper we will achieve the maximum error if in reality it turns out that

$$\frac{f_{p\,max}}{f_p} = 2$$

or that

$$\frac{f_{p\,max}}{f_p} = 3$$

We will assume that the actual spring deflection will be such that

$$\frac{f_{p\,max}}{f_p} = 2$$

i.e. that

$$f_p = \frac{f_{p\,max}}{2} = \frac{0{,}66\,a}{2} = 0{,}33\,a$$

The angle α corresponding to this deflection will be determined by the following equation:

$$0{,}33 = \cos(75° - 10°) - \cos(75° + \alpha).$$

The result is that:

$$\cos(75° + \alpha) = 0{,}926,$$

$$\alpha = 9°\,40'.$$

The value of the apparent spring temper will be expressed in the following equation:

$$h = h_p \frac{a^2}{b^2} \frac{0{,}66 \sin (75^\circ + 9^\circ 40')}{2 (\sin 9^\circ 40' + \sin 10^\circ) \cos 9^\circ 40'} = 0{,}975\, h_p \frac{a^2}{b^2}.$$

Accordingly, the maximum error in the determination of the value of assumed spring temper will be represented in the following equation:

$$\frac{0{,}975\, h_p \frac{a^2}{b^2} - 0{,}950\, h_p \frac{a^2}{b^2}}{0{,}975\, h_p \frac{a^2}{b^2}} \cdot 100 = 2{,}6\%.$$

If the actual deflection of the springs was such that $f_{p\,max}/f_p = 3$, this maximum error would be 2.15%.

In ordinary calculations such errors are completely permissible.

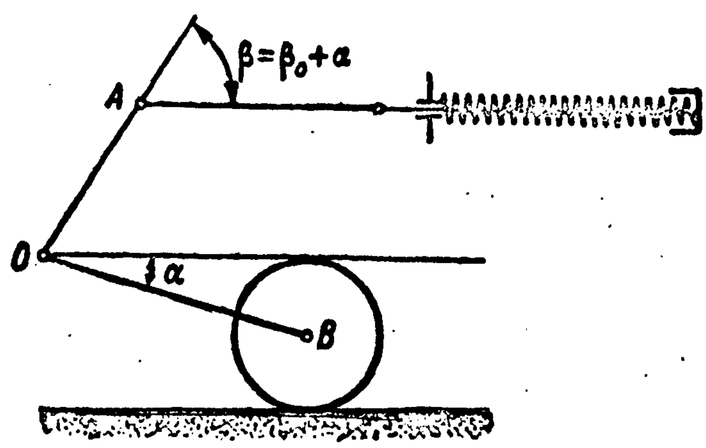


Figure 18. Anchoring of the Road Wheel on the Tank Hull

In the case of a swing arm support with individual springs, as depicted in Fig. 3, we will encounter the same formulas that we determined with regard to individual springs on Figs. 2 and 17, if we designate the angle between lever O B (Fig. 18) and the plane of travel as α and the angle between the spring axis and the lever O A at $\alpha = 0$ as β_o and proceed on the same considerations.

5. Determination of the Road Wheel Loads on Bogie Wheel Suspension Frames

For the purpose of determining the road wheel loads on bogie wheel suspension frames we wish to first determine the vertical loads of individual bogie wheel suspension frames.

As soon as we have determined the vertical loads of bogie wheel suspension frames, we determine the bogie wheel loads as well, where we take the equilibrium of each individual unit as a basis.

We now wish to determine the vertical loads T_1 and T_2 of the axes of the front and rear bogie wheel suspension frame (Fig. 19).

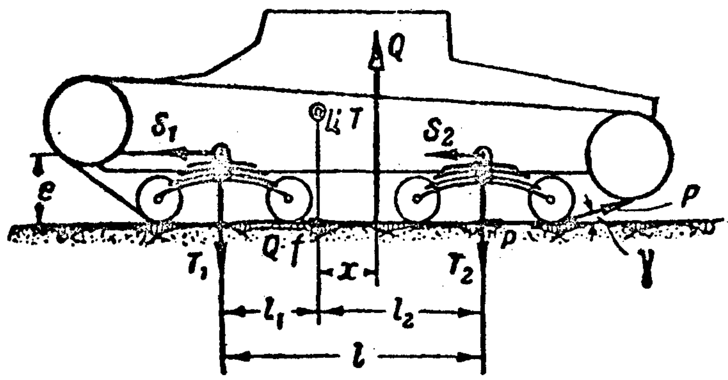


Figure 19. Forces Exerted on the Bogie Wheel Suspension Frame

For this purpose we utilize an equation for the moments of forces which are exerted on the support of the last road wheel, as well as a second equation for the front bogie wheel suspension frame axis. These equations are

$$\left.\begin{aligned} -T_1 l + Q(l_2 - x - f_c) + P(e-k) = 0, \\ T_2 l - Q(l_1 + x + f_c) + P(e - k - l\sin\gamma) \end{aligned}\right\}. \qquad \text{(a)}$$

k is the length of the plumb line which we suspend from the axis of the rear bogie wheel suspension frame onto the plane of the climbing track.

From Figs. 20 and 21 it is easy to see that the following equation applies for the bogie wheel suspension frame with two road wheels:

$$k = r + \alpha_4 \sin\gamma\,(c - r)\cos\gamma$$

and for the bogie wheel suspension frame with four road wheels:

$$k = r + (a_8 + b_4)\sin\gamma + P\,(c - r)\cos\gamma.$$

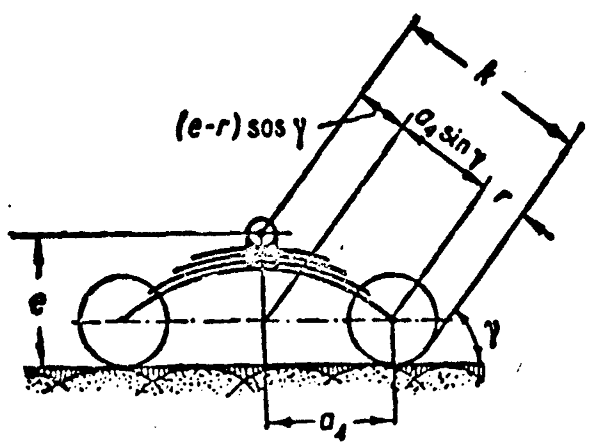
From these equations (a) we obtain the following:

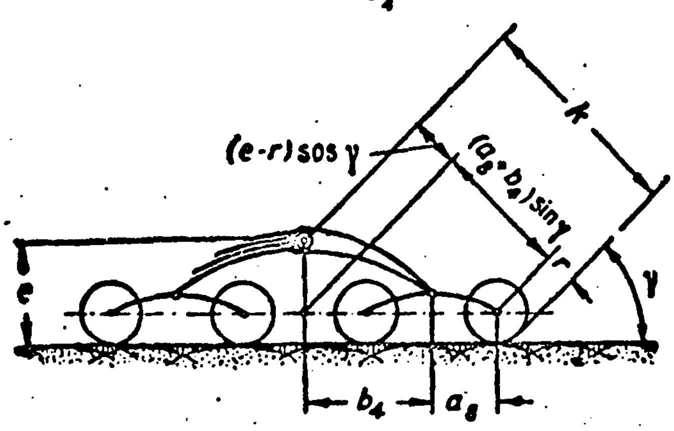
$$\left.\begin{aligned} T_1 &= \frac{Q(l_2 - x - f_c) + P(e - k)}{l} \\ T_2 &= \frac{Q(l_1 + x + f_c) - P(e - k - l)\sin\gamma}{l}. \end{aligned}\right\} \qquad (12)$$

The normal loads T_1 and T_2 are distributed onto the bogie wheel suspension frame on the left and right side of the tank in direct ratio to the loads Q' and Q'' by the weight of the tank on the right and left track; this means that:

and

$$\left.\begin{aligned} \frac{T_1'}{T_1} = \frac{T_2'}{T_2} = \frac{Q'}{Q} \\ \frac{T_1''}{T_1} = \frac{T_2''}{T_2} = \frac{Q''}{Q} \end{aligned}\right\} \qquad (b)$$





Figures 20 and 21. Determination of the Value of k.

T_1' and T_2'' are the vertical loads on the right front and rear of the bogie wheel suspension frame: T_1'' and T_2'' are the vertical loads on the left front and rear side of the bogie wheel suspension frame.

On the other hand, the vertical loads Q' and Q'' through the weight of the tank on the right and left tank track are determined by the following equations:

$$\left.\begin{aligned} Q\left(\frac{B}{2} - y\right) &= Q'B \\ Q\left(\frac{B}{2} + y\right) &= Q''B \end{aligned}\right\} \qquad (c)$$

The result is

$$\left.\begin{aligned} \frac{Q'}{Q} &= \frac{1}{2} - \frac{y}{B} \\ \text{and } \frac{Q''}{Q} &= \frac{1}{2} + \frac{y}{B} \end{aligned}\right\} \qquad (d)$$

Consequently,

$$\left.\begin{aligned} \frac{T_1'}{T_1} &= \frac{T_2'}{T_2} = \frac{1}{2} - \frac{y}{B} \\ \frac{T_1''}{T_1} &= \frac{T_2''}{T_2} = \frac{1}{2} + \frac{y}{B} \end{aligned}\right\} \qquad (e)$$

Accordingly we determine:

$$\left.\begin{aligned} T_1' &= T_1\left(\frac{1}{2} - \frac{y}{B}\right); \quad T_1'' = T_1\left(\frac{1}{2} + \frac{y}{B}\right) \\ T_2' &= T_2\left(\frac{1}{2} - \frac{y}{B}\right); \quad T_2'' = T_2\left(\frac{1}{2} + \frac{y}{B}\right) \end{aligned}\right\} \qquad (13)$$

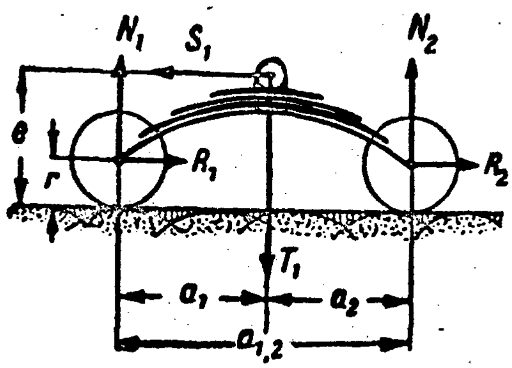


Figure 22. Forces Exerted on the Front Bogie Wheel Suspension Frame

Because the forces being exerted on the bogie wheel suspension frame -- right and left side -- are similar in nature, in our further studies we will limit ourselves to considering only one side; thus the designation "omitted".

We wish to determine the longitudinal load S_1 of the axle on the front of the bogie wheel suspension frame (Fig. 22). To achieve this, we set up equations for the horizontal forces:

$$S_1 = R_1 + R_2 = fQ_1 + fQ_2 = P_1 + P_2;$$
$$N_1 + N_2 = Q_1 + Q_2 = T_1.$$

In addition, we have the following equations on the basis of Equation (1):

$$Q_1 = N_1;\ R_1 = fQ_1;$$
$$Q_2 = N_2;\ R_2 = fQ_2.$$

Consequently:

$$S_1 = R_1 + R_2 = f(Q_1 + Q_2) = f(N_1 + N_2) = fT_1 \qquad (14)$$

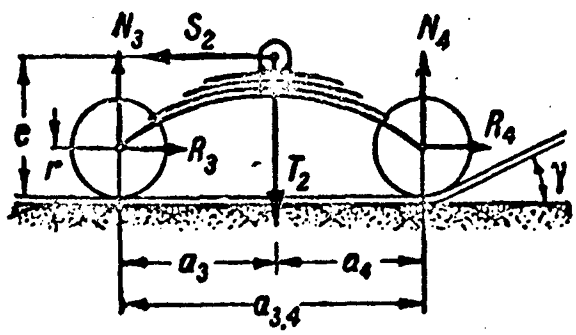


Figure 23. Forces Being Exerted on the Rear of the Bogie Wheel Suspension Frame

Then we wish to consider the longitudinal load S_2 on the rear of the bogie wheel suspension frame (Fig. 23). We set up equations for the horizontal forces:

$$\left.\begin{array}{l} S_2 = P_3 + P_4 \\ N_3 + N_4 = T_2 \end{array}\right\} \qquad (f)$$

In addition, we have the following equations on the basis of Equations (1) and (2):

$$
\begin{gathered}
Q_3 = N_3, \\
Q_4 = N_4 - P \sin \gamma, \\
P_3 = f Q_3, \\
P_4 = f Q_4 - P (1 - \cos \gamma).
\end{gathered}
$$

Consequently:

$$
\begin{aligned}
S_2 = P_3 + P_4 &= f (Q_3 + Q_4) - P (1 - \cos \gamma) \\
&= f (N_3 + N_4 - P \sin \gamma) - P (1 - \cos \gamma) \\
&= f (T_2 - P \sin \gamma) - P (1 - \cos \gamma).
\end{aligned} \tag{15}
$$

If we know the vertical and longitudinal loads on the bogie wheel suspension frame axis, we can determine the vertical loads on the road wheel axis.

We now wish to select an equation for the moments of the forces being exerted on the front of the bogie wheel suspension frame around the axis of the second road wheel, and a second equation for the moments around the axis of the first road wheel (Fig. 22):

$$
\left.
\begin{aligned}
N_1 a_{1,2} - T_1 a_2 - S_1 (e - r) = 0 \\
- N_2 a_{1,2} + T_1 a_1 - S_1 (e - r) = 0
\end{aligned}
\right\} \tag{g}
$$

From this we determine

$$
N_1 = \frac{T_1 a_2 + S_1 (e - r)}{a_{1,2}} \quad \text{and} \quad N_2 = \frac{T_1 a_1 - S_1 (e - r)}{a_{1,2}} \tag{16}
$$

In precisely the same way we now determine the vertical loads of the road wheel axis on the rear of the bogie wheel suspension frame:

$$
N_3 = \frac{T_2 a_4 + S_2 (e - r)}{a_{3,4}} \quad \text{and} \quad N_4 = \frac{T_2 a_3 - S_2 (e - r)}{a_{3,4}} \tag{17}
$$

After we have determined the vertical loads on the road wheels we can also determine the vertical reactions Q_1 of the ground under the road wheels and then also the longitudinal loads P_1 of the road wheel axes by using formulas (1) and (2).

The road wheel loads with which the bogie wheel suspension frame has 4 road wheels (Figs. 5 and 24) were determined in the same manner as road wheel loads where the bogie wheel suspension frame had two road wheels.

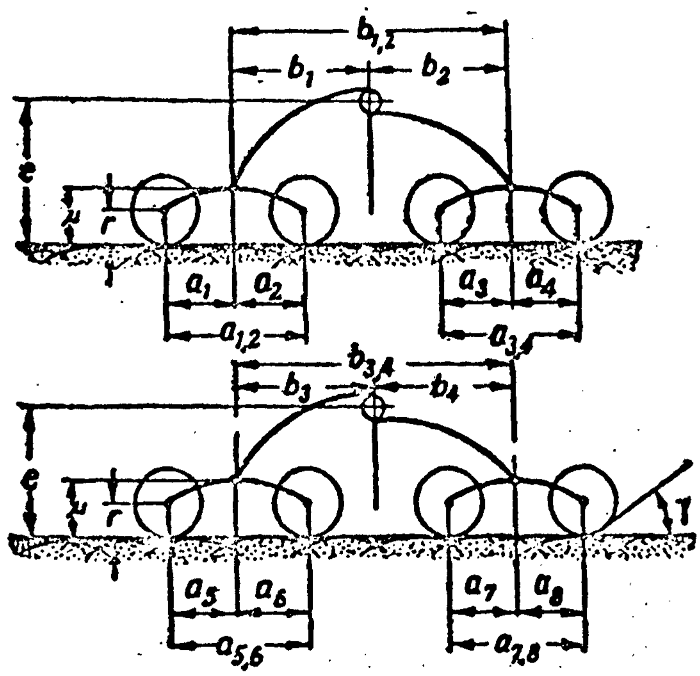


Figure 24. Bogie Wheel Suspension Frame

After we have first determined the vertical loads T_1 and T_2 of the front and rear bogie wheel suspension frame according to formulas (12), we now wish to determine the distribution of these loads on the right and left bogie wheel suspension frame of the suspension according to formulas (13).

Then we wish to compile the equations for the horizontal forces which are exerted on each of these bogie wheel suspension frames and to determine the longitudinal loads S_1 and S_2 of the bogie wheel suspension frame axes so as to determine finally the normal loads $T_{1.2}$, $T_{3.4}$, $T_{5.6}$ and $T_{7.8}$ on the axis of the center of the bogie wheel suspension frame $a_{1.2}$, $a_{3.4}$, $a_{5.6}$ and $a_{7.8}$ which the road wheels compile in pairs.

After we have determined the vertical loads on the axis of the center bogie wheel suspension frame, we determine the longitudinal loads $S_{1.2}$, $S_{3.4}$, $S_{5.6}$ and $S_{7.8}$ on the axis of this bogie wheel suspension frame and in addition, the vertical loads on the road wheels whereby we proceed from the equilibrium conditions of all auxiliary swing arms individually.

The final expressions for determining the loads of the track and suspension axis and of the road wheels are compiled in the following table without derivations.

Formulas for determining the loads of the swing arms and on the road wheels of bogie wheel suspension frames with four road wheels:

<u>Front Bogie Wheel Suspension Frame</u>

$$S_1 = f\,T_1$$

$$T_{1,2} = \frac{T_1 b_2 + S_1 (e - \mu)}{b_{1,2}} \qquad N_1 = \frac{T_{1,2} a_2 + S_{1,2} (\mu - r)}{a_{1,2}}$$

$$S_{1,2} = T_{1,2} f \qquad N_2 = \frac{T_{1,2} a_1 - S_{1,2} (\mu - r)}{a_{1,2}}$$

$$T_{3,4} = \frac{T_1 b_1 - S_1 (e - \mu)}{b_{1,2}} \qquad N_3 = \frac{T_{3,4} a_4 + S_{3,4} (\mu - r)}{a_{3,4}}$$

$$S_{3,4} = T_{3,4} f \qquad N_4 = \frac{T_{3,4} a_3 - S_{3,4} (\mu - r)}{a_{3,4}}$$

<u>Rear Bogie Wheel Suspension Frame</u>

$$S_2 = f\,(T_2 - P \sin\gamma) - P\,(1 - \cos\gamma)$$

$$T_{5,6} = \frac{T_2 b_4 + S_2 (e - \mu)}{b_{3,4}} \qquad N_5 = \frac{T_{5,6} a_6 + S_{5,6} (\mu - r)}{a_{5,6}}$$

$$S_{5,6} = T_{5,6} f \qquad N_6 = \frac{T_{5,6} a_5 - S_{5,6} (\mu - r)}{a_{5,6}}$$

$$T_{7,8} = \frac{T_2 b_3 - S_2 (e - \mu)}{b_{3,4}} \qquad N_7 = \frac{T_{7,8} a_8 + S_{7,8} (\mu - r)}{a_{7,8}}$$

$$S_{7,8} = f\,(T_{7,8} - P \sin\gamma) - P\,(1 - \cos\gamma) \qquad N_8 = \frac{T_{7,8} a_7 - S_{7,8} (\mu - r)}{a_{7,8}}$$

6. Hull Vibrations

a) <u>The most important types of tank vibrations and their influence on firing.</u>

During travel, the tank rolls over uneven spots on the road. Thereby it is subjected to shocks which set off vibrations in the tank hull and in its track and suspension. These vibrations are of a complex nature, because they are made up of various motions, the mutual influence of which depends on the coupling factor.

The track and suspension of recent tanks are built in such a way that the tank hull vibrates around three axes due to the elastic deformation of the springs.

In accordance with these three axes of vibration, the tank is subjected to three basic types of vibrations:

1. Vertical vibrations of the tank center of gravity.
2. Pitching vibrations around its transverse axis.
3. Torsional vibrations around its longitudinal axis.

These three types of vibration occur simultaneously. They reduce the effectiveness of the weapons considerably during movement, lead to an abrupt increase of equipment wear and cause the tank crew to tire more quickly.

Vertical vibrations of the tank hull center of gravity, if there are no torsional vibrations, cause a continuous shift of the weapons barrel upward and downward.

Such weapons vibrations would cause no significant misses if the gunner himself were not subject to their effects.

The significant forces of these alternating accelerations of the tank hull which are caused by vertical vibrations of the hull center of gravity, hinder the gunner and keep him from maintaining the target in his view so that the weapon is incorrectly aimed, thus reducing weapon efficiency.

The torsional vibrations around the longitudinal axis cause the barrel to vibrate in the direction of the lowered trunnion whereby the miss may be considerable if the shot was fired at great elevation.

The angle of elevation is usually small when firing from a moving tank because only small ranges are used. Consequently, the tilt of the trunnion which is caused by torsional vibrations is of insignificant influence on effective firing when firing from movement.

The hit effect when firing from a moving tank is sharply reduced by the vibrations of the barrel in the traversing plane.

When the weapons barrel undergoes pitching vibrations, changes occur in the elevation angle which may cause the shot to miss by a considerable distance.

Pitch vibrations of the barrel can also cause a considerable miss of the target when the shot was fired by direct vision. This phenomenon is explained by the fact that the shot was not fired at once but after a certain span of time, usually designated as firing lag.

During the course of this short time span the angle of elevation may change from its original value.

Of course we must not conclude from this information that firing must necessarily be inaccurate from movement.

A well trained gunner will always be able to adjust to the situation, and fire an accurate shot at the appropriate time. In addition there are special measures for increasing the hit effect when firing from a moving tank and for easing the duties of the gunner.

From this information it is clear that tank hull vibrations are of considerable significance. Thus this area should be discussed in more detail in conjunction with differential equations for free vibrations of the tank hull.

b) Differential equation for free vibrations of the tank hull in the case of swing arm suspension of the tank.

We wish to introduce the following designations:

z = vertical shift of the hull center of gravity (Fig. 25).
ψ = pitching vibration of the tank hull around its transverse axis passing through the center of gravity.
Q = torsional vibration of the tank hull around its longitudinal axis X, which passes through the center of gravity (Fig. 26).

The coordinates z, ψ and Q are read from the vibration center point, where we proceed on the assumption that the angles ψ and Q are positive values which are transmitted to the rear and left side by the torsional vibrations of the tank hull.

If the tank hull deviates from the position of equilibrium, the springs are additionally deflected and generate the rectifying force N and the rectifying moments Mx and My, which demonstrate a tendency to bring the tank hull back into a position of equilibrium:

$$\left.\begin{aligned} N &= -c(f_1' + f_2' + f_1'' + f_2'') \\ M_x &= \frac{cB}{2}(f_1' + f_2' - f_1'' - f_2'') \\ M_y &= m[l_1(f_1' + f_1'') - l_2(f_2' + f_2'')] \end{aligned}\right\} \qquad (a)$$

f_1' and f_2' are the dynamic deflections of the springs of the bogie wheel suspension frame on the right side of the tank and

f_1'' and f_2'' are the same on the left side of the tank, while

c is the spring temper of the bogie wheel suspension frame.

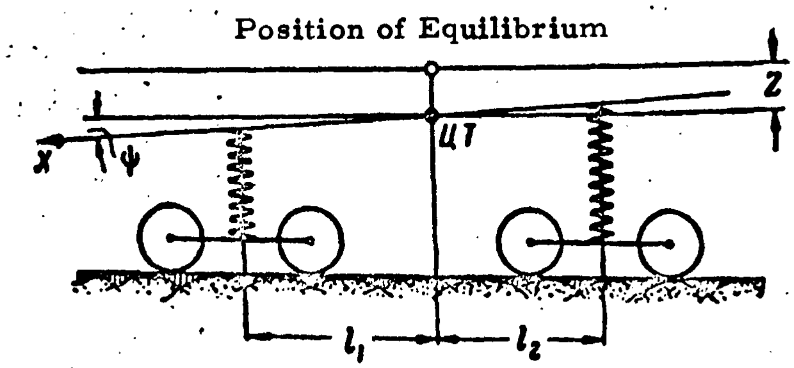


Figure 25. Pitching Vibrations of the Tank Hull

To obtain differential equations of the free vibrations of the tank hull, we wish to determine the dynamic deflections of the springs by the coordinates z, ψ and Θ where, with consideration of the small values of angles ψ and Θ, we are assuming that sin $\psi = \psi$ and $\Theta = \Theta$.

$$\left.\begin{aligned} f_1' &= z - l_1\psi - \frac{B}{2}\theta \\ f_2' &= z + l_2\psi - \frac{B}{2}\theta \\ f_1'' &= z - l_1\psi + \frac{B}{2}\theta \\ f_2'' &= z + l_2\psi + \frac{B}{2}\theta \end{aligned}\right\} \qquad \text{(b)}$$

When we substitute the values under (b) of the dynamic spring deflection into formulas under (a), we will determine the following equations:

$$\begin{aligned} N &= -4\,m\,z + 2\,m\,(l_1 - l_2)\,\psi; \\ M_y &= 2\,m\,(l_1 - l_2)\,z - 2\,m\,(l_1^2 + l_2^2)\,\psi; \\ M_z &= -m\,B^2\theta \end{aligned} \qquad \text{(c)}$$

The rectifying force and the rectifying moments will be equalized at an arbitrary point of time by the force of inertia and the moments of inert forces of the tank:

$$\left.\begin{aligned} N - \frac{G}{g}\frac{d^2 z}{dt^2} &= \theta \\ M_y - J_y\frac{d^2\psi}{dt^2} &= \theta \\ M_z - J_z\frac{d^2\theta}{dt^2} &= \theta \end{aligned}\right\} \qquad \text{(d)}$$

where J_x and J_y are the moments of inertia of the tank hull around its longitudinal and transverse axis which pass through the center of gravity.

Position of the Center of Equilibrium

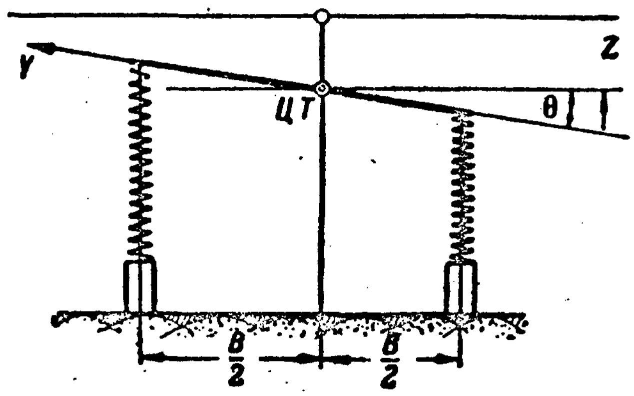


Figure 26. Transverse Vibrations of the Tank Hull

G -- is the sprung weight of the tank.

If we substitute the expressions under (c) into the equations (d) and for purposes of simplification substitute the letters z, ψ and Θ for the accelerations $\frac{d^2 z}{dt^2}$, $\frac{d^2 \psi}{dt^2}$ and $\frac{d^2 \Theta}{dt^2}$ we obtain the differential equations for the free vibrations of the tank hull.

$$\frac{G}{g}\ddot{z} + 4cz - 2c(l_1 - l_2)\psi = 0 \quad \ldots\ldots\ldots \quad (18)$$

$$J_y\ddot{\psi} - 2c(l_1 - l_2)z + 2c(l_1^2 + l_2^2)\psi = 0 \quad \ldots\ldots \quad (19)$$

$$J_x\ddot{\Theta} + cB^2\Theta = 0 \quad \ldots\ldots\ldots\ldots\ldots \quad (20)$$

We will now begin with the solution of these differential equations by first solving equation (20).

We set $\frac{cB^2}{J_x} = k^2$

$$\ddot{\Theta} + k^2\Theta = 0 \quad (21)$$

We compile a characteristic equation $\lambda^2 + k^2 = 0.$

The roots of this characteristic equation are imaginary:

$$\lambda = \pm k\sqrt{-1}$$

Consequently, the general solution of our differential equation (21) will be as follows:

$$\theta = A_1 \cos k t + A_2 \sin k t \tag{22}$$

In this equation A_1 and A_2 are the arbitrary constant values.

We are assuming that

$$A_1 = A \sin \alpha$$

and

$$A_2 = A \cos \alpha$$

and write the general equation in the following form:

$$\theta = A \sin (k t + \alpha) \tag{22'}$$

in which A and α are arbitrarily constant values.

A and α are the vibration amplitude and the phase of the torsion-transverse vibrations; they were determined according to the initial preconditions.

We are assuming that $t = 0$; $\Theta = \Theta_0$ and $\dot{\Theta} = \dot{\Theta}_0$.

When we substitute these basic values into Eq. (22') and the equation $\dot{\Theta} = A\ k \cos (kt + \alpha)$ which we obtain by differentiating Eq. (22') according to time, we obtain the following equation:

$$\left.\begin{aligned} A &= \sqrt{\theta_0^2 + \frac{\dot{\theta}_0^2}{k^2}} \\ \operatorname{tg} \alpha &= k \frac{\theta_0}{\dot{\theta}_0} \end{aligned}\right\} \tag{23}$$

The movement represents a harmonic vibrator of the vibration rate k, where

$$k = \sqrt{\frac{c B^2}{J_x}} = B \sqrt{\frac{c}{J_x}}$$

We will now attempt to solve Eqs. (18) and (19). We note that $l_1 - l_2 = 0$ and that the following Eqs. (18) and (19) were simplified if the tank hull center of gravity lies above the center of the tank support surface:

$$\frac{G}{g} \ddot{z} + 4 c z = 0 \tag{18'}$$

$$J_y \ddot{\psi} + 2 c (l_1^2 + l_2^2) \psi = 0 \tag{19'}$$

We introduce the following designations:

$$\frac{4cg}{G} = p_1^2,$$

$$\frac{2cl_1^2 + l_2^2}{J_\varphi} = p_2^2.$$

After substituting these designations into formulas (18') and (19') this will have the following appearance:

$$\left.\begin{aligned} \ddot{z} + p_1^2 z = 0 \\ \ddot{\varphi} + p_2^2 \varphi = 0 \end{aligned}\right\} \qquad (24)$$

The equations (24) were solved in the same manner as Eq. (21) and will have the following appearance:

$$\left.\begin{aligned} z = C_1 \sin(p_1 t + \beta_1) \\ \varphi = C_2 \sin(p_2 t + \beta_2) \end{aligned}\right\} \qquad (25)$$

where C_1, β_1 and C_2, β_2 are arbitrarily constant values which were determined according to the initial conditions.

We are assuming that $t = 0$ and that therefore the following equations result:

$$z = z_0; \quad \varphi = \varphi_0;$$

$$\dot{z} = \dot{z}_0; \quad \dot{\varphi} = \dot{\varphi}_0.$$

With these initial preconditions, the arbitrarily constant values C_1, C_2, β_1 and β_2 will be as follows:

$$\left.\begin{aligned} C_1 = \sqrt{z_0^2 + \frac{\dot{z}_0^2}{p_1^2}} \\ C_2 = \sqrt{\varphi_0^2 + \frac{\dot{\varphi}_0^2}{p_2^2}} \end{aligned}\right\} \qquad (26)$$

$$\left.\begin{aligned} \operatorname{tg} \beta_1 = p_1 \frac{z_0}{\dot{z}_0} \\ \operatorname{tg} \beta_2 = p_2 \frac{\varphi_0}{\dot{\varphi}_0} \end{aligned}\right\} \qquad (27)$$

In this manner the vertical vibrations of the center of gravity become harmonic vibrations of the vibration rate ω_1 if the tank center of gravity lies above the center surface of the tank, where

$$\omega_1 = \sqrt{\frac{4cg}{G_n}}$$

The pitch vibrations of the tank hull in this case will occur with the frequency ω_2.

$$\omega_2 = \sqrt{\frac{2c\,(l_1^2 + l_2^2)}{J_y}}$$

We will now turn to the general case when the center of gravity does not lie on the center surface of the tank hull and consequently:

$$l_1 - l_2 \neq 0.$$

In consideration of this general case, we will select the following designations:

$$\frac{4cg}{G} = a;$$
$$-\frac{2c(l_1 - l_2)g}{G} = b$$
$$\frac{2c(l_1 - l_2)}{J_y} = u$$
$$-\frac{2c(l_1^2 + l_2^2)}{J_y} = e.$$

After substituting these designations into Eqs. (18) and (19)

$$\left.\begin{aligned} \ddot{z} + az + b\psi = 0 \\ \ddot{\psi} + cz + e\psi = 0 \end{aligned}\right\} \qquad (28)$$

If the movement is of a vibrational kind, we obtain the solutions of Eqs. (28) in the following form:

$$\left.\begin{aligned} z = C_1 \cdot \sin(\omega t + \beta) \\ \psi = C_2 \cdot \sin(\omega t + \beta) \end{aligned}\right\} \qquad (29)$$

When we substitute these values z and ψ as well as their secondary derivations into Eq. (28) and shorten with $\sin(\omega t + \beta)$, we obtain:

$$\left.\begin{aligned} C_1\,(a - \omega^2) + C^2 b = 0 \\ C_1 u + C_2\,(e - \omega^2) = 0 \end{aligned}\right\} \qquad (30)$$

If we eliminate the constant values C_1 and C_2 from this Eq. (30), we obtain

$$(a - \omega^2)\,(e - \omega^2) - bu = 0 \qquad (31)$$

The roots of Eq. (31), which is considered an equation for ω^2, are as follows:

$$\left.\begin{aligned} \omega_1^2 = \frac{1}{2}(\alpha + e) + \sqrt{\frac{1}{4}(\alpha - e)^2 + bu} \\ \omega_2^2 = \frac{1}{2}(\alpha + e) - \sqrt{\frac{1}{4}(\alpha - e)^2 + bu} \end{aligned}\right\} \qquad (32)$$

We make reference to the fact that these roots are always real and positive. The proof of this relation can be found in any instruction book on vibrations.

If we extract the square root from the positive values ω_1' and ω_2', we obtain two solutions $\omega = \omega_1$ and $\omega = \omega_2$, from which we can determine special solutions of the type (29) according to the equation system (28). The root ω_1 will correspond to this special solution:

$$\left.\begin{aligned} z^{(1)} &= C_1^{(1)} \sin(\omega_1 t + \beta^{(1)}) \\ \psi^{(1)} &= C_2^{(1)} \sin(\omega_1 t + \beta^{(1)}). \end{aligned}\right\} \quad (33)$$

The root ω_2 will correspond to another special solution:

$$\left.\begin{aligned} z^{(2)} &= C_1^{(2)} \sin(\omega_2 t + \beta^{(2)}) \\ \psi^{(2)} &= C_2^{(2)} \sin(\omega_2 t + \beta^{(2)}). \end{aligned}\right\} \quad (34)$$

The initial conditions do not influence phase (B). For this reason the constant values $\beta^{(1)}$ and $\beta^{(2)}$ remain arbitrary. On the other hand, the constant values $C_1^{(1)}$, $C_2^{(1)}$ and $C_2^{(2)}$ must correspond to equations (30).

We select one of the roots, i.e. the root ω_1', substitute it into Eq. (30) and determine that this root is of the corresponding ratio C_1/C_2

$$\frac{C_1^{(1)}}{C_2^{(1)}} \quad - \frac{b}{\alpha - \omega_1^2} = \frac{c - \omega_1^2}{c}.$$

From this it is logical, if we assume that

$$C_1^{(1)} = D_1\, n$$

and

$$C_2^{(1)} = - D_1 (\alpha - \omega_1')$$

where D_1 is an arbitrarily constant value , that Eqs. (30) can be solved.

Of course we could assume just as well that

$$C_1^{(1)} = D_1' D_1 (\alpha - \omega_1')$$

and

$$C_2^{(1)} = - D_1' C$$

where

D_1' is an arbitrarily constant value.

In a similar manner we determine the values of the constant quantities C_1 and C_2 with $\omega = \omega_2$:

$$C_1^{(2)} = D_2 b$$

$$C_1^{(2)} = - D_2 (\alpha - \omega_1^2)$$

where D_2 is an arbitrarily constant value.

In this manner, special equations (33) and (34) will finally have the following form:

$$\left.\begin{aligned} z^{(1)} &= D_1 b \sin(\omega_1 t + \beta^{(1)}) \\ \psi^{(1)} &= - D_1 (\alpha - \omega_1^2) \sin(\omega_1 t + \beta^{(1)}) \end{aligned}\right\} \tag{33'}$$

$$\left.\begin{aligned} z^{(2)} &= D_2 b \sin(\omega_2 t + \beta^{(2)}) \\ \psi^{(2)} &= - D_2 (\alpha - \omega_2^2) \sin(\omega_2 t + \beta^{(2)}) . \end{aligned}\right\} \tag{34'}$$

According to the known characteristic of the linear equations, the sum of the special solutions is likewise a solution. Consequently, the total solution of the equation system (28) will be with four arbitrarily constant values:

$$\begin{aligned} z &= D_1 b \sin(\omega_1 t + \beta^{(1)}) + D_2 b \sin(\omega_2 t + \beta^{(2)}) \\ \psi &= - D_1 (\alpha - \omega_1^2) \sin(\omega_1 t + \beta^{(1)}) - D_2 (\alpha - \omega_2^2) \sin(\omega_2 t + \beta^{(2)}) . \end{aligned} \tag{35}$$

From this it is clear that each of the coordinates z and ψ gives that vibrational movement of the tank at which it is subjected to two harmonic vibrations: one with the vibration rate ω_1 and the other with the vibration rate ω_2 whereby these harmonic vibrations overlap.

c) The differential equations of the free vibrations of the tank hull with the road wheels having individual springs.

We will now proceed to the consideration of free vibrations of the tank hull when the road wheels have individual springs and again designate the vertical shift of the tank hull center of gravity from the position of equilibrium as z (Fig. 27) and as ψ and Θ the angular deflection of the tank hull towards the position of equilibrium during tipping around the transverse and longitudinal axis X and Y, which pass through the center point of vibration (Figs. 27 and 28).

When the tank hull deviates from the position of equilibrium, it will feel the effect of the rectifying force N and of the rectifying moments M_x and M_y which tend to bring it back into a position of equilibrium.

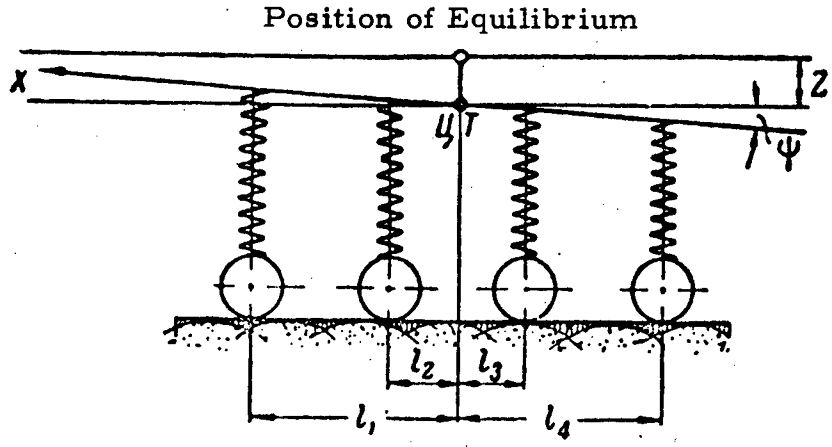


Figure 27. Vibrations of the Tank Hull in the Longitudinal Plane of the Tank, the road wheels having individual springs.

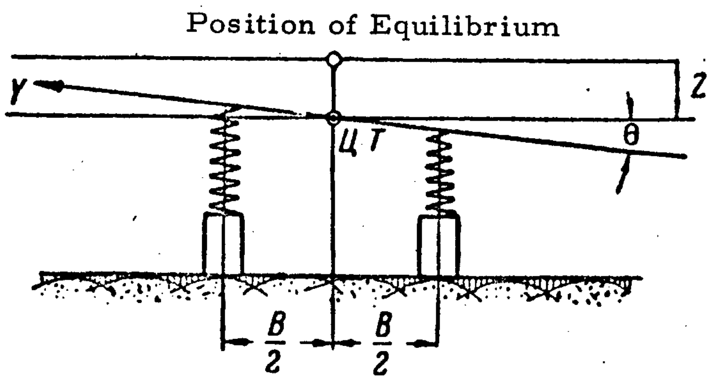


Figure 28. Transverse Vibrations of the Tank Hull, the road wheels having individual springs.

$$\left.\begin{aligned} N &= -c \sum (f'_i + f''_i) \\ M_x &= \frac{cB}{2} \sum (f'_i + f''_i) \\ M_y &= c \sum (f'_i + f''_i)\, l_i \end{aligned}\right\} \qquad \text{(a)}$$

where f'_1 and f''_1 -- the dynamic deflections of the vehicle springs on the right and left side,

n -- the number of road wheels on one side, and

l_i -- distances from the transverse surface of the tank hull through the center of gravity up to the road wheel axes. The distances l_i in the direction toward the tank front are positive, while those directed toward the rear will be considered as negative.

c -- is the apparent spring temper.

B -- is the tank track width.

We will designate the additional spring deflections by the coordinates z, ψ and Θ, then the following equations will be formed:

$$\left.\begin{aligned} f_i &= z - l_i\psi - \frac{B}{2}\Theta \\ f_i' &= z - l_i\psi + \frac{B}{2}\Theta \end{aligned}\right\} (i = 1,2 \cdots n) \qquad \text{(b)}$$

If we substitute values (b) of the additional spring deflections into the formulas (a), we will obtain the following equations:

$$\left.\begin{aligned} N &= -2cnz + 2c\psi \sum l_i \\ M_x &= -\frac{cnB^2}{2}\Theta \\ M_y &= 2cz \sum l_i - 2c\psi \sum l_i \end{aligned}\right\} \qquad \text{(c)}$$

The rectifying force and the rectifying moments will be equalized at an arbitrary point in time by the force of inertia and the moments of the force of inertia of the tank hull; this means that:

$$\left.\begin{aligned} N - \frac{G}{g}\ddot{z} &= 0 \\ M_x - J_x\ddot{\Theta} &= 0 \\ M_y - J_y\ddot{\psi} &= 0 \end{aligned}\right\} \qquad \text{(d)}$$

If we substitute expressions (c) for the rectifying force and the rectifying moments into equations (d), we obtain the differential equations of the free vibrations of the tank hull.

$$\left.\begin{aligned} &\frac{G}{g}\ddot{z} + 2cnz - 2c\psi \sum l_i = 0 \\ &J_y\ddot{\psi} - 2cz \sum l_i + 2c\psi \sum l_i^2 = 0 \\ &J_x\ddot{\Theta} + \frac{cnB^2}{2} = 0 \end{aligned}\right\} \qquad (36)$$

When we introduce the expressions:

$$\frac{2cng}{G} = a$$

$$-\frac{2c\sum l_i g}{G} = b$$

$$-\frac{2c\sum l_i}{J_y} = u$$

$$\frac{2c\sum l_i^2}{J_y} = e$$

$$\frac{cnB^2}{2J_x} = k^2$$

we will be able to rewrite Eqs. (36) in the following form:

$$\left.\begin{array}{l} \ddot{z} + az + b\psi = 0 \\ \ddot{\psi} + cz + e\psi = 0 \\ \Theta + k^2\Theta = 0 \end{array}\right\} \qquad (36')$$

We refer to the fact that the obtained equations (36') are in no way different in outer appearance from the differential equations (21) and (28) of the free vibrations of the tank hull with swing arm suspension.

For this reason we do not wish to dwell on the solution of equations (36'), for otherwise we would have to repeat all our considerations which we have already used in the solution of equations (21) and (28).

d) Estimation of Driving Characteristic

Because the tank is primarily a platform for firing weapons, its driving characteristic must be judged according to the effect of the weapons under movement.

It has already been mentioned that the effect of the weapons from movement is reduced mainly for the following reasons:

a) by a change of the weapon's elevation angle during firing,
b) by the gunner's eye moving on the eyepiece of the telescopic sight.

The change of the weapon's angle of elevation while the shot is being fired corresponds to the product of the angular mean velocity of the tank's vibrational movement and the time for the shot to be fired.

It is completely clear that with the reduction of velocity of the torsional, longitudinal and transverse vibrations of the tank hull, the change of the weapon's elevation angle during the time the shot is being fired will be reduced.

The movement of the gunner's eye from the eyepiece of the telescopic sight is stipulated by significant changes (accelerations) of the vertical vibrations of the tank hull center of gravity. With a reduction of this shift acceleration, the movement of the gunner's eye from the eyepiece of the telescopic sight will also be reduced.

Accordingly, the values of angular velocities of the pitching and transverse vibrations of the tank hull as well as the value of the angular acceleration of vertical vibrations of the tank hull must be considered important for evaluating the uniformity of tank travel.

A tank with road wheels having individual springs should serve as an example (Figs. 27 and 28).

We wish to consider the angular velocities of the longitudinal and transverse vibrations of the tank hull as well as the acceleration of vertical vibrations of the center of gravity which occur during the rolling of the road wheel, i.e. the first road wheel on the right side over some ground unevenness.

When the road wheel rolls over the elevation, the spring will be compressed by the amount f_1'.

In the same measure as the road wheel rolls over the elevation, the additional deflection of the spring f_1' will change and can be considered as a certain function of time t.

$$f_1' = \varphi(t).$$

At the prevailing tank traveling speed, the type of the function $\varphi(t)$ will be stipulated exclusively by the longitudinal profile of the ground elevation over which the road wheel rolls.

When spring deflection changes, some kind of reaction occurs which affects the tank hull along the spring axis and equals the product of spring temper and the additional spring deflection.

Consequently, the right halves of the differential equations of vibration movement of the tank hull will no longer exhibit the value zero, but instead of Eqs. (36) will exhibit the following values:

$$\left.\begin{aligned} &\frac{G}{g}\ddot{z} + 2cnz - 2c\psi\sum l_i = c\varphi(t) \\ &J_y\ddot{\psi} - 2cz\sum l_i + 2c\psi\sum l_i^2 = cl_1\varphi(t) \\ &J_x\ddot{\Theta} + \frac{cnB^2}{2}\Theta = \frac{cB}{2}\varphi(t) \end{aligned}\right\} \qquad (37)$$

We will assume that the tank hull is initially located in a position of equilibrium and that the velocities of its torsional, longitudinal and transverse vibrations equal zero.

In addition we will assume that the tank is traveling at a high speed and that the ground elevation is short.

If this is the case, then the coordinates z, ψ and Θ in Eqs. (37) equal zero. With the considerable tank traveling speed and the brevity of the ground elevation, the time span t during which the road wheel rolls over this elevation will be short. Consequently, the deviations of the tank hull from the position of equilibrium during this short time span will be insignificant.

Consequently, we are dealing with the following equations:

$$\left.\begin{aligned} \frac{G}{g}\ddot{z} &= c\varphi(t) \\ J_y\ddot{\psi} &= cl_1\varphi(t) \\ J_x\ddot{\Theta} &= \frac{cB}{2}\varphi(t) \end{aligned}\right\} \quad (38)$$

from which we determine the following values:

$$\left.\begin{aligned} \ddot{z} &= \frac{cg}{G}\varphi(t) \\ \dot{\psi} &= \frac{cl_1}{J_y}\int_0^{t_1}\varphi(t)\,dt \\ \dot{\Theta} &= \frac{cB}{2J_x}\int_0^{t_1}\varphi(t)\,dt \end{aligned}\right\} \quad (39)$$

When we check expressions (39) we find that the function φ(t) with various tanks which drive over the same road at the same speed remains the same, since its character is determined exclusively by the longitudinal profile of the ground elevation over which the road wheel rolls.

On the other hand, the coefficients cg/G, cl_i/J_y and $cB/2J_x$ are determined exlusively by the suspension design of the tank.

For this reason it is advisable not to evaluate the drive characteristics of the tank according to the values $\ddot{z}$, $\dot{\psi}$ and $\dot{\Theta}$, but according to the coefficients cg/G, cl_i/J_y and $cB/2J_z$ the corresponding values of which stand in direct ratio to the values $\ddot{z}$, $\dot{\psi}$ and $\dot{\Theta}$ but are not determined by any factors other than the type of track and suspension.

We wish to name these coefficients "coefficients of uniform tank travel". The smaller the value of such a coefficient, the more uniform the tank will travel. The expressions for the coefficients of uniform tank travel, if it is equipped with swing arm suspension, are derived in the same manner as in the case of a tank having road wheels with individual springs.

Without going further into these methods of derivation, we wish to state here that when the tank is equipped with a bogie wheel suspension frame (Fig. 25) the following coefficients of tank travel uniformity will come under consideration:

$$\frac{cg}{iG};\quad \frac{cl_1}{iJ_y};\quad \frac{cB}{2iJ_x},$$

where the relationship between the vertical shift of the road wheel and the deflection of the spring which causes this shift is designated by i.

If the swing arm mechanisms have the same levers, then the value of the bogie wheel suspension frames having two road wheels each (Fig. 25) is i = 2. In the case of swing arm suspensions with four road wheels (Fig. 5) it is i = 4.

An increase of the ratio i results in a decrease of the values of the ratios of tank travel uniformity; consequently the tank will travel in a more uniform manner.

This is due to the favorable influence of the swing arm mechanisms which aid the uniformity of tank travel.

e) Load on the Road Wheels with Consideration of the Influence of Tank Hull Vibrations.

In Section 3 we discussed road wheel load when the road wheels have individual springs. The following equations (6) were determined for the purpose of determining normal load on the road wheels:

$$N_i' = c\left[z_0 - (\alpha_i - \alpha)\,k_x - \frac{B}{2}\,k_y + u_i'\right]$$

$$N_i'' = c\left[z_0 - (\alpha_i - \alpha)\,k_x + \frac{B}{2}\,k_y + u_i''\right].$$

We now wish to introduce an improvement into these equations which is important for the influence of vibrations on the tank hull.

We will refer to the fact that the values z_o, k_x and k_y indicate the lowering of the tank hull center of gravity and its longitudinal and transverse differences from the position of equilibrium. If, however, the tank hull is subject to a vibration, then the actual lowering of the center of gravity, likewise its longitudinal and transverse differences, will be:

$$z_0 + z$$
$$k_x + \psi$$
$$k_y + \Theta$$

where z, ψ and Θ are calculated according to Eqs. (36).

If we substitute the values $z_o + z$, $k_x + \psi$ and $k_y + \Theta$ into the equations (6) instead of the values z_o, k_x and k_y, we obtain equations of the normal loads on road wheels with consideration of tank hull vibrations.

$$N_i' = c\,(z_0 + z - (\alpha_i - \alpha)\,k_x + \psi) - \frac{B}{2}(k_y + \Theta) + u_i'$$
$$N_i'' = c\,(z_0 + z - (\alpha_i - \alpha)\,kx + \psi) + \frac{B}{2}(k_y + \Theta) + u_i''.$$

We now wish to turn to swing arm suspension. In Section 5 equations (13) were set up for the purpose of determining the normal loads on road wheels without considering tank hull vibrations. These equations were:

$$T_1' = T_1\left(\frac{1}{2} - \frac{y}{B}\right); \quad T_2' = T_2\left(\frac{1}{2} - \frac{y}{B}\right)$$
$$T_1'' = T_1\left(\frac{1}{2} + \frac{y}{B}\right); \quad T_2'' = T_2\left(\frac{1}{2} + \frac{y}{B}\right).$$

The road wheel springs were additionally deflected when tank hull vibrations occurred. These deflections were determined according to Eqs. (c).

$$f_1' = z - l_1\psi - \frac{B}{2}\Theta; \quad f_2' = z + l_2\psi - \frac{B}{2}\Theta$$
$$f_1'' = z - l_1\psi + \frac{B}{2}\Theta; \quad f_2'' = z + l_2\psi + \frac{B}{2}\Theta.$$

The values z, ψ and Θ which occur in these equations were determined according to equations (35) and (22').

Aside from the changes of spring deflections, each of the road wheels will be subjected to the effect of an additional vertical load which is equal to the product of spring temper and additional deflection.

Consequently, the vertical loads of the road wheels with consideration of tank hull vibrations are determined according to the following equations:

$$T_1' = T_1\left(\frac{1}{2} - \frac{y}{B}\right) + c f_1' \qquad T_1'' = T_1\left(\frac{1}{2} + \frac{y}{B}\right) + c f_1''$$
$$T_2' = T_2\left(\frac{1}{2} - \frac{y}{B}\right) + c f_2' \qquad T_2'' = T_2\left(\frac{1}{2} + \frac{y}{B}\right) + c f_2''.$$

We shall refer to the fact that while the tank is traveling, especially over uneven ground, vibrations can increase to such an extent that spring deflections can reach an extreme, which is to be considered in the design.

It was already stated in Section 4 that the static deflection of the springs by the weight of the tank being imposed on the springs usually makes up one-third to one-half of the maximum deflection, which is to be considered permissible according to the track and suspension design.

Consequently, tank hull vibrations which occur within the elasticity limit of springs can have such an effect that the vertical load of the road wheels in comparison to static load is doubled or tripled.

7. Impact Stress of the Track and Suspension

The travel stroke of the road wheel in comparison to the tank hull is usually limited by a stop.

During rapid travel when the tank travels over any ground elevation which is higher than the travel stroke of the road wheel, the tank will be subjected to an impact and the suspension system will be subject to the effect of the impact stress, which may cause some parts to fracture.

The answer to the question to what extent it is possible to foresee such a fracture is of very practical interest.

We will now consider more closely the indicated phenomenon in the case of a tank having road wheels with individual springs.

We will assume that the tank has traveled over a rather high ground elevation with a pair of its road wheels in such a way that the moment arm OC (Fig. 29) which joins the road wheel axis to the tank hull, has already struck against stop B while the road wheels move further upward and roll over the ground elevation.

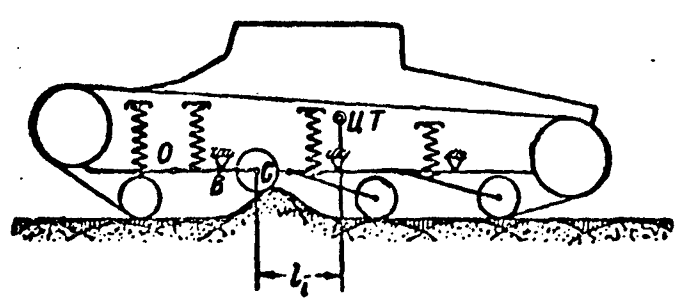


Figure 29. Road Wheel Running Over a Ground Elevation.

Moment arms OC strike against stop B and are deformed to a certain extent; the type of deformation is determined by the design.

We will assume thatthis deformation appears in the form of a bend. The force of the bent moment arm becomes a motive force of the tank hull due to the pressure on the stops and on the joints which connect the moment arms to the hull.

If we were in a position to determine in some way the increase of motive force which is communicated to the tank hull when the moment arm strikes against the stops, then we would know how great the deformation of the moment arm is; we would do this by comparing the increase of motive force on the tank hull with the force deforming the moment arm.

When we had determined the extent of deformation, it would not have been difficult to calculate the stress on the moment arm according to known formulas for the resistance force of materials.

Our task may be summarized as follows: to determine the increase of motive force on the tank hull when moment arm OC strikes against stops B.

The increase of motive force on the tank hull when the moment arm strikes against the stops is determined by the speed of the vertical shift of the road wheels at the moment of impact.

On the other hand, the speed of the vertical shift of the road wheels when rolling over the ground elevation is related to the speed of forward movement and with the nature of the ground elevation.

We will designate the speed of forward tank movement as v, the maximum angle between the tangential surface to the front inclined surface of the ground elevation and the direction of the speed v as φ_{max} (Fig. 30).

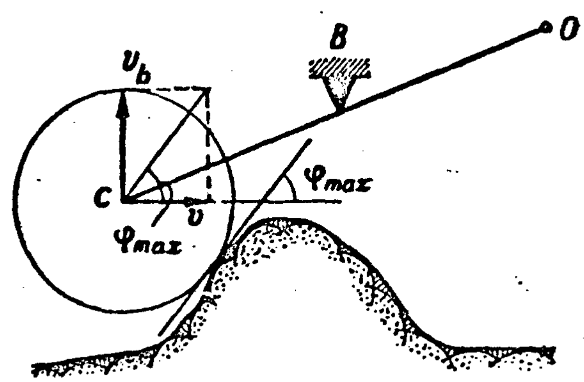


Figure 30. Road Wheel Running Over a Ground Elevation.

Then the speed of the vertical shift of the road wheels the moment the arm touches the stops is expressed in the following equation:

$$v_B \leq v \operatorname{tg} \varphi_{max} \tag{a}$$

We will refer to the fact that the road wheels axes, when the moment arm OC strikes against the stops, can be considered as being firmly connected to the tank hull because the shift of the axes due to deformation of the moment arm will be insignificant.

Under this assumption we can relate the elevating speed of the road wheels with the elevating speed of the center of gravity and with the angular velocity of the tank hull by the following equation:

$$v_B = \dot{z}_1 + l_i \dot{\psi}_1 \tag{b}$$

where the elevating speed of the tank hull and its angular velocity due to the moment arm striking the stops is expressed by $\dot{z}_1$ and $\dot{\psi}_1$.

When the moment arm strikes against the stops, impact stresses occur on the road wheel axes, the impulse of which is indicated by R.

Under the effect of these shock stresses, the tank hull undergoes a change of motive "energy" and a change of the moment of this motive "energy". These changes are expressed by the following formula:

$$\left.\begin{aligned} \frac{G}{g}(\dot{z}_1 - \dot{z}_0) &= R \\ J_y(\dot{\psi}_1 - \dot{\psi}_0) &= R\,l_i . \end{aligned}\right\} \tag{c}$$

The initial values of elevating speed of the tank hull center of gravity and of its angular velocity around the transverse axis when the moment arm strikes the stop are designated by $\dot{z}_0$ and $\dot{\psi}_0$.

When we eliminate the value R from equation (c), we obtain the following equation:

$$\frac{G}{g}(\dot{z}_1 - \dot{z}_0)\,l_i - J_y(\dot{\psi}_1 - \dot{\psi}_0) = 0. \tag{d}$$

When we solve equations (b) and (d) together, we obtain:

$$\left.\begin{aligned} \dot{z}_1 &= \frac{J_y(v_B - l_i\dot{\varphi}_0) + \frac{G}{g} l_i^2 \dot{z}_0}{J_y + \frac{G}{g} l_i^2} \\ \text{and} \qquad \dot{\varphi}_1 &= \frac{\frac{G}{g}(v_B - \dot{z}_0) l_i + J_y \dot{\varphi}_0}{J_y + \frac{G}{g} l_i^2}. \end{aligned}\right\} \tag{e}$$

When we have determined the value of $\dot{z}_1$ and $\dot{\Psi}_1$, we can easily determine the increase of motive energy in the tank hull ΔE.

$$\Delta E = \frac{1}{2}\left[\frac{G}{g}\left(\dot{z}_1^2 - \dot{z}_0^2\right) + J_y\left(\dot{\varphi}_1^2 - \dot{\varphi}_0^2\right)\right] \tag{f}$$

If we substitute the values $\dot{z}_1$ and $\dot{\Psi}_1$, we obtain the equation:

$$\Delta E = \frac{v_B^2 - (\dot{z}_0 + l_i\dot{\varphi}_0)^2}{2\left(J_y + \frac{G}{g} l_i^2\right)} J_y \frac{G}{g} \tag{42}$$

We now compare the moment of resistance against movement of the arms which connect the road wheel pair under consideration with the tank hull with the increase of hull motive energy:

$$2V = \frac{v_B^2 - (\dot{z}_0 + l_i\dot{\varphi}_0)^2}{2\left(J_y + \frac{G}{g} l_i^2\right)} J_y \frac{G}{g} \tag{g}$$

If we substitute the value v_B obtained from Eq. (a) into the obtained equation (g), we obtain finally:

$$2V \leq \frac{v^2 \operatorname{tg}^2 \varphi_{max} - (\dot{z}_0 + l_i\dot{\varphi})^2}{2\left(J_y + \frac{G}{g} l_i^2\right)} J_y \frac{G}{g}.$$

In the special case when the initial preconditions are such that

$$\dot{z}_0 = 0 \text{ and } \dot{\varphi}_0 = 0$$

Eq. (43) is simplified:

$$2V \leq \frac{v^2 \operatorname{tg}^2 \varphi_{max}}{2\left(J_y + \frac{G}{g} l_i^2\right)} J_y \frac{G}{g} \tag{44}$$

We will refer to the fact that for the center road wheels the value l_1^2 is smaller than for the front and rear road wheels. The resistance against movement of the moment arms is increased by the decline of the value l_1^2 .

This means that at otherwise equal preconditions the impact stress of the center road wheels will be greater than the corresponding loads on the front or rear road wheels.

After the resistance against the moment arm has been determined in this manner and for reasons of simplification we assume that this moment arm strikes with its center against the stop, we can easily determine the deflection of the moment arm according to the following formula:

$$f = \sqrt{\frac{V L^3}{24 E J}} .$$

L = the length of the moment arm OC,
E = the modulus of elasticity,
J = the moment of inertia of the arm cross section.

The bending stress of the arm corresponding to this deflection will amount to:

$$\sigma_{max} = f \frac{12 \alpha E}{L^2}$$

α signifies the distance of the neutral axis from the outer edge of the arm.

Section V.

AMPHIBIOUS TANKS

1. Amphibiousness

a) General Description

Amphibiousness is the ability of a body not to sink. The section of the water surface with the floating object is called the water line, which divides the body into an unsubmerged and a submerged part related to the principle of Archimedes.

If we consider a certain quantity of water in a solid container, the liquid will always be in a state of rest without outside influences, aside from air pressure; i.e. we will observe neither vertical nor horizontal currents or eddies in the liquid. This fact is explained by the position of equilibrium of all fluid particles.

This means that forces from all sides affect each layer of the liquid, that these are of equal intensity; the direction in which these forces are exerted is the same, but are directed against each other.

The force which impels a separately considered layer upward is equal to the weight according to the water column located over it. We now wish to remove this water column and replace it by some other body of equal weight.

In this case, two forces will be exerted on the vertical axis line z of this body: G weight of the body which is exerted at the center of gravity of the body and Q the buoyancy of the displaced water which is exerted upward and in the center of gravity of the displaced water mass -- the center point of the buoyancy.

If G is larger than Q, the body will sink; on the other hand it will float if Q is greater than G.

From this we can conclude that the buoyancy force Q is as great as the weight of the floating body G.

If the weight of the body is increased, this body will sink deeper until Q equals the new weight G.

The amphibious reserve of a tank means the amount of load (in kg) which can be imposed until the tank sinks down to the open hatches (ventilation hatches, radiator shutter, driver's hatch). The amphibious reserve of a fully equipped tank is designated as combat amphibious reserve.

In the case of small amphibious tanks the amphibious reserve is 200-400 kg. When the hatches, covers and trap doors are closed and when the space between the turret and the turret support ring is sealed, the amphibious reserve is two to three times as great as when the maximum submersibility of the tank reaches up to the roof.

It is particularly important to know the amphibious reserve of a tank.

If, for example, a large body of water is to be crossed and water suddenly penetrates into the vehicle and this excess cannot be removed by the pump, it is possible to judge whether one should proceed if the amphibious reserve of the tank is known.

It is particularly important to know the amphibious reserve of a tank when the tank is used to haul troops. From the previous data it is clear that 2-3 men can be hauled in the small tank.

Insofar as ship principles are used in the construction of amphibious tanks, it appears quite practical to maintain these principles. Consequently, we wish to apply the following principles for amphibious tanks:

1. The center longitudinal line divides the tank lengthwise into 2 symmetrical parts. It is designated as the longitudinal plane.
2. The surface which goes through the contact line of the water surface (in rest position) with the tank, is called the width (water line diagram) and its half, with consideration of the symmetry, is called the half-width.
3. The surfaces which lie vertical to the two induced ones, is designated as maximum width at its widest location.
4. The sections of the tank with the surfaces parallel to the water level are called the water lines and the framework vertical to them. It is clear that the frameworks are approximately equal with all cross sections. The water line which a tank exhibits under all loads is called the load line.

   The framework rib selected at the widest location is called the center framework rib.

b) Determination of the Load Line

Before we distribute the loads finally in the tank hull with a given outline and provide the openings for air intake to ventilate and cool the crew compartment and before we provide the driver's

escape hatch and the viewing slits, it is first necessary to determine the water line.

The normal water line must be designated as the one at which the tank exhibits rear load distribution of 5-10°.

At normal floating position (with the usual load in water with slight waves), the air vents, the driver's hatch and the viewing slots must lie so far above the loading hatch that no water can penetrate into the chassis from without.

In ship construction this detail is attended to with extreme care and the method used can also be used to solve our problem. But nevertheless it appears practical to deviate from the ship building method.

Since the shape of a tank deviates from the outlines of a ship, it would save time to determine the loading line according to the following simple method.

We have before us the outline of a tank with the center of gravity indicated (Fig. 1). Now we wish to calculate the loading line.

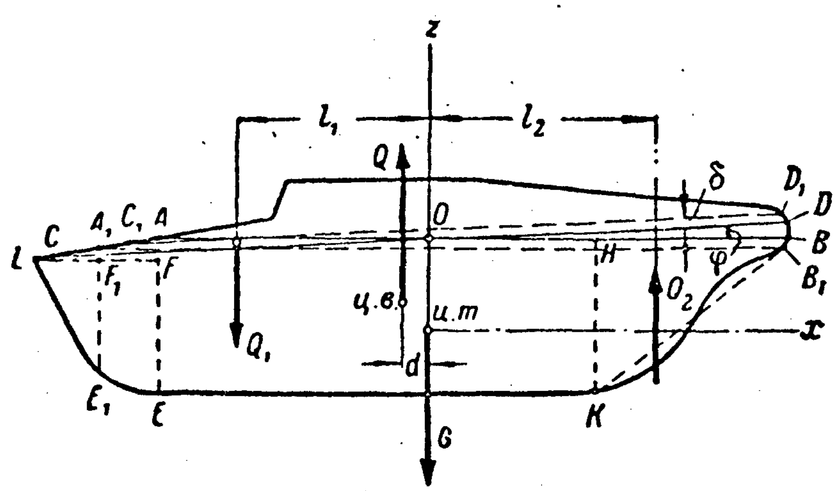


Figure 1.

The first question is: What do we do about the track and suspension which usually is fully submerged in the water ?

Here we can find two different solutions:

1. Determine the buoyancy of each individual part by means of water penetration and determine the point at which it is exerted and thus determine the resultant force and the buoyancy center point.

2. Calculate the weight of each individual part of the track and suspension and divide the result by the specific average weight of the construction material and in this way determine the total force of buoyancy. It can be assumed with sufficient accuracy that the buoyancy center point of the track and suspension coincides with the tank center of gravity.

As a basis of this assumption we use the fact that when the tracks are freed from the ground while the tank is floating, no visible change of the water line can be observed and thus we feel ourselves justified in using the method mentioned twice before.

Consequently, the weight of the tank to be calculated is:

$$O = G_0 - \frac{G_2}{\gamma_0}.$$

G_o = the weight of the tank chassis.

G_2 = the weight of all individual parts of the track and suspension.

γ_o = the specific mean weight of the construction material, i.e.

$$\gamma_0 = \frac{\gamma_1 + \gamma_2 + \gamma_3}{n} \cdots \gamma_n.$$

Numerically, G in t is equal to the volume of water penetration V in m^2.

Buoyancy Q = G, on the other hand, is G = V = SB, where B is the width of the chassis and S is the surface of the longitudinal rib to the water line.

As we see, our main task will be to draw a line AB in such a way that the surface which it intersects equals S.

We wish to determine this surface by quoting the calculation for an even keel, i.e. by assuming that the trim equals zero.

We will be able to determine the height h between the assumed water line and the ground by the following equation:

$$h = \frac{S}{L}(1 \div 1{,}07),$$

where L is the mutual distance between the axes of the idler wheel and the drive sprocket.

We will assume that we obtain the water line AB. In order to be able to judge the correctness of the selection, we must calculate the determined surface.

For this purpose we will employ the following method:

a) We set the outline of the stern in place of its curve through the straight line BK in such a way that the intersected and the accessory surfaces are equal.

b) In the bow we draw a vertical line to the ground A. E. in the intersected part A E L. Then we obtain three geometrical figures: the triangles ALF and LF_1E_1 as well as the trapezoid FEE_1F_1.

As we see, the S_1 intersected by the water line will be as large as the sum of the surfaces of the geometrical figures.

Almost all dimensions of the named figures can be determined by calculating with the inserted dimensions from the drawings; very little calculating need be done with the ruler and the compass.

We will assume that the S_1 intersected by the water line AB is S. To equalize this difference $S - S_1$ we proceed as follows:

From the obtained water line AB we divide upward and downward new water lines (AB) in intervals of 1 cm each and add or subtract the corresponding surfaces until we achieve an equality of S and S_1 $(S = S_1)$.

We are assuming that as a result we will achieve the water line AB at which buoyancy is $Q = G$.

The buoyancy center point is determined from two equations of moments opposite some axis (for example the one which passes through the center point of gravity vertical to the drawing surface).

The first equation is written down when the forces run parallel to the weight line and the second when the forces have turned at an angle of 90°.

As a result, we obtain the following equations:

$$x = \frac{\sum Q_i x_i}{Q}$$

$$z = \frac{\sum i Q_i z_i}{Q} \quad \text{(1)}$$

Q_i equals the surfaces of the individual figures into which the entire surface is divided.

x_i and z_i are the coordinates of the centers of gravity in these figures.

As a rule, the buoyancy center of gravity does not correspond with the weight center of gravity; it lies somewhat above or below it.

In the case of a tank, only the second position must be considered admissible, because the first position causes top-heaviness.

Ships traveling on the sėa or ocean are built with even keel or with rear heaviness, because if a ship were topheavy, the bow would be forced deeper into the water, increasing the surface in the water and thus increasing travel resistance; while the "water column" over the ship's propeller would be lower (see "selection of propeller" in the following section).

River ships, on the other hand, often have the bow deeper in the water (are topheavy) so that the keel does not hit a reef.

Trimming a tank for rear heaviness is attributable to the effort to keep the top of the tank dry because this makes it easier for the driver to steer. In addition, turbulence build-up is reduced, the propeller lies deeper in the water and the tank can aim easier.

Therefore, after the calculation if Q is shifted toward the rear, the load must be regrouped or the chassis changed in such a way that Q and G exchange places.

We will assume that Q lies to the left of G. It is understandable that the moment Qd would turn the tank hull at some angle φ so that one keel OCA is out of the water while the other keel QDB is immersed.

The angle of rotation of the tank hull and the relative sizes of the surfaces of the keels out of the water and submerged must be correct for the following preconditions:

$$Qd = (l_1 Q_1 + l_2 Q_2);$$
$$Q_1 = Q_2.$$

In order that these preconditions might be fulfilled, we proceed as follows: we draw the water line CG through point O at an angle of $\varphi = 5$. Then we obtain the triangles ODB and OAC.

We are assuming that the surface of the triangle $ODB = S_2$ and the surface of the triangle $OAC = S_1$ and $S_2 \neq S_1$. Then we proceed with the determination of the difference between S_1 and S_2 as follows: we divide $S_1 - S_2 = 1\,S$ by the length of the projection of the water line DC and in this way obtain the thickness of the adjusted layer δ, i.e.:

$$\delta = \frac{S_1 - S_2}{DC}$$

The water line must be raised or lowered to this level in order to achieve an equality between the amount of surfaces of the keels out of the water and submerged.

Then we determine the point at which the forces Q_1 and Q_2 are exerted and again calculate the following moment:

$$M = Q_1 l_1 + Q_2 l_2.$$

If M is not equal to Qd, then we draw a new water line with the angle $\varphi = 10$ and repeat all calculations in the previously given sequence.

Usually both these calculations are sufficient if we achieve the following relative sizes:

$$M_1 < Qd < M_2.$$

The moments M_1 and M_2 correspond to the angle of rotation $\varphi_1 = 5^\circ$ and $\varphi_2 = 10^\circ$.

With this we conclude the determination of the loading line.

2. Inertia (Stability)

By the inertia of a tank we mean its ability to float in the position imparted to it (center longitudinal diagram vertical to the nonmoving surface of the water) and its ability to return to this position as soon as these exterior forces are no longer exerted which have caused it to deviate from its original position.

Inertia is important for the ability of the tank to go into the water at a certain angle. The crew (or the load) can be regrouped (i.e. the crew can leave the tank and position itself in the rear) and in this way change the weight, (fuel consumption). In addition, inertia is important for the dynamic loads (firing, going over waves) without having to fear the danger of tipping.

Inertia must be calculated both crosswise and lengthwise.

Here we will cite the solution of the problem (list and trim) in succession for a small and a large angle of tilt, while assuming that the cause for the tank being brought out of its position of equilibrium is of a static nature.

a) Tank Inertia at a Small Angle of List

We will proceed on the assumption that the tank is inclined to one side at a small angle θ (Fig. 2).

In order to be able to judge the degree of inertia of the tank in the prescribed position, we must know the position of three points in relation to each other: the center of gravity O, the weight center of gravity C and the metacenter M.

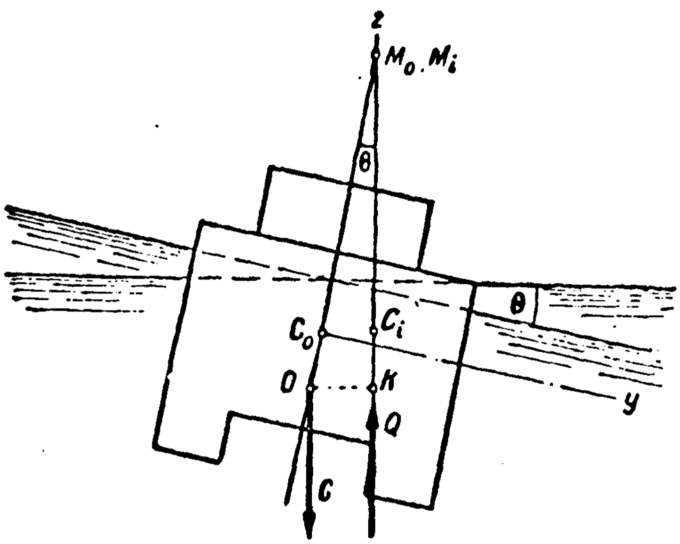


Figure 2. Position of the Metacenter at Slight List

The tank is the only amphibious vehicle with which point O lies deeper than point C.

The metacenter is the point of intersection of the plumb line through the center of buoyancy and the center longitudinal axis and is a measure for stability. Stability increases with the distance of the metacenter from the center of gravity. When the vehicle is tilted, the metacenter is located in the point of intersection of a rectangle through the displacement center to the inclined water line with a rectangle through the vehicle weight center of gravity to the construction water line.

Of course the weight center of gravity will be grouped correspondingly with further turning of the chassis while it describes a curve, the curvature radii of which form sections $M_i C_i$ -- the metacentric radius.

These radii will be directed vertically to the tangent to the curve of the weight center of gravity in the corresponding points and to the water line producing the effect.

We wish to introduce the following designations:

the initial metacentric radius,
the height of the center of gravity,
metacentric height.

In order for the tank to resume its original position, it is necessary for the forces G and Q to form a force pair which turns the tank into the opposite position (from tilt). In this case the metacenter would be located nearer to point O.

Of course the size of the moment of the rectifying force pairs $M = G \cdot OK$ will be greater (and thus the tank inertia as well) the higher point M is located.

In order to determine line O K, we will utilize the character of the point M, according to which, when the tank lists slightly, this does not change its position, since the amount of the keel which is immersed and withdrawn from the water in this case is usually equal.

On the basis of this information we obtain the equation:

$$OK = OM_0 \sin \Theta = (a + \varrho_0) \sin \Theta,$$

because at small angles

$$\sin \Theta = \frac{\Theta^\circ}{57{,}3}$$

if

$$OK = \frac{\Theta^\circ}{57{,}3} (a + \varrho_0) \qquad (2)$$

and the rectifying moment:

$$M = G (a + \varrho_0) \frac{\Theta}{57{,}3} \qquad (3)$$

The equation thus obtained will be designated as the formula of the metacentric inertia.

Since tanks have a low freeboard, the dimensions of both keels (which go in and out of the water) are unequal even at slight list, so that the application of the metacentric formula is very limited.

The metacentric formula can be used to determine the angle of list in the case of asymmetrical consumption of the load (fuel consumption) opposite the center of gravity or in the case of rearranging the load (regrouping the crew) as well as to determine the height of the center of gravity if M, θ^{o} and ϱ_{o} are given and α is being sought.

Without coming to conclusions, we will refer to the fact that the initial metacentric radius is

$$\varrho_0 = \frac{J_0}{V_0} \tag{4}$$

J_o is the moment of inertia of the water line surface opposite axis X or G (the axial lines pass through the center of gravity of the named surface) in relation to which inertia is to be determined, transverse or longitudinal stability;

V_o is the water displacement of the tank.

If we substitute its value for ϱ_o according to equation (4), we obtain the following equation:

$$M = \frac{\theta^{o}}{57,3}\left(a + \frac{J_0}{V_0}\right)G. \tag{5}$$

b) Influence of Liquid Ballast on Tank Inertia

This situation is important for the amphibious tank if the part of the chassis below water is not completely tight and if the pump is not capable of removing the water which has penetrated inside.

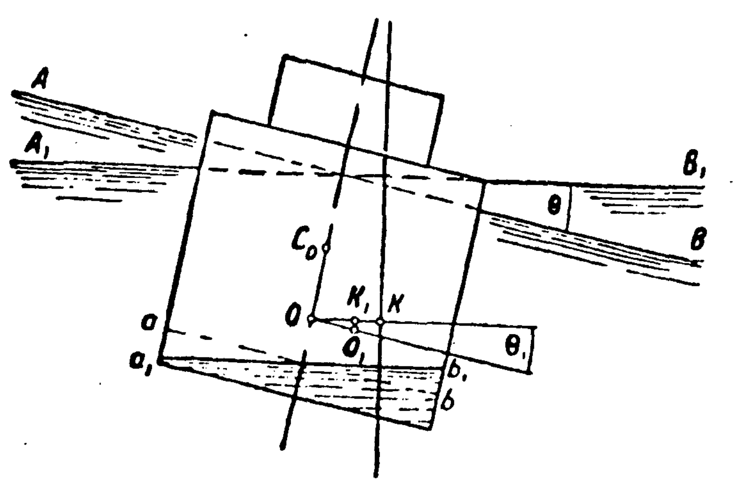


Figure 3. Stability of the Tank with Water in the Hull

We will assume that water has accumulated on the floor of the tank (Fig. 3). If the tank is listing at an angle of Θ, the surface of the water will assume the position in which the water center of gravity is shifted to the right which causes a general shift of the tank center of gravity from point O to point O_1. The result is that the line of the rectifying moment will be expressed in the following formula:

$$K_1\overline{K} = O\overline{K} - O\overline{O}_1 \cos\Theta .$$

After a few transformations, the difference given can be written in the following form:

$$K_1\overline{K} = O\overline{K} - \frac{J_1}{V_0}\cos\Theta_1 \,\mathrm{tg}\,\Theta$$

or, if

$$O\overline{K} = \overline{OM}\sin\Theta$$

then

$$K\overline{K}_1 = \left(O\overline{M} - \frac{J_1}{V_0}\frac{\cos\Theta_1}{\cos\Theta}\right)\sin\Theta \qquad (6)$$

In this case J_1 is the moment of inertia of the cross section of the water accumulated on the floor of the tank.

Thus, when the water has flooded up to a certain level of the entire floor, $J_1 = J$ and $\varrho_1 = \varrho_o$. Consequently, the following formula results:

$$K_1\overline{K} = \left(\varrho_o + a - \varrho_0\frac{\cos\Theta_1}{\cos\Theta}\right)\sin\Theta = \left[a + \varrho_0\left(1 - \frac{\cos\Theta_1}{\cos\Theta}\right)\right]\sin\Theta \qquad (7)$$

If, however,

$$\Theta_1 < \Theta,$$

then

$$K_1\overline{K} < \alpha\sin\Theta,$$

i.e. the tank will exhibit very slight stability.

c) Stability at Large Angles of List

The reason why the metacentric formulas cannot be used at large angles of list is that with the inequality of the volume of the keel being immersed and coming out of the water, the weight center of gravity cannot be determined as before and the position of the metacenter is not precise.

Before we calculate the rectifying force pair, it is first necessary to solve the following two problems: to find the effective water line (Fig. 4) and to determine the position of point C_1.

We are assuming that the volume O_1AA_1 of the keel coming out of the water v_4 and of that being immersed in the water $O_1B_1B = v_2$, and that v_1 is larger than v_2. Clearly the effective water line A_2B_2 will lie

$$\delta = \frac{v_1 - v_2}{S_1}$$

higher than the water line A_1B_1.

S_1 is the surface of the water line.

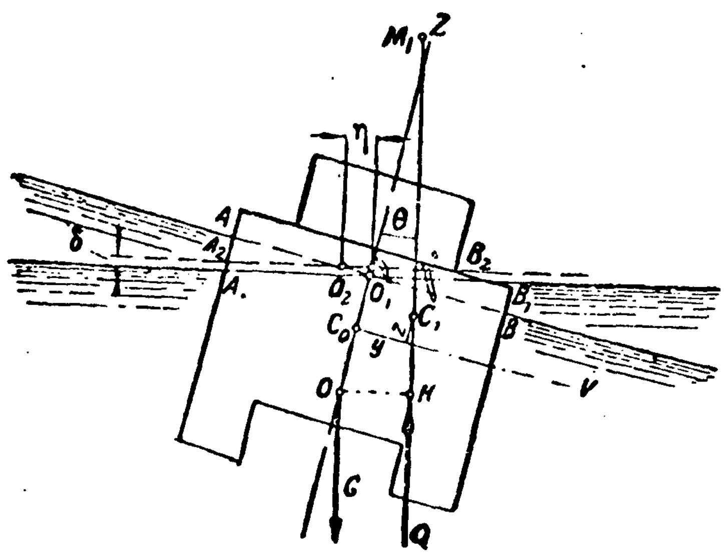


Figure 4. Weight Center of Gravity at Large Angles of List

The shift of the water line center of gravity to the right or left is determined according to the equation:

$$\frac{M_1 - M_2}{S_2} = \eta$$

Where:

M_1 is the static moment of the water line surface left of O_2.

M_2 is the same to the right of O_2.

S_2 is the surface of the water line A_2B_2.

These calculations must be used in sequence for all angles of list.

To determine the coordinates y and z of point C_4 we draw the part of drawing 4 individually as it is shown in drawing 5. On the basis of this drawing we can write the following equation:

$$O\overline{K} = \alpha \sin \Theta + y \cos \Theta + z \sin \Theta .$$

Since, however

$$dy = C_1 D \text{ and } dz = D C_1'$$

or

$$dy = C_1 C_1' \cos \Theta = \varrho \cos \Theta \, d\Theta$$

and

$$dz = C_1 C_1' \sin \Theta = \varrho \sin \Theta \, d\Theta$$

then ,

$$\left. \begin{aligned} y &= \int_0^\Theta \varrho \cos \Theta \, d\Theta \\ \text{and} \quad z &= \int_0^\Theta \varrho \sin \Theta \, d\Theta \end{aligned} \right\} \quad \ldots . \qquad (8)$$

These equations cannot be integrated at once because $\varrho = f$ (Q, where this function is not known).

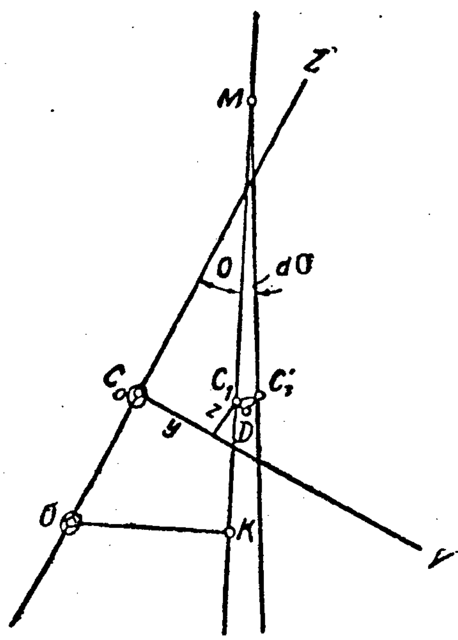


Figure 5. Grouping of the Nadirs in the Coordinate System

Usually integration is performed either graphically or numerically.

Prof. Krylow has made the suggestion to conduct the calculation of the rectifying force pairs by filling out Table 1, which puts us in a position to solve the problem rapidly.

Explanations to Table 1 (for Tables see supplement to the translation).

The formula $\frac{\pi}{180} \cdot \frac{\theta}{2}$ (5) shows us that this multiplier (coefficient) must be multiplied by the number cited in column 5, which corresponds to the basic values for θ. The other numbers in parentheses exhibit a corresponding meaning (7); (8); (9); (11); (12) and (13) which are in the other columns.

As an example we cite the calculation of stability in the design of an amphibious tank made by the Academy for Mechanization and Motorizing (WAMM).

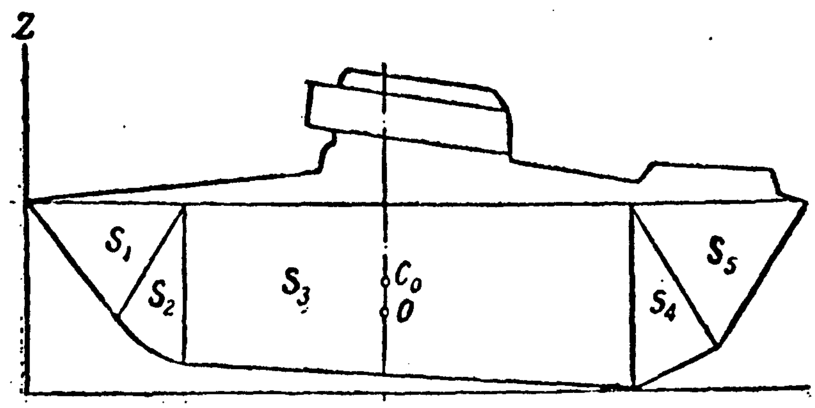


Figure 6. Sectioning of the Tank Body

The calculation is conducted according to the following plan:

1. The weight center of gravity of the submerged part of the longitudinal rib (Fig. 6) was determined.
2. The thickness of the correction layer δ and the shift of the center of gravity of the water line surface η is determined for all angles of list because these values are necessary for the calculation of ϱ of the effective water line.
3. The stability lines $O\overline{K} = r_i$ were calculated for the same tank angle of list.
4. Ried made diagrams of the longitudinal and transverse stability which indicated the curves $r_i = f(O)$.

For the purpose of determining the weight center of gravity, the tank is depicted in the shape shown on Fig. (6).

We divide up the hull below the water into five simple figures and calculate their surfaces. Then we obtain

$$S_1 = 2,260 \text{ cm}^2$$
$$S_2 = 1,480 \text{ cm}^2$$
$$S_3 = 12,000 \text{ cm}^2 \qquad \text{Total: } S = 18,410 \text{ cm}^2$$
$$S_4 = 870 \text{ cm}^2$$
$$S_5 = 1,800 \text{ cm}^2$$

The volume of the hull is $V_1 = S\,B = 18,410 \cdot 175 = 3,221,750$ cm^2. Now we calculate the volume of the water displaced by the submerged parts of the track and suspension.

$$\left.\begin{array}{ll} \text{2 tracks} & = 25,700 \cdot 2 = 51,400 \text{ cm}^3 \\ \text{2 idler wheels} & = 5,000 \cdot 2 = 10,000 \text{ cm}^3 \\ \text{2 drive sprockets} & = 2,000 \cdot 2 = 4,400 \text{ cm}^3 \\ \text{4 road wheels} & = 16,000 \cdot 2 = 32,000 \text{ cm}^3 \end{array}\right\} \text{ total } 141,400 \text{ cm}^2$$

The total volume of the submerged tank parts amounts to $V_o = v_1 + v_2 = 3,351,150 \text{ cm}^3$, i.e. the water displacement amounts to 3.35 m^3.

From Fig. 6 we determine the weight center of gravity for the hull and the track and suspension individually. For the hull we obtain:

$$z_k = \frac{\sum S_i z_i}{\sum S_i}; \quad x_k = \frac{\sum S_i x_i}{\sum S_i}.$$

If we measure the coordinates z_i and x_i according to the drawing, we obtain:

$$\begin{array}{ll} z_1 = 36.2 \text{ cm} & x_1 = 31.3 \text{ cm} \\ z_2 = 19.2 \text{ ,,} & x_2 = 67.5 \text{ ,,} \\ z_3 = 29.8 \text{ ,,} & x_3 = 182 \text{ ,,} \\ z_4 = 20.0 \text{ ,,} & x_4 = 300 \text{ ,,} \\ z_5 = 47.3 \text{ ,,} & x_5 = 310 \text{ ,,} \end{array} \qquad \sum S_i = 17\,600 \text{ cm}^2.$$

If we substitute the values of z_1 and x_1 into the equations of the coordinates z_k and x_k we obtain the following equation:

$$z_k = \frac{2260 \cdot 36.2 + 1,480 \cdot 19.2 + 12,000 \cdot 29.8 + 870 \cdot 20 + 1,800 \cdot 47.3}{18,410} = \text{cm}.$$

$$x_k = \frac{2260 \cdot 31.3 + 1,480 \cdot 67.5 + 12,000 \cdot 182 + 870 \cdot 300 + 1,800 \cdot 310}{18,410} = \text{cm}.$$

We determine the coordinates of the weight center of gravity for the entire tank

	Volume in cm^3	z cm	x cm
Body	3,222,000	00.0	000.0
Track	51,400	00.0	267.0
All individual parts of the track and suspension ..	90,000	-5.0	168.0

$$x_T = \frac{51{,}400 \cdot 167 + 3{,}222{,}000 \cdot 000 + 90{,}000 \cdot 178}{3{,}241{,}000} = 000 \text{ cm};$$

$$z_T = \frac{51{,}400 \cdot 25 + 90{,}000 \cdot 5 + 3{,}222{,}000 \cdot 000}{3{,}241{,}000} = 00{,}0 \text{ cm}.$$

Table a

Calculation of the static inertia r_i for the trim of a tank

Θ	$\varrho_i = \frac{J_i}{r_*}$	$\varrho_i \cos\Theta$	Sum (3) in pairs from above	Sum (4) from above	$y_i = V\,0{,}0435$	$\varrho_i = \sin\Theta$	Sum (7) in pairs from above	Sum (8) from above	$z_i = 9 \cdot 0{,}0435$	$z_i \sin\Theta$	$y_i \cos\Theta$	$z_i \sin\Theta + y_i \cos\Theta = 11 + 12$	$\alpha \sin\Theta$	$r_i = z \sin\Theta + y_i \cos\Theta + \alpha \sin\Theta = 13 + 14$
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0°	163	163,0	241,0	0	0	0	6,8	0	0	0	0	0	0	0
5	78	78,0	116,4	241,0	10,7	6,8	13,6	6,8	0,3	0,026	10,6	10,63	0,24	10,87
10	39	38,4	75,2	357,4	15,7	6,8	16,6	20,4	0,9	0,156	15,5	15,66	0,48	16,14
15	38	36,8	70,8	342,6	19,1	9,8	22,1	37,0	1,6	0,414	18,5	18,91	0,71	19,62
20	36	34,0	59,4	503,4	22,2	12,3	24,2	59,1	2,6	0,89	20,5	21,40	0,95	22,35
25	28	25,4	47,1	562,8	25,0	11,9	24,4	83,3	3,5	1,48	22,7	24,20	1,17	25,37
30	25	21,7	37,2	609,9	27,0	12,5	23,4	107,7	4,7	2,35	23,4	25,75	1,37	27,12
35	19	15,5	27,0	647,1	28,4	10,9	19,2	131,1	5,7	3,27	23,2	26,5	1,59	28,08
40	13	11,5	17,9	674,1	30,0	8,3	14,6	150,3	6,5	4,17	23,0	27,2	1,77	28,97
45	8,9	6,4	11,9	692,0	30,4	6,3	12,6	164,9	7,2	5,1	21,5	26,6	1,93	28,53
50	8,2	5,3	9,3	703,9	31,2	6,3	12,0	177,5	7,7	5,9	20,0	25,9	2,10	28,00
55	7,0	4,0	6,6	713,2	31,6	5,7	10,2	189,5	8,2	6,6	18,2	24,8	2,25	27,05
60	5,2	2,6	4,5	719,8	31,8	4,5	8,6	199,7	8,7	7,5	16,0	23,5	2,38	25,88
65	4,5	1,9	3,3	724,3	32,0	4,1	8,0	208,3	8,8	8,0	13,5	21,5	2,50	24,00
70	4,1	1,4	2,38	727,6	32,3	3,9	7,6	216,3	8,8	8,3	11,0	19,3	2,58	21,88
75	3,8	0,98	1,59	730,0	32,3	3,7	7,1	223,9	8,8	8,5	8,3	16,8	2,66	19,46
80	3,5	0,61	0,89	731,6	32,4	3,4	6,6	231,0	8,8	8,7	5,6	14,2	2,70	16,90
85	3,2	0,28	0,28	732,5	32,4	3,2	6,3	237,6	8,9	8,8	2,7	11,5	2,70	14,20
90	3,2	0		732,8	32,5	3,2		243,9	8,9	8,9	0,0	8,9	2,75	11,65

$r_i = 11 + 12 + \alpha \sin\Theta$ — when the center of gravity lies deeper than the center point.
$r_i = 13 - \alpha \sin\Theta$ — when the center of gravity lies higher than the center point

Table b

Calculation of inertia r_i in a bank.

θ	$e = \frac{J}{c_0}$	$e \cos\theta$	Sum (3) in pairs from above	Sum(4) from above	$y_i = V \frac{\cdot}{180}$ $\frac{\theta}{2} = V\,0.0435$	$e \sin\theta$	Sum(7) in pairs from above	Sum (8) from above	$z_i = 9 \cdot 0.0435$	$z_i \sin\theta$	$y \cos\theta$	11 + 12	$\alpha \sin\theta$	$r_i = z \sin\theta + y \cos\theta + \alpha \sin\theta = 13 + 14$
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0°	43,3	43,3	95,3	0	0	0	4.5	0	0	0	0	0	0	0
5	52,3	52,0	99,5	95,3	4,15	4,5	12,9	4,5	0,2	0,017	4,14	4,16	0,24	3,96
10	48,3	47,5	93,9	194,8	8,3	8,4	20,9	17,4	0,7	0,122	8,16	8,28	0,48	7,80
15	48,0	46,3	84,3	288,6	12,5	12,5	26,3	38,3	1,7	0,44	12,0	12,44	0,71	11,73
20	40,2	38,0	65,3	372,9	16,2	13,8	26,6	64,6	2,8	0,96	15,2	16,16	0,95	15,21
25	30,1	27,3	48,9	438,2	19,0	12,7	25,2	91,1	4,0	1,70	17,2	18,90	1,17	18,73
30	25,0	21,6	43,4	487,1	21,2	12,5	27,7	116,3	6,0	2,50	18,4	20,90	1,37	19,53
35	26,5	21,8	34,0	530,5	23,1	15,2	25,5	144,0	6,5	3,72	19,0	22,72	1,58	21,14
40	16,0	12,2	20,0	564,5	24,5	10,3	18,1	169,5	7,4	4,75	18,8	23,55	1,77	21,78
45	11,0	7,8	14,4	584,5	25,4	7,8	14,5	187,6	8,1	5,70	18,0	23,70	1,93	21,77
50	8,7	5,6	10,2	598,9	26,0	6,7	13,2	202,1	8,8	6,73	16,8	23,53	21,00	21,43
55	8,0	4,6	7,6	609,1	26,5	6,5	11,8	215,3	9,4	7,7	15,2	22,90	2,25	20,61
60	6,1	3,0	5,2	616,7	26,8	5,3	10,1	227,1	9,9	8,6	13,4	22,00	2,38	19,62
65	5,2	2,2	3,8	621,9	27,0	4,8	9,2	237,2	10,4	9,5	11,5	21,4	2,50	18,50
70	4,7	1,6	2,7	625,7	27,2	4,4	8,55	246,4	10,7	10,1	9,3	19,8	2,58	16,82
75	4,3	1,1	1,8	628,4	27,3	4,15	8,15	255,0	11,1	10,7	7,1	17	2,66	15,14
80	4,1	0,7	1,05	630,2	27,4	4,00	8,00	263,1	11,4	11,2	4,78	16	2,70	13,30
85	4,0	0,35	0,35	631,2	27,5	4,00	8,00	271,1	11,8	11,7	2,40	14	2,70	11,30
90	4,0	0,0		631,5	27,6	4,00		278,1	12,1	12,1	0,00	12	2,75	9,25

In determining the longitudinal and transverse inertia we are assuming that during list (tilt) no water penetrates into the tank.

Table c

Calculation of the additional layer δ and η for the trim of a tank.

θ	a_i	b_i	$\Sigma(a_i+b_i)$	$\frac{a_i^2}{100}$	$\frac{b_i^2}{100}$	$\frac{1}{2}\sum\frac{a_i^2-b_i^2}{100}$	Sum (7) in pairs from above	Sum (8) from above	$9-0.0435\times100$	$\delta=\frac{10}{4}$	$\eta=3-2$
1	2	3	4	5	6	7	8	9	10	11	12
0	166,0	167	—	—	—	—	82.5	—	—	—	—
5	98,0	162	260	96,0	263	82,5	188,0	82,5	360	1.4	31,6
10	62.0	158	220	38,5	250	105,5	213.7	270,5	1170	5.3	48,0
15	48.0	155	203,5	23,5	240	108.2	209.2	693.4	2100	9.0	54.0
20	48,0	150	198,0	23,0	225	101,0	192.0	885.4	3020	13.5	51,0
25	48.0	143	191,0	23,0	205	91.0	164.0	1049.4	3860	20.0	47.0
30	49,0	130	179,0	24.0	170	73.0	130,0	1179.4	4500	25.3	41,0
35	50,0	118	168,0	25,0	140	57.0	99.0	1278.4	5100	30.4	34.0
40	51,0	105	156,0	26.0	110	42.0	76,0	1354.4	5600	37.0	27.0
45	53,0	97,5	150,5	27,0	95	34.0	59,5	1413.9	5900	39.0	22,2
50	49,0	87,5	136,5	24.0	75	25.5	49,0	1462.9	6150	45.0	18,6
55	458,0	82,6	128,4	21.0	68	23,5	43.7	1506.6	6350	50.0	18.3
60	43,2	77,5	120,7	19,6	60	20.2	39.7	1546.3	6550	54,5	16,6
65	41.2	75,0	162,2	17,0	56	19,5	36.0	1582.3	6700	58.0	16.2
70	40,0	70,0	110,0	16,0	49	16.5	31.7	1613,3	6850	62,0	15,0
75	38,8	67,5	106,3	15,0	45,5	15.2	27.5	1640.8	7000	66.0	14.4
80	38,0	62,5	100,5	14,4	39,0	12,3	24,5	1665.3	7050	70.0	12.2
85	37,5	62,0	99,5	14,0	38,5	12.2	23,8	1688,3	7100	71,0	12.2
90	37,5	61,0	98,5	14,0	37,2	11,6		—	7150	72,0	11,8

If we know δ and η we determine the effective water line and the center of gravity, with the aid of which we also determine the moment of inertia of the water line surface and thus also the metacentric radius.

Table d

Calculation of the additional layer η while the tank is tilted.

θ	a_i	b_i	$\frac{a'_i}{100}$	$\frac{b'_i}{100}$	$a+b$	$\frac{1}{2}\frac{a'_i-b'_i}{100}$	Sum (7) in pairs from above	Sum (8) from above	$9-0.0435\times100$	$\delta=\frac{10}{6}$	$\eta=3-2$
1	2	3	4	5	6	7	8	9	10	11	12
0°	88,0	88,0	77,4	774,0	176,0	0.0	9,4	0	0	0	0
5	88,2	98,4	77,7	96,6	186,6	9,4	13,3	9,4	41	0.22	5,1
10	89,0	93,4	79,2	87,0	182,4	3,9	3,9	22,7	98	0,535	2,2
15	91,0	91,0	82,7	82,7	182,0	0,0	14,1	26,6	111	0,61	0,0
20	76,4	93,2	58,4	86,7	169,6	14,1	44,7	40,7	177	1,05	8,4
25	62,0	96,8	32,2	93,5	158,8	30,6	80,2	85,4	372	2,36	18,4
30	52,2	113,0	27,3	127,0	165,2	49,6	83,3	165,6	720	4,36	29,9
35	45,4	94,0	20,7	88,2	139,4	33,7	59,2	248,9	1090	7,8	24,3
40	40,6	82,4	16,5	67,7	123,0	25,5	46,8	308,1	1340	10,9	21,0
45	36,8	75,0	13,5	56,2	111,8	21,3	39,3	354,9	1550	13,8	19,1
50	34,0	69,0	11,6	47,7	103,0	18,0	33,6	394,2	1720	16,8	17,5
55	32,2	64,6	10,4	41,7	96,8	15,6	29,6	427,8	1860	19,2	16,3
60	30,2	60,8	9,1	37,0	91,0	14,0	27,4	457,4	2000	22,0	15,3
65	29,0	58,4	8,4	32,2	87,4	13,4	25,4	484,8	2120	24,2	14,7
70	28,0	56,4	7,8	31,9	84,4	12,0	23,3	510,2	2250	26,6	14,2
75	27,0	54,8	7,3	30,0	81,8	11,3	22,3	533,5	2350	28,8	13,9
80	26,6	53,8	7,1	29,0	80,4	11,0	21,7	555,8	2450	30,4	13,6
85	26,2	53,2	69,0	28,3	79,4	10,7	21,3	577,5	2510	31,6	13,5
90	26,0	51,0	6,8	28,0	79,0	10,6		598,8	2600	32,8	12,5

Note: The ordinates b' and a' were divided by 100 to reduce the obtain numbers and consequently to make it easier to work and to formulate the assumptions more easily.

As we note, the consideration of the size of the track and suspension individual parts exerts a very slight influence on the coordinates of the weight center of gravity of the whole tank. Consequently, to save calculating time, we can neglect this and limit ourselves to considering the weight center of gravity for the hull.

The thickness of the additional layer δ and the shift of the center of gravity η of the effective water line is determined according to Fig. 7.

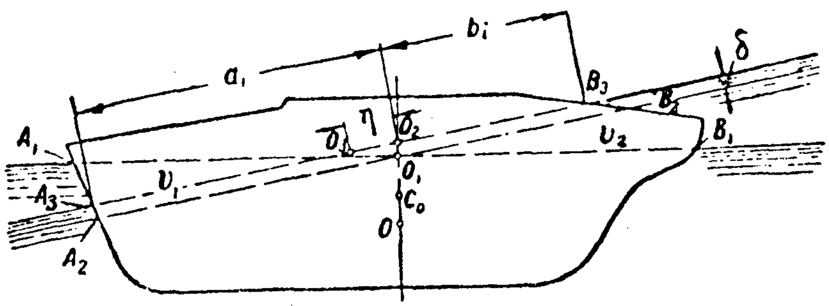


Figure 7. Calculating the Weight Center of Gravity for the Hull.

In this case:

a_i = the ordinates,

v_1 = the volume of the keel out of the water,

b_i = the ordinates and

v_2 = the volume of the keel submerged in the water.

As already mentioned above,

$$\delta = \frac{v_1 - v_2}{S_i};$$

but,

$$v_1 - v_2 = \frac{1}{2}\int_{-\frac{L}{2}}^{+\frac{L}{2}}\int_{0}^{\Theta}(a_i^2 - b_i^2)\,d\Theta\,dx; \quad S_i = \int_{-\frac{L}{2}}^{+\frac{L}{2}}(a_i + b_i)\,dx.$$

so that

$$\delta = \frac{\frac{1}{2}\int_{0}^{\Theta}\sum(a_i^2 - b_i^2)\,d\Theta}{\sum(a_i + b_i)} \qquad (9)$$

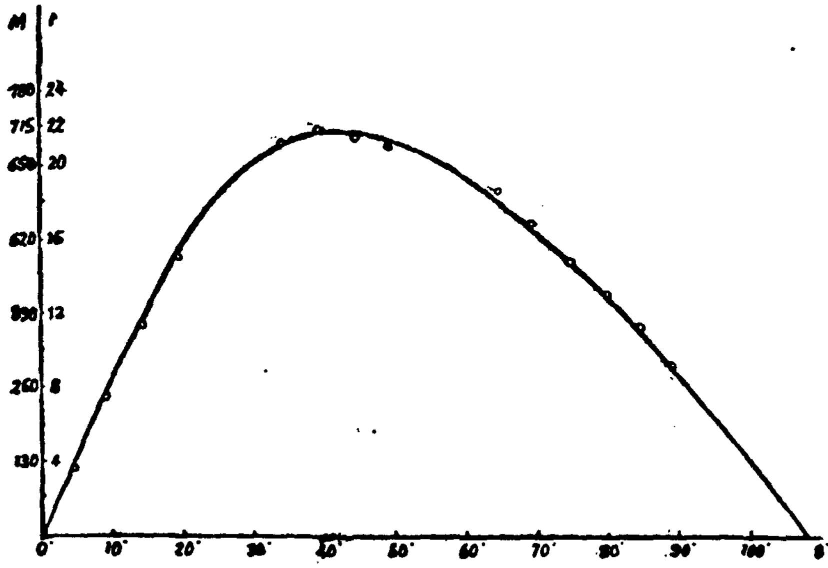


Figure 8. Diagram by Ried of the Longitudinal Inertia (stability) of the Floating Tank.

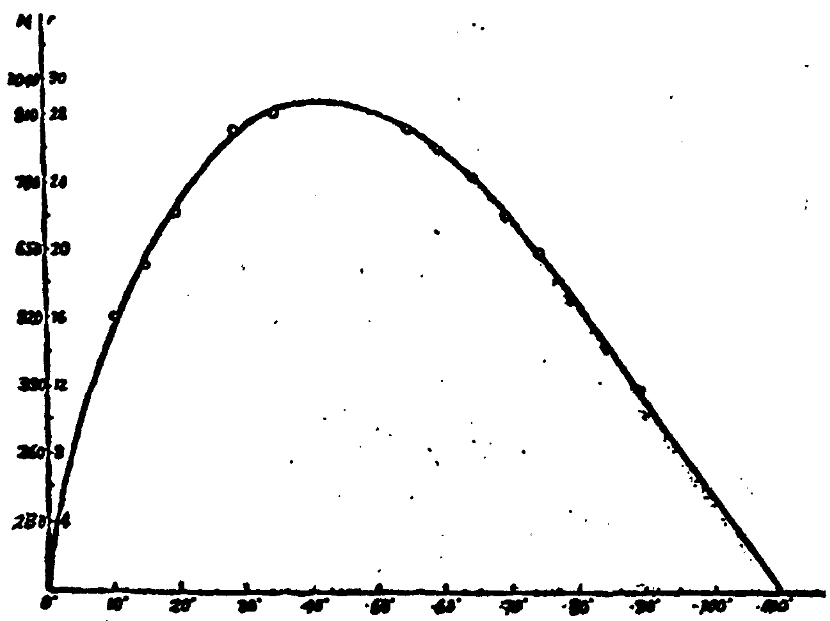


Figure 9. Diagram by Ried of the Transverse Inertia (stability) of the Floating Tank.

If a_i and b_i are a function of Θ, which are difficult to determine, the equation written here must be integrated according to the trapezoid method, i.e. fill out the table which comes into consideration in the calculation of r_i.

Since the width of the tank hull is a constant value over its entire length, the vertical lines through O to O_3 will shift by:

$$\eta = \frac{a_i - b_i}{2}.$$

We now cite the table for the calculation of δ and η according to which the submerged surface is calculated and in addition ϱ as well as the line of the restoring moment O K = r_i. According to the data in the table we have the diagrams of Ried (Figs. 8 and 9.).

3. Dynamic Static Stability

By dynamic static stability of a tank we mean its ability to offset the effect of the tilting (tipping, listing) force pair and to return to its original position as soon as the tilting force pair ceases to exert an influence.

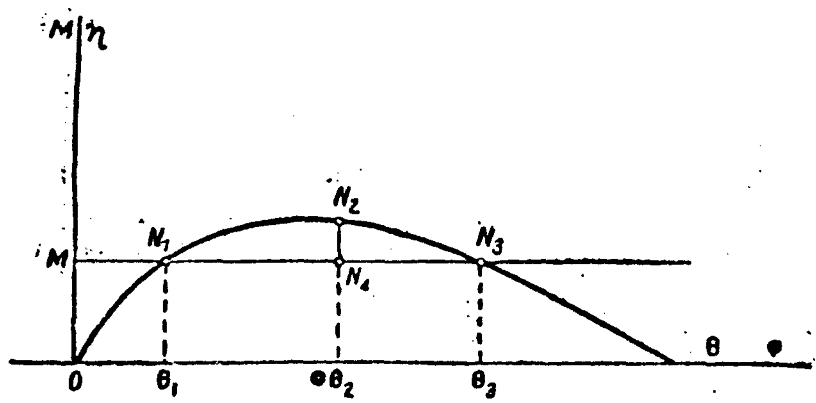


Figure 10. Ried Diagram of Static Stability

We are assuming that we have the Ried diagram of static inertia before us (Fig. 10) and that a force pair with constant moment M is being exerted on the tank. We determine the list angle of the tank under the influence of the force pair being exerted. Obviously, the tank will exhibit two angles of list:

Θ_1 indicates the static position of equilibrium at which the moments of the listing and of the restoring force pairs are equal;

Θ_2 indicates the dynamic position of equilibrium when the effects of the listing and rectifying force pair are equal.

Consequently, the tank will first turn at the angle Θ_2, at which the surfaces $OMN_1\Theta_2$ (exertion of the listing force pair) and $ON_1N_2\Theta_2$ (exertion of the rectifying force pair) are equal. Then the tank will list to the opposite side (due to the inequality of the moment ordinates Θ_2N_2 and Θ_2N_4) and when the vibrations have ceased, the tank will stop at an inclination at the angle Θ_1 at which the moments of both force pairs are equal.

If the surface $N_1N_2N_4$ is smaller than the surface OMN_1, then two things can occur:

1. The surface OMN_1 is larger than the surface $N_1N_2N_3N_4$, then the tank will capsize.
2. The surface OMN_4 is as large as the surface $N_1N_2N_3N_4$, then the tank will lay on one side at an angle Θ_3 and stay in this position until the listing force pair stops.

We now shall write down the formula for the rectifying force pair.

In normal position, the weight center of gravity and the displacement center of gravity are located at points C_o and O (Fig. 5). At a tilt angle Θ, the buoyancy center of gravity shifts to C_1. Of course while the listing force pair is being exerted during tilt at the angle Θ, the product of the rectifying force pair Q and the difference between the distances of the vertical lines between the displacement center of gravity and the weight center of gravity will be equal, i.e.:

$$-W = Q(K\,C_1 - O\,C_0);$$
$$K\,C_1 = a\cos\Theta - y\sin\Theta + z\cos\Theta;$$
$$O\,C_0 = a.$$

Consequently,

$$-W = Q\,[(z\cos\Theta - y\sin\Theta - a(1-\cos\Theta)] = +Q\,h_i \tag{10}$$

Using this equation we can plot the Ried diagram of the dynamic inertia of the tank while we divide the angle of list Θ on the abscissa axis and the line of inertia h_i on the line of ordinates.

It is not difficult to determine the relation between the lines of static and dynamic stability on the tank, i.e. between r_i and h_i.

The rectifying moment is $Q\,r_i$. The exertion of this moment during list at the angle Θ is expressed in the following equation:

$$W_i = \int_0^\Theta Q\,r_i\,d\Theta$$

but on the other hand,

$$W_i = Q\,h_i,$$

consequently,

$$h_i = \int_0^{\Theta} r\,d\Theta$$

i.e. the line of dynamic stability is in itself the integral curve of the static diagram by Ried.

The type of the curve $h_i = f(\Theta)$ of the Ried diagram of dynamic stability is determined in accordance with the following equations:

$$\frac{d\,h_i}{d\,\Theta} = r_i$$

and

$$\frac{d^2 h_i}{d\,\Theta^2} = \frac{d\,r_i}{d\,\Theta}.$$

From this it is clear that h_i will exhibit its maximum value at $r_i = O$. Curve $h_i = f(\Theta)$ exhibits its turning point at r_{max}.

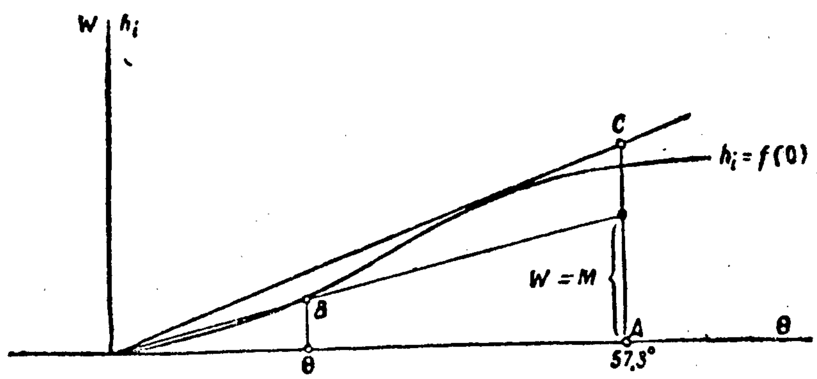


Figure 11. Ried Diagram of Dynamic Inertia

To obtain the necessary basic values to draw the Ried diagram, we must fill out the following table:

1	2	3	4	5	6	7	8	9
$\Theta_\circ$	y	z	$z\sin\Theta$	$z\cos\Theta$	V—IV	$a\cos\Theta$	$a - a\cos\Theta$	h_i = VI—VIII

At the angles Θ we must proceed in the same way as when drawing the Ried diagram of static stability; a, y and z are taken from the tables, according to which the value of r_i is determined.

We now wish to show how the Ried diagram of dynamic stability is to be used in solving the following problems:

1. Using the diagram, determine the angle of list for M = const.
2. Determine the tank tipping moment.

Figure 11 shows the Ried diagram of dynamic stability of the tank in which we can read the ordinates of dynamic stability h_i or the exertion $W = Qh_i$ ofthe rectifying force pair.

Since the listing force pair is expressed by the equation

$$W = M\theta \qquad (11)$$

it will appear at M - const. in the ordinates θ and W in a straight line with an angle coefficient M.

If we draw a vertical line on the abscissa axis at a point corresponding to the first straight line (57.3°) at which $W = M_1$, and divide off a section M on it in the scale of W and connect its end with the beginning of the coordinates, we obtain the diagram of the listing force pair.

The point of intersection with curve h_i will of course yield the point at which the listing and the rectifying force pair are equal.

Consequently, it will suffice to plot the tank angle of tilt at point A and to connect the end with the beginning of the coordinates if we wish to determine the tank angle of tilt under the influence of the force pair. Then the point of intersection B of the straight line with the curve of the tank stability line will indicate the angle of tilt.

The answer to the second question is given if we draw a tangent from the beginning of the coordinates to the curve of the Ried diagram and extend it further to the point of intersection with the ordinate at A. The section AC thus obtained will represent the size of the tipping moment which we are seeking.

In conclusion we cite an example for utilizing the table to calculate h_i and to draw the Ried diagram of dynamic stability, the calculation of which is given in the section on static stability.

By comparing the static and the dynamic stability, we reach the following conclusion:

To increase static stability it is sufficient if we have a high value of the stability ordinates on the Ried diagram, even if it is only on a very small section.

In the case of dynamic stability it is very important that the Ried diagram extend to the maximum possible section of the abscissa axis. This means that the surface of the Ried diagram in the first-mentioned case is not important, but that it is important to have as high an ordinate as possible.

Table e.

Calculations of the Stability h_i of the Dynamic Ried Diagram. Longitudinal Stability.

θ	y_i	z_i	$y_i \sin \theta$	$z_i \cdot \cos \theta$	5—4	$a \cos \theta$	$a - a \cos \theta$	$h_i = 6-8$
1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	2,75	0	0
5	10,7	0,3	0,93	0,3	0,63	2,71	0,04	0,67
10	15,7	0,9	2,74	0,8	1,94	2,70	0,05	1,99
15	19,1	1,6	4,45	1,55	2,90	2,66	0,10	3,00
20	22,2	2,6	7,6	2,45	5,15	2,58	0,17	5,32
25	25,0	3,5	10,6	3,2	7,4	2,50	0,25	7,65
30	27,0	4,7	13,5	4,1	9,4	2,38	0,37	9,77
35	28,4	5,7	16,3	4,7	11,6	2,25	0,50	12,10
40	30,0	6,5	19,3	5,0	13,8	2,10	0,65	14,45
45	30,4	7,2	21,5	5,1	16,4	1,94	0,81	17,21
50	31,2	7,7	24,0	4,9	19,1	1,77	0,93	20,00
55	31,6	8,2	25,5	4,7	20,8	1,57	1,18	21,98
60	31,8	8,7	27,4	4,4	23,0	1,37	1,38	24,38
65	32,0	8,8	29,0	3,7	25,3	1,16	1,59	26,90
70	32,2	8,8	30,3	3,0	30,0	0,95	1,80	31,80
75	32,3	8,8	31,2	2,3	30,3	0,71	2,04	32,30
80	32,4	8,8	32,0	1,5	30,5	0,48	2,27	32,77
85	32,4	8,9	32,3	0,77	31,5	0,24	2,51	34,00
90	32,5	8,9	32,5	—	32,5	—	2,75	35,25

$a = 2{,}75$ cm

Table f.

Calculation of the Stability Lines h_i of the Ried Diagram

θ	y_i	z_i	$y_i \sin\theta$	$z_i \cos\theta$	5-4	$\alpha\cos\theta$	$\alpha-\alpha\cos\theta$	$h_i = 6-8$
1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	2,75	0	0
5	4,15	0,2	0,39	0,20	0,19	2,71	0,04	0,23
10	8,3	0,7	1,44	0,69	0,74	2,70	0,05	0,79
15	12,5	1,7	3,23	1,65	1,58	2,66	0,10	1,68
20	16,2	2,8	5,56	2,63	2,93	2,58	0,17	3,10
25	19,0	4,0	8,0	3,63	4,37	2,50	0,25	4,62
30	21,2	5,0	10,6	4,33	6,30	2,38	0,37	6,67
35	23,1	6,5	13,3	5,30	8,00	2,25	0,50	8,50
40	24,5	7,4	15,8	5,70	10,10	2,10	0,65	10,75
45	25,4	8,1	18,0	5,70	12,30	1,94	0,81	13,11
50	26,0	8,8	20,0	5,65	14,35	1,77	0,93	15,28
55	26,5	9,4	21,0	5,40	15,60	1,57	1,18	16,78
60	26,8	9,9	23,2	5,00	18,20	1,37	1,38	19,58
65	27,0	10,4	24,5	4,40	20,10	1,16	1,59	21,70
70	27,2	10,7	25,7	3,66	22,10	0,95	1,80	23,90
75	27,3	11,1	26,3	2,73	23,60	0,71	2,04	25,64
80	27,4	11,4	27,0	2,00	25,00	0,48	2,27	27,30
85	27,5	11,8	27,3	1,03	26,30	0,24	2,51	28,80
90	27,6	12,1	27,6	0	27,60	0	2,75	29,35

4. Entry and Exit of the Amphibious Tank or Amphibious Vehicle into and From the Water.

When the preceding problem has been solved, we can proceed from two points of departure:

1. The maximum entry and exit angle is determined under the precondition that no hatch is submerged beneath the surface of the water.

2. The same, but under the precondition that maximum power efficiency is used and the vehicle does not slip.

Since determining the sought angle under the precondition that no hatch goes under water depends on checking the power and ground adhesion; one condition becomes a part of the second and no longer applies as an independent precondition.

We will begin with the solution under a special precondition.

We determine the maximum exit angle of an amphibious, rectangular surface on which we bestow all the running characteristics of a tank or of a vehicle. It is more convenient to divide the exit of the rectangle into two time intervals:

1. from the moment in which it comes into contact with the soil to the point in time that its entire underpart is on the surface of the ground.

2. from landing until exit from the water.

Propellers as well as tracks or wheels can be used simultaneously for all amphibious vehicles. In the first point in time the main track and suspension, the propeller and momentum are all being exerted, whereas in the point in time of ground contact the track and the wheel are being exerted. This is explained by two circumstances:

1. The entire driving coefficient of conversion and of the track is several times (4-5) greater than the propeller, and consequently the propeller thrust which is sufficient in the water is insufficient to overcome the vehicle gravity components and the driving resistance from the soil and the water when the tank is exiting.

2. The further the tank comes out of the water, the more the propeller becomes free and thereby breaks off the blades on the ground. Consequently, when propeller thrust proves to be insufficient, the forward thrust is converted to the tracks as soon as they gain traction on land.

But ground traction is closely related to vertical reaction. Consequently, this ground reaction plays a decisive role in determining the maximum permissible gradient of the river bank.

As is clear from Fig. 12, we can achieve each additional position of the rectangle as a result of three movements: thrust s, stroke h and turn a.

During displacement (beginning with ground contact) the following forces will be exerted:

G = Gravity of the rectangle,
F = Shearing force of the propeller,
Q = Buoyancy,
R_S = Water resistance,
N = Vertical reaction of the ground,
P = Tractive effort of the track,
R = Ground resistance against movement of the rectangle.

Buoyancy Q can be expressed as a function of h and α in an equation as long as the surface of the rectangle is not submerged and as long as point D does not emerge from the water.

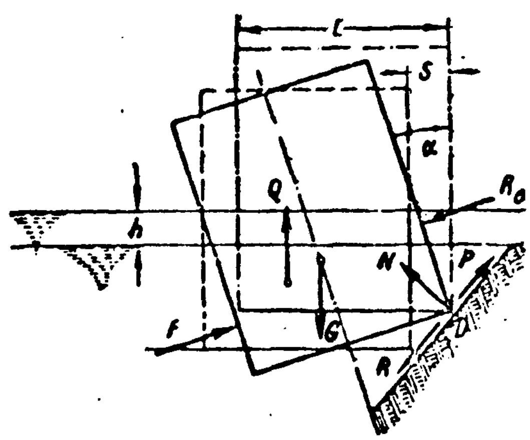


Figure 12. Forces being Exerted on the Rectangle

The following unknown values will appear during the movement of the rectangle: F, P, h, α and N, i.e. five values, but:

$$\frac{F_v}{270\,\eta_B} + \frac{P_v}{270\,\eta} = N_e \qquad (12)$$

and at the maximum value of angle α according to:

$$P = N \cdot \varphi. \qquad (13)$$

where:

v = the velocity of the rectangle,
ηB = the efficiency of the propeller,
η = the efficiency of the rectangle track and suspension,
φ = the adhesion coefficient of the track.

Consequently, having equations:

$$\left.\begin{array}{l}\Sigma x = 0 \\ \Sigma y = 0 \\ \Sigma z = 0\end{array}\right\} \qquad (14)$$

and Functions (12) and (13), we can determine the value of the angle α_{max}.

But this solution has only a theoretical value because we are considering here the movement of a body whose water displacement can be expressed mostly by a single equation, which frees us of the greatest difficulty of solving the problem.

We will now proceed to answer the question in general.

a) Determining the Angle α_{max} When Exiting from the Water and Entering the Water.

After the tank or vehicle has come on ground and has moved forward, a part of the hull will have emerged from the water while the other part is still submerged (Figs. 13 and 14). The water displacement and the center of gravity will change and thereby the ground reaction N will occur.

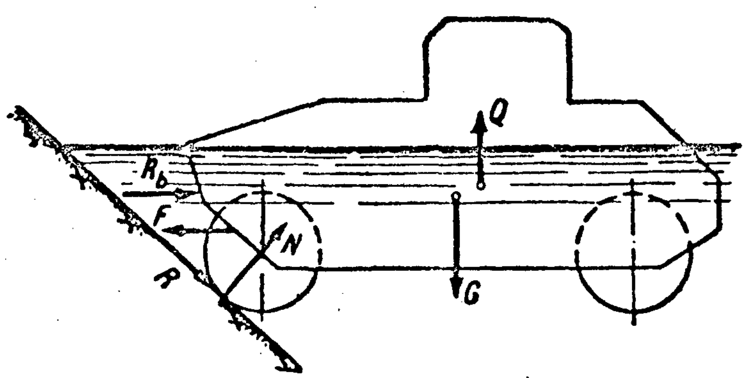


Figure 13. When emerging from the water, the vehicle comes into contact with the ground.

Consequently, we must determine Q and the coordinates of the weight center of gravity in order to learn the change of N. This presents the main difficulty in solving the problem.

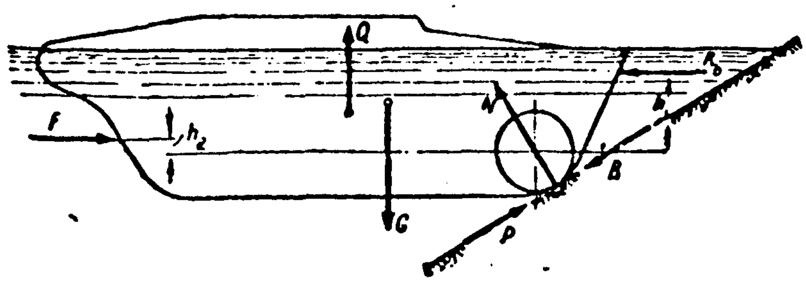


Figure 14. When emerging from the water, the tank comes into contact with the ground.

The first time interval of movement will be the most important because N will increase steadily after landing, while resistance will remain almost constant. In this interval of time a difficulty will arise with the force F in addition to the difficulties with Q and the point at which this force is exerted, it being variable in size because both tracks are operating simultaneously.

In order to solve this problem, we set up the following calculation (see Fig. 15).

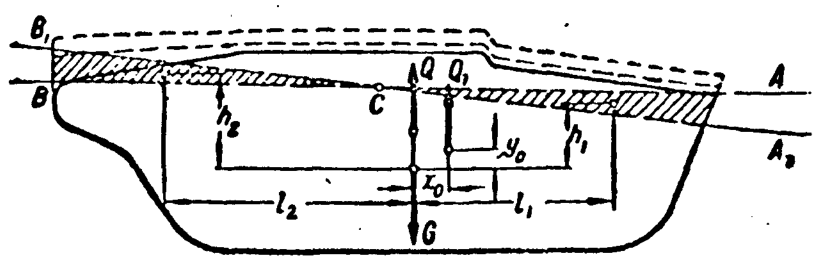


Figure 15. Side Profile of the Tank

We are taking the side profile of the amphibious vehicle hull, the load line of which is indicated by AB and we are calculating the water displacement of the pontoons and of the suspension. Then we make additions to the drawing in such a way that the ordinates multiplied by one-half of the hull indicates the water displacement of the parts; we eliminate the pontoons along with the track and suspension and consider the weight as well as the coordinates of the center of gravity to be invariable.

We will assume that we have obtained the sketch on Fig. 15 as a result of the information. Since the angle of the river bed which we are seeking is not greater than 15-20°, we proceed as follows:

We draw a series of lines through an arbitrary point C in succession with angles 5°, 10° and 15° to the horizontal plane.

The first line must be drawn in such a way that the volume of the keel ACA_1 jutting out of the water is somewhat larger than the volume of the keel BCB_1 submerged in the water.

We designate the water displacement of the first keel by v_1 and the second by v_2 and the coordinates of the center of gravity of water displacement from the earlier weight center of gravity accordingly by h_1, l_1 and h_2 and l_2. Then the new water displacement would be

$$Q_1 = Q - (v_1 - v_2)$$

and we determine the change of the coordinates according to the equations:

$$\left.\begin{aligned} x_0 &= \frac{v_1 l_1 + v_2 l_2}{Q_1} \\ y_0 &= \frac{v_1 h_1 + v_2 h_2}{Q_1}. \end{aligned}\right\} \qquad (15)$$

After we have taken all these measures at all water lines and the indicated series of angles, we will be in the position to prepare the drawing $M = f(Q_i)$ (Fig. 16), on which $M = Q_i l_i$ is the moment around the axis of the front road wheel.

Then we divide the moments of the weight force of the same angles (5-10-15°), points m, n and k on the ordinate axis. On these same points we erect vertical lines to the point of intersection with the curves and obtain points α_1, α_2 and α_3.

When we project these points on the abscissa axis, we obtain intersection points Oq_1, Oq_2 and Oq_3 and water displacement for the angles 5°, 10° and 15°.

When we plot G to the left of points q_1, q_2 and q_3, we obtain the points b_3, b_2 and b_1.

When points p_1, p_2 and p_3 on the ordinate give the end points of the water line angle of gradient which has been divided off according to scale, then we obtain the curve $G - Q_i = f_2(\alpha)$.

If we know Q_i and have the connection between F and P, we can determine the values N and P according to the equations for the projections of the forces on the axial lines X and Y and we can plot the curve $P/N = f_3(\alpha)$ (Fig. 17).

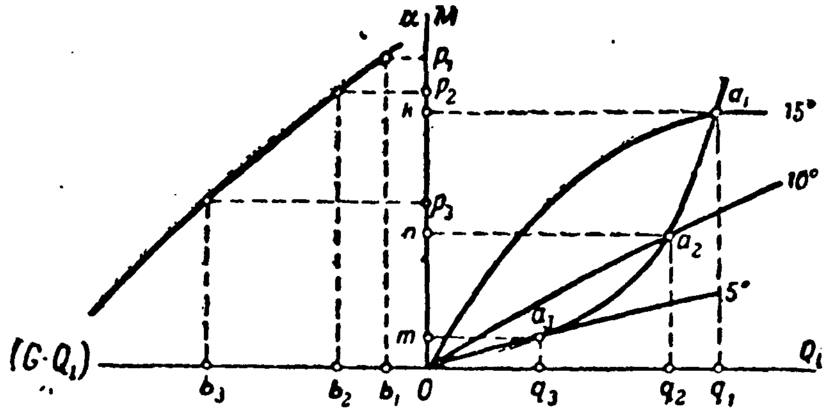


Figure 16. $M = f_1(Q_i)$

If we divide off the point on the ordinate axis which corresponds to the value of the difference between the coefficients $\varphi - f$ and at this point erect a vertical line to the point of intersection with the curve, we obtain a point which we project on the axis of the abscissa and in this way obtain α_{max}.

As we see, the determination of α_{max} is a very involved and tedious affair.

The preconditions under these circumstances are very different for a tank and for a vehicle. As a result of the double steering, the vehicle travels straight ahead in water with the propeller.

Consequently, the propeller can be put into operation as soon as it is submerged, which is impossible in the case of a tank, since on it the propeller is in the rear.

In addition, when a tank enters the water, the front part will begin to vibrate and will change its water displacement in time intervals which makes it much more difficult to calculate the tractive effort of the track. But in view of the slight influence of vibrations on the adhesion weight, we can neglect this phenomenon.

In general it is necessary to calculate the short gradient when exiting. The angle which the tank surmounts when traveling up the bank can certainly be surmounted more easily when traveling down into the water.

If an amphibious vehicle is completely tight, from a theoretical standpoint, it can travel from the shore into the water at any arbitrary angle by using the momentum of downward movement, (if there are no uneven spots on the road and the ground does not slope laterally).

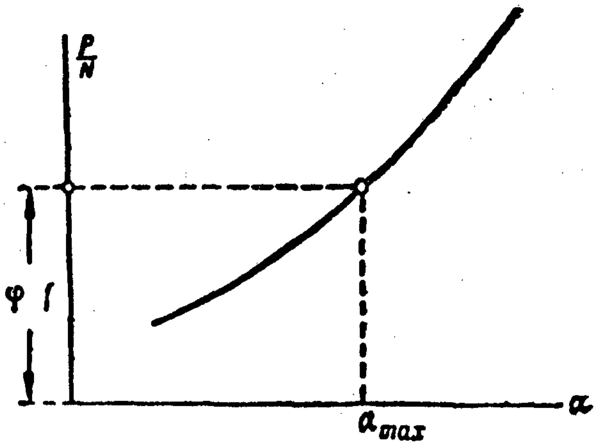


Figure 17. $\frac{P}{N} = f_3(\alpha)$

Even if the vehicle comes with the bow onto land in the first moment, in the next moment it will regain its amphibiousness.

5. Water Resistance Against Tank Movement

The complexity of the phenomena which appear when water surrounds the hull and the difference in these characteristics make it impossible to consider flow as a whole.

Thus we can set up no uniform equation which reflects the flow pattern uniformly.

Consequently, only one way is left open to us, that of considering each individual phenomenon separately and considering the origin and effect of these phenomena which are independent of each other.

The resistance of the water against the movement of a body is caused entirely by the generated movement resistance of the liquid.

Since now the motion of the liquid and the forces being exerted on the body in question are two interacting sides of the same phenomenon, it is possible to determine the other side -- force -- even if we know only one of these sides -- motion, and conversely.

The quantitative relationship between the individual forms of motion and the forces corresponding to them depends on the nature of the liquid and of the amphibious body.

The nature of the fluid includes the following factors: moisture, elasticity, viscosity and specific weight. The characteristics of the floating body include: inertia, shape of the submerged part, the nature of its surface and its speed.

But not all properties of the liquid are causes in the same measure of resistances which occur when the body moves; the main properties will be the viscosity and the specific weight of the water.

We can list all types of liquid motion when it flows around a body:

1. The progressive motion of the liquid layer drawn along by the body -- boundary layer flow,
2. The rotational motion of the liquid -- turbulence,
3. Change of the surface of the water -- wave formation.

In relation to the forms of liquid motion, the following forces occur:

1. Tangential force -- friction resistance,
2. Normal force -- body resistance,
3. Normal force -- resistance of the waves.

The factors influenced by liquid properties are: at point 1 and 2 viscosity, at point 3 weight.

To illustrate more clearly we will use the method of Professor Pawlenko and compile this information in a table.

Designation of Resistance	Property of the water	Direction of Forces	Type of Water Motion
Friction resistance	Viscosity	Tangential	Boundary layer flow
Body resistance	Viscosity	Normal	Single eddies
Wave resistance	Weight	Normal	Waves

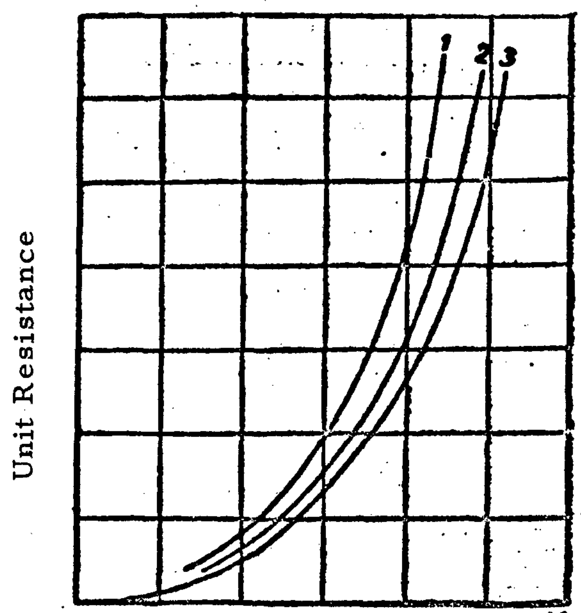


Figure 18. 1) $\frac{L}{B} = 9$; 2) $\frac{L}{B} = 7$; 3) $\frac{L}{B} = 5$

The relationship between the resistances is determined by the speed, the shape of the submerged part and the surface smoothness of the floating body.

If we know the type of resistance forming the main resistance and know the cause, it is possible to take all precautions to reduce resistance. The influence of the properties of the body on the relationship of the resistance components is regarded objectively as follows:

1. If there are no projecting parts on the body and we are dealing with an average speed (2-6 m/sec) -- river ships -- it is predominently friction and body resistance.
2. If no parts project and speed is high, i.e. 6-13 m/sec (seagoing ships) friction and wave resistance will play an important role. Consequently, at the same water displacement resistance will be less, the smaller and smoother the body in both cases.

Insofar as we are concerned with the influence of the ratios between width B, height H and length L on resistance, when the ratio between L/B is increased, specific resistance decreases (Fig. 18) but apart from basic values it is difficult to determine any mathematical interrelationship.

Only one thing is certain: a stream-lined shape of the body reduces resistance.

3. Too many projecting parts and the lack of a streamline shape (tank) increases the resistance of shape and wave resistance at the low speed of 2-3 m/sec. Consequently, encasing the projecting parts and streamlining of the hull is the only way to reduce resistance.

b) Methods for Calculating Resistance

Before we present the most applicable formulas of power requirements for ship propulsion, it seems advisable to us to establish the basic rules for setting them up.

Test data forms the basis of all calculation which express friction resistance, shape and wave resistance:

$$\left.\begin{aligned} R_1 &= C_1 S v^{1.83} \\ R_2 &= C_2 S v^2 \\ R_3 &= C_3 S v^4 \end{aligned}\right\} \qquad (16)$$

R = resistance in kg,
C = experimental coefficients,
S = the submerged surface and
v = speed in m/sec.

Of the three equations, the one with the best foundation from the theoretical standpoint and the best confirmed one by careful checking is the equation of friction resistance which was established by Froude.

Although the Froude method has recently been made "obsolete" by the studies of Schlichting (with stream-lined bodies) I consider it fully sufficient for our calculations.

Aside from the fact that the equations are present for each kind of resistance, these are ordinarily not used due to the difficulties of determining the coefficients C_1, C_2 and C_3.

Frequently the method is used by which we determine experimentally the full resistance (by towing the model) and when the difference between the determined value and that calculated by the equation $R_1 = C_1 S_v{}^{1.83}$ gives the sum of the shape and wave resistance.

In the case of low speed and uniform shapes, wave resistance can be neglected and in this manner the shape resistance can be determined. In the case of steamships and sailing ships, this resistance is in the ratio of 5-15% to friction resistance.

At high speeds, at which we run up against full resistance and the resistance of shape in an additional percentage, we must calculate wave resistance in addition.

c) Calculating the Resistances against Tank Forward Movement.

As we have already stated, the resistances of ships can be divided into two groups:

1. Friction and shape resistances,
2. Friction and wave resistances.

Correspondingly, equations for the total resistance have been also set up for ships on the basis of test results.

Since the resistance against the movement of a tank is mainly determined by the resistances of shape and waves, the equations are suited for ships and not for tanks.

As proof of this we will select the equation used mostly for ship drive calculations and calculate the resistance for a tank, the basic dimensions of which are known to us.

We have the equations:

1. By Froude -- for the determination of friction resistance:

$$R = f S_v^{1.83} \tag{17}$$

where f equals 0.14 for steel ships.

2. By Eiffel for calculating the resistance of the plate shape which lies transverse to the stream:

$$R = 0{,}6 \varrho S v^2 \tag{18}$$

where

$$\varrho = \frac{1000}{9{,}81}$$

3. By Gebers -- for the unlimited medium (water)

$$R = (f\,S + \varphi\,S_0)\,v^{2,25} \tag{19}$$

where $f = 0.14$; $\varphi = 3.5$ and S_o for the center frame.

4. By Gebers -- for flat sections of water

$$R = (f_1\,S_1 + f_2\,S_2 + \varphi\,S_0)\,v^{2,25} \tag{20}$$

where S_1 and S_2 is the side and undersurface which is submerged. f_1 is 0.14; $\varphi = 3.5$ and f_2 is dependent of h, the distance of the bottom of the ship to the base of the river bed.

h (m)	1,00	0,75	0,50	0,25
f_2.................	0,140	0,185	0,250	0,250

In our case we will assume that $h = 1$, consequently, $f_2 = f_1$ and the equation (20) is written in the following form:

$$R = (f_1\,S + \varphi\,S_0)\,v^{2,25},$$

that is the same as in the third case.

5. The equation by Middendorf

$$R = R_1 + R_2 = 0{,}17\,S\,v^{1,83} + \frac{k\,S_0\,v^{2,5}}{\sqrt{1 + a\left(\frac{L}{B}\right)^2}}. \tag{21}$$

where the coefficients k and a are taken from a special table.

L is the length,
B is the width.

We are assuming that in our case $S = 20\ m^2$. The speed changes from 0 to 2.5 m/sec and $S_o = 1.15\ m^2$.

We will substitute these values into Eqs. (17) to (21) and calculate the speeds in the range from 0 to 2.5 m/sec; then we will plot the curve of resistance $R = f(v)$. In this way we obtain the graphical representation on Fig. 19.

The actual resistance against tank movement on the basis of the actual equipment and the model can be calculated with sufficient accuracy, in my view, by the following formula:

$$R = 0{,}005\,v^{1,5}\,G \tag{22}$$

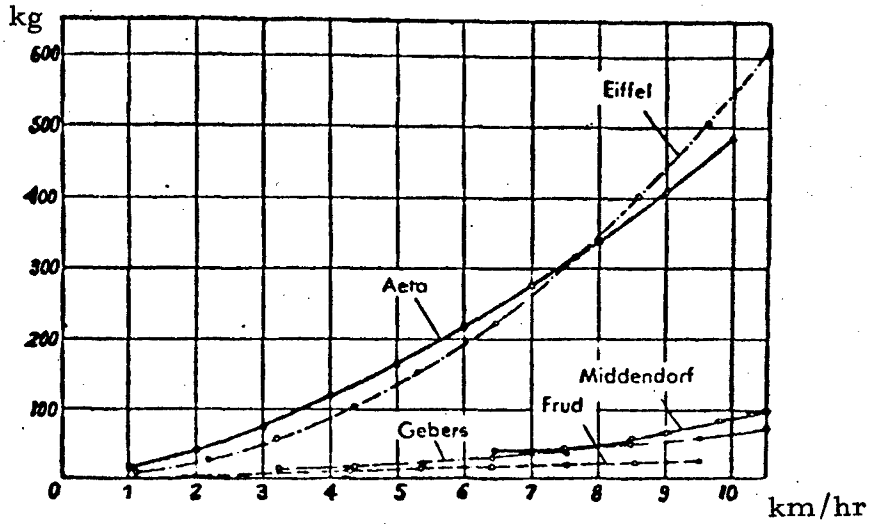


Figure 19.

We now wish to calculate the resistances at the same speed range and plot them on Fig. 19.

As we see, none of the equations used for calculating ship propulsion are used for determining the drive resistance of a tank.

Only the equation of Eiffel for the plate comes near to the actual curve of tank resistance.

These facts are also confirmed in tests with submarines, which were conducted by the French and which showed an increase of resistance up to 16-79% when the projecting parts were submerged, whereas the submerged surface showed almost no change.

Consequently, the resistance of shape plays an important role on the tank.

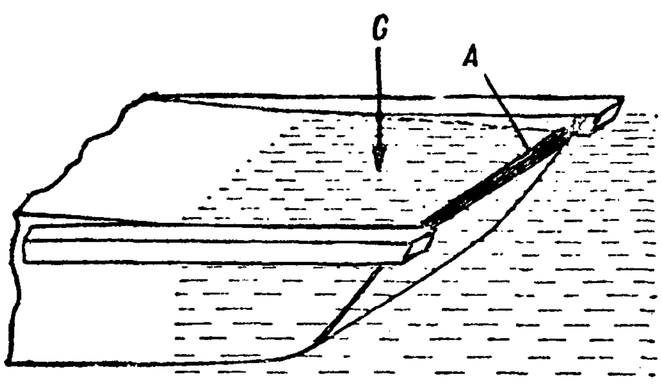


Figure 20. Tank with a Flat Bow

In general we can set up a three-member equation relatively easy for determining the resistance on the basis of the curves (Fig. 19); since, however, this is too large and involved to be used, it seems more practical to use it in Eq. (22) which we also wish to use in our further considerations.

As is evident from Eq. (22), resistance increases with an increase of speed; however, in spite of the low efficiency of the ship's propeller (normally 0.20-0.25) and the high travel resistance, maximum tank velocity is not the result of engine operational efficiency but of tank floating position.

This is due to the fact that at high speed the bow wave submerges the bow plate; thus the bow goes deeper into the water, submerging the hatches, which usually causes a reduction of tank speed.

We will not consider what measures have been taken by the manufacturer to overcome this deficiency. The best example in this regard is the Vickers Armstrong tank.

In its first design this vehicle had a flat bow (A). It was soon proven, however, that this was a poor arrangement. The wave splashing over the bow plate filled the tank and due to weight G of the accumulated water forced the tank down deeper.

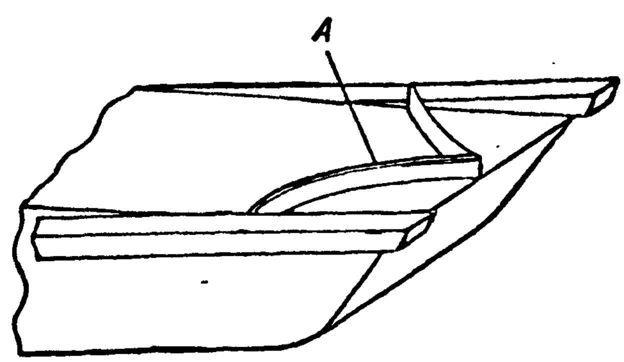


Figure 21. Tank with Wave Breaker

The Poles mounted a wave breaker (A) on this same model, which has the deficiencies of the bow plate and which gives the water free access between the wave breaker and the body and led to the formation of turbulence.

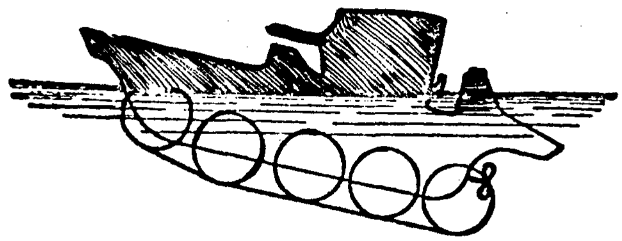


Figure 22. Tank with the Stern Down

According to the last edition of Heigl (Pocket Book of Tanks) the English have found a new method; they have made the tank stern heavy, thus lowering the propeller and increasing resistance. This method seems less effective because it is invariable.

The most effective method of counteracting the waves has not yet been discovered.

6. Testing Methods of Amphibious Tank Models

A model is understood to be another body, similar to the design, in which the relative dimensional size is a constant value in comparison to the other.

In other words, the model and the actual equipment must exhibit geometrical similarity.

In order to be able to transfer the test results from the model to the actual equipment, it is necessary that both bodies exhibit a dynamic similarity.

Two bodies are dynamically similar if the paths of motion of the medium particles which flow around them are similar. The speeds at which the dynamic similarity is present are designated as corresponding.

According to the location at which the tests are conducted, they can be designated as testing laboratory and track and suspension tests.

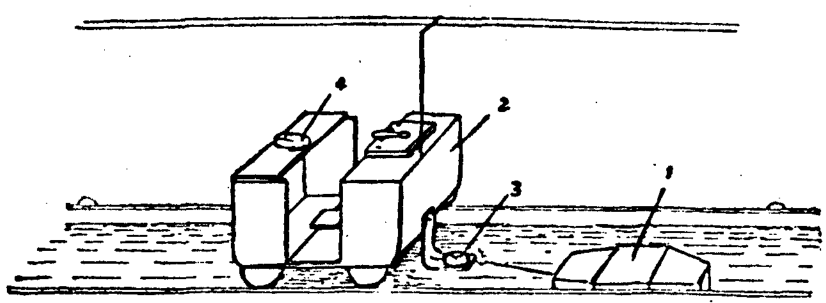


Figure 23. Towing the Model in the Channel

The tests for determining water resistance against forward movement are conducted in a channel.

To exclude the influence of the channel walls, the channel must be at least twice as wide as the test model.

In regard to the use on a tank, this requirement is very inconvenient because too small a scale makes the manufacture of the model with its involved track and suspension more difficult with regard to precision.

The length of this channel is determined by the level of towing speed.

The test model is usually constructed of paraffin, thus it is easier to cast and the same material can be used several times in succession.

The model is towed by a small vehicle (2) (Fig. 23) which runs on rails along the canal edges. The vehicle is coupled with the model in direction of travel.

The power efficiency in most cases is recorded by an automatic device (3) and the speed measured by an electrical clock (4) i.e. by a clock which turns on and off automatically when passing through a measuring distance (in the time spans in which the forward movement is regarded as uniform).

There is also another method for determining these values, but basically the aforementioned one is standard.

The run tests depend on the location at which the tests are undertaken (in a pond, lake); thus they are different from the laboratory tests as to how they are conducted.

The run tests can be divided into two groups according to how the model moves:

1. with the model being pushed,
2. with the model being pulled.

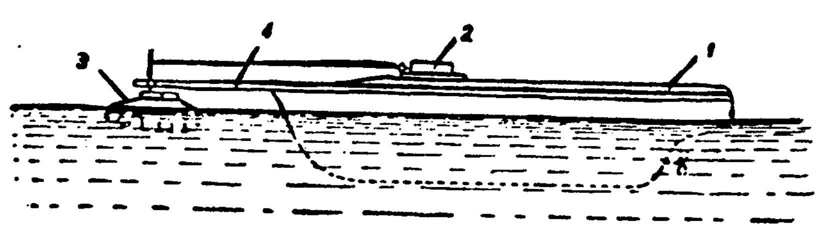


Figure 24. Model in Front of a Towing Device

In both cases the equipment is the same; as towing device (1) we can use an amphibious tank or a motor boat. The resistance is measured by an automatic recording device (2). In regard to speeds, a difference will be dependent on the size of the model.

In the first case the speed as average speed is determined by recording the time it passes over the measuring distance and in the second case with a drum having an automatic speed gauge and according to the length of a thread.

The first method has two main disadvantages.

To exclude the influence of the towing vehicle bow wave, a long lever (4) must be arranged on the rear of the model to change the pressure there (Fig. 24). This, however, is difficult to do, because the draft of the model changes due to pitching which occurs when the towing vehicle changes speed.

The second disadvantage is that it is difficult to measure (to record) the speed of the model itself. Because of the difference between the dimensions of the towing vehicle and of the model this speed difference at non-uniform travel will be exhibited at a given point in time.

Consequently, the recording of the towing vehicle speed will not indicate the type of forward movement of the model, and for this reason it no longer seems practical to record the temporal curve of speed; we content ourselves with measuring average speed.

In the case of the second method, the influence of the propeller is difficult to exclude because a very long tow rope (4) cannot be used due to slack (Fig. 25).

On the basis of a series of considerations, towing seems to be more practical as it is depicted in Fig. 26. The movement of the boat (1) is transmitted from the shore to the towing vehicle which is connected to it by a rope.

A recording device (2) is installed on the rear of the boat which is connected (coupled) with the model (3) by a pull rod (4). In the preceding case, speed is measured with a speed recorder (5) which is set up on the shore opposite the towing vehicle.

After some practice by the towing vehicle operator, several points can be achieved with this method when traveling on a small body of water.

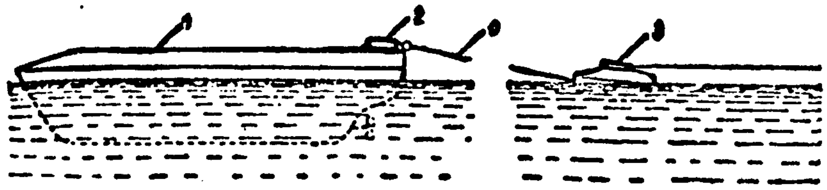


Figure 25. Model in Tow

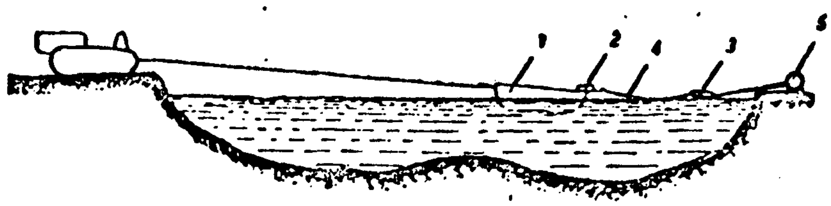


Figure 26. Towing Test with Drive from Land

For reasons of economy and also to simplify the required measurements, it is advisable to build the model of wood. Low cost, quick manufacture and remodeling offset the insignificant disadvantages of the wood model (the chief disadvantage is water absorption during long tests).

In the preceding section we mentioned the fact that the resistance of the tank against forward movement at a speed of 0-10 km/h can be calculated with sufficient accuracy according to one of the following equations:

either the Eiffel equation for the plate:

$$R = 0{,}6\, \rho\, S\, v^2.$$

or my equation:

$$R = 0{,}005\, G\, v^{1{,}5}.$$

We will now check which of these equations most correctly reflects the flow phenomenon in our case.

Since the first equation justifies the turbulence resistances caused by forces of viscosity, the relationship between the speeds of the equipment itself and the model is determined by the Reynolds number, i.e.

$$\frac{l_1 v_1}{\mu_1} = \frac{l_2 v_2}{\mu_2} \tag{23}$$

l_1 and l_2 = the length of the model and of the actual equipment,

v_1 and v_2 = travel speeds,

μ_1 and μ_2 = are the viscosity coefficients of the liquid in which the tests are conducted.

If, however, the numerical coefficient in the formula is not related to the Reynolds number, as is the case in the Eiffel equation, the relationships between the equipment and the model are determined with the following formula:

$$\frac{R_n}{R_m} = \frac{A\,S_n\,v_n^2}{A\,S_m\,v_m^2}. \qquad (24)$$

But since at $\mu_1 = \mu_2$, $S_n v_n^1 = S_m v_m^1$ the travel resistance of the tank is due to water turbulence and wave motion, i.e. which is caused by the force of gravity, and the dynamic similarity in this case for speeds, which is determined by the Froude number (law of similarity)

$$\frac{v_1}{\sqrt{l_1}} = \frac{v_2}{\sqrt{l_2}}. \qquad (25)$$

Consequently, we cannot use Eqs. (24) and (25) to transfer the test results to the equipment itself.

According to the second equation, we will obtain the relative sizes between the equipment and the model in the following form:

$$\frac{R_n}{R_m} = \frac{A\,G_n\,v^{1,5}}{A\,G_m\,v^{1,5}} = \frac{G_n}{G_m}, \qquad (26)$$

i.e. the resistances behave in relation to each other like weights (at the same speed) a fact which does not conform to the Froude law of similarity.

Consequently, the second equation must be regarded as empirical; it completely reflects the type of body movement through the water stream.

According to this information, all that is left to us is to apply the Froude method to recalculate the basic values from the model to the equipment.

If the general law of travel resistance against movement of a body in liquid is expressed by the following equation:

$$R = A\,S\,v^2 f\left(\frac{v}{\sqrt{l}}\right) \qquad (27)$$

and turbulence resistance by

$$R = A S v^2$$

then the Froude law of similarity is applicable not only to resistances caused by viscosity forces, but also to the forces in the liquid due to gravity.

According to the Froude law, the final resistances (wave and turbulence) of the equipment and of the model behave in relation to each other as weights:

$$\frac{R_n}{R_m} = \frac{G_n}{G_m}.$$

We are assuming that r, r_v, r_f and r_w as well as R, R_v, R_f and R_w -- are the complete resistances of the turbulence, friction and wave resistances of the model and thus of the equipment itself.

Then, maintaining the mutual relationships between the equipment and the model, i.e. at $\frac{v}{\sqrt{l}}$ = constant, we will obtain the following equation:

$$r_v \frac{G_n}{G_m} = r_v \frac{l_n^3}{l_m^3} = R_v,$$

i.e.

$$R = (r - r_f)\frac{G_n}{G_m} + R_f = (r - r_v - r_f)\frac{G_n}{G_m} + R_v + R_f. \qquad (28)$$

Formula (28) proves that in order to make the transition from the results of towing the model to the equipment itself, we must calculate the resistance of the model and of the equipment according to the Froude Formula $R_f = fS_v^{1.83}$.

Friction resistance must be subtracted from the results of calculating the resistance which was determined experimentally at the appropriate speed, then the remainder $r - r_f$ must be multiplied by G_n/G_m and added to the results of the determined equipment resistance. In the results we will then have the complete resistance of the equipment.

To illustrate more clearly, we will now recalculate the basic values of the model onto the tank.

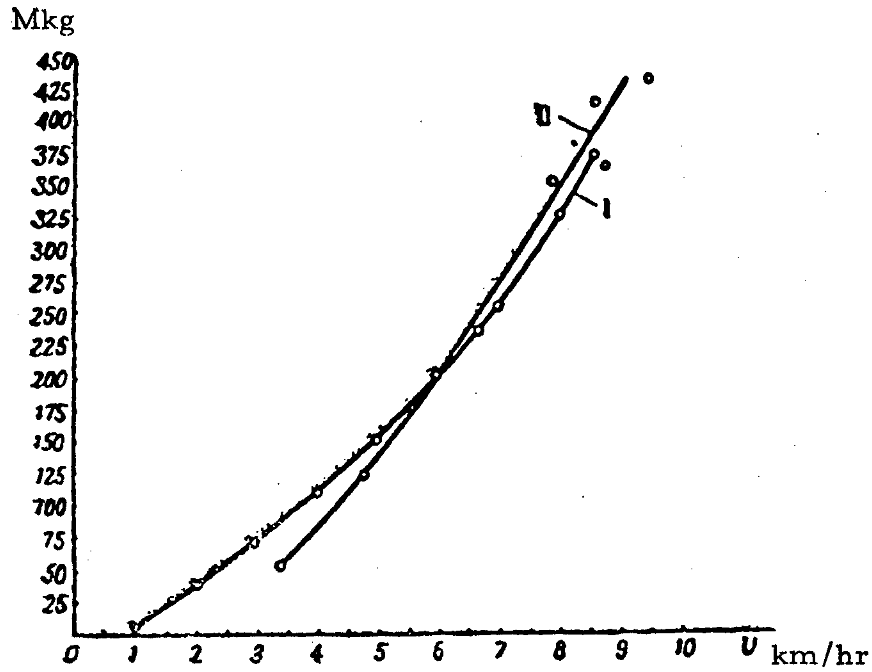


Figure 27. Resistance Curves

When the model is being towed with a water displacement of Q_m = 180 kg, and a submerged surface of $S_m = 3.2$ m^2, the following resistances were determined as a function of velocity:

Speed of Model being Towed km/h	Determined Resistance	Friction Resistance according to Froude	Final Resistance
2	3.5	0.2	3.3
3	8.2	0.4	7.8
4	15.1	0.7	14.4
5	22.7	1.0	21.7
6	30.0	1.5	28.5

On the basis of the determined values we now wish to plot curve II (Fig. 27) and then plot curve I on the basis of this drawing:

$$R = 0{,}005\,G\,v^{1.5}.$$

As we see, the curve transmitting the resistances from the model to the equipment is very similar to the actual curve. Consequently, the Froude method for processing the test results on models (in the given scale) of small tanks is completely applicable.

Towing Speed of the Model	Corresponding Speeds of the Equipment according to the Froude number	Final Resistance of the Equipment	Friction Resistance according to Froude	Complete Resistance
Km/h	km/h	kg	kg	
v_m	$v_n = \sqrt{v_m^2 \frac{l_n}{l_m}}$	$R_n - R_f = (r - r_f)\frac{G_n}{G_m}$	R_f	R_n
2	3,3	51,5	2,4	54,0
3	4,75	122,0	4,4	126,4
4	6,6	225,0	10,5	235,5
5	7,9	339,0	15,0	354,0
6	9,5	445,0	20,0	465,5

7. Propeller

The propeller is composed of the hub and the blades (Fig. 28); it can be cast in one piece or separately. The blade forms the main part.

To obtain a concept of the blade, we must know the type of surface from which it is cut, its position, its shape and the profile of its cut.

The surface of a propeller blade with uniform pitch is obtained by uniformly progressing and uniformly turning movement of the straight line OA (Fig. 29). In this case point O slides along line O_1O_2.

One piece is cut from the surface obtained in this manner according to a certain shape so as to obtain a smaller surface (to reduce friction resistance) and a short hub.

The propeller is divided into the following parts: "the root" which is to say the part of the blade on the hub, the end of the blade (the upper edge), the front leading edge and the rear trailing edge.

The surface of the blade turned toward the tank is called the front surface and the opposite side, the rear or working surface.

In the selection of blade shape we refer to the available prototypes. The blade shape exhibits various and different shapes. The oval surface has found the most widespread use.

The motive for the selection of shape is based on a series of considerations such as the hub length which is permissible according to local conditions, the thickness of the "root", the thickness of the blade at the base and the specific surface load.

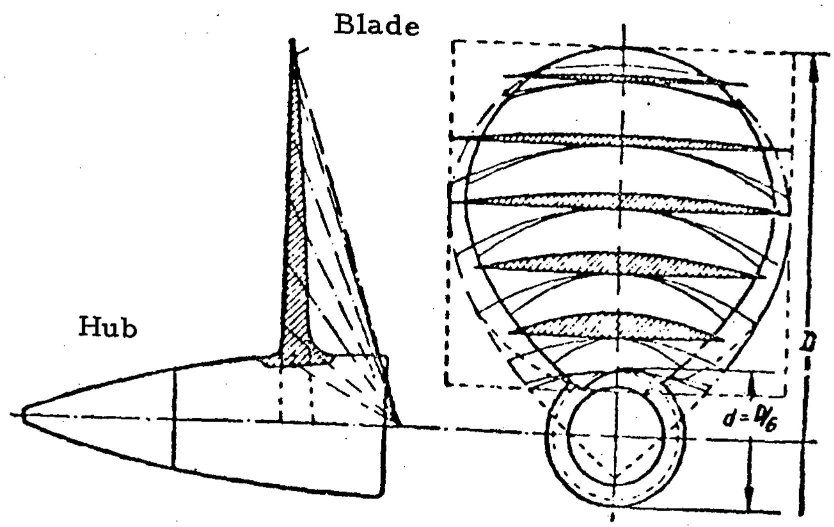


Figure 28. Propeller

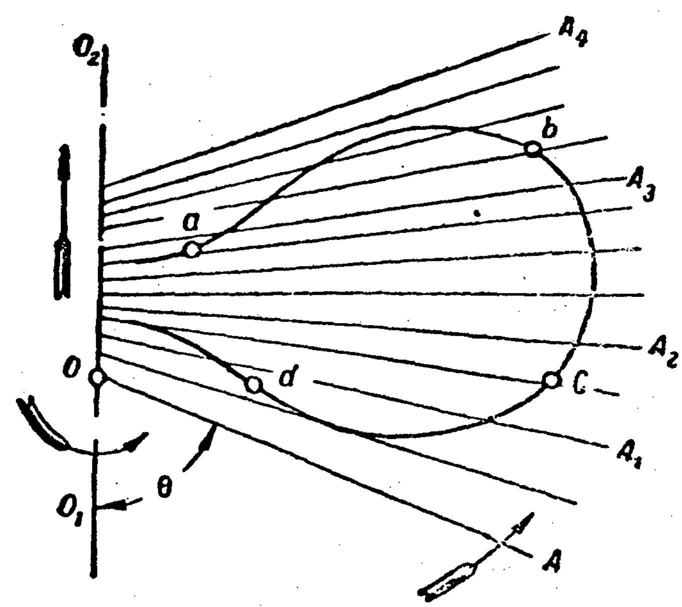


Figure 29. Formation of the Hub Surface

We must add parenthetically that the shape of the blade exerts no substantial influence on the efficiency of the propeller.

By "profile" of the propeller section we mean the cylindrically shaped blade section, the line of axis of which corresponds with the line of axis of the propeller.

In everyday usage the following profiles have been introduced (Fig. 30):

a) the circular segment profile (easy to manufacture and most widely used),

b) in the shape of an aircraft propeller blade,

c) vehicle support surface shape,

d) double convex.

As tests of the Central Office for Aero-hydrodynamics have shown (Fig. 31), the section profile shape of the propeller blade exerts no great influence on the propeller.

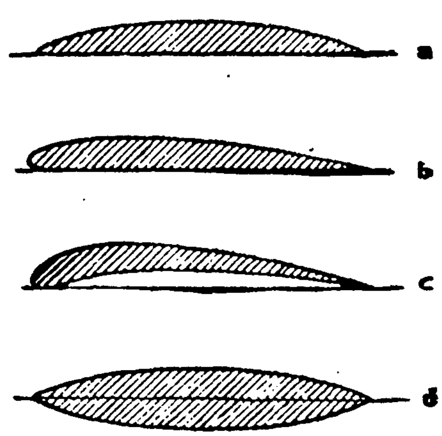
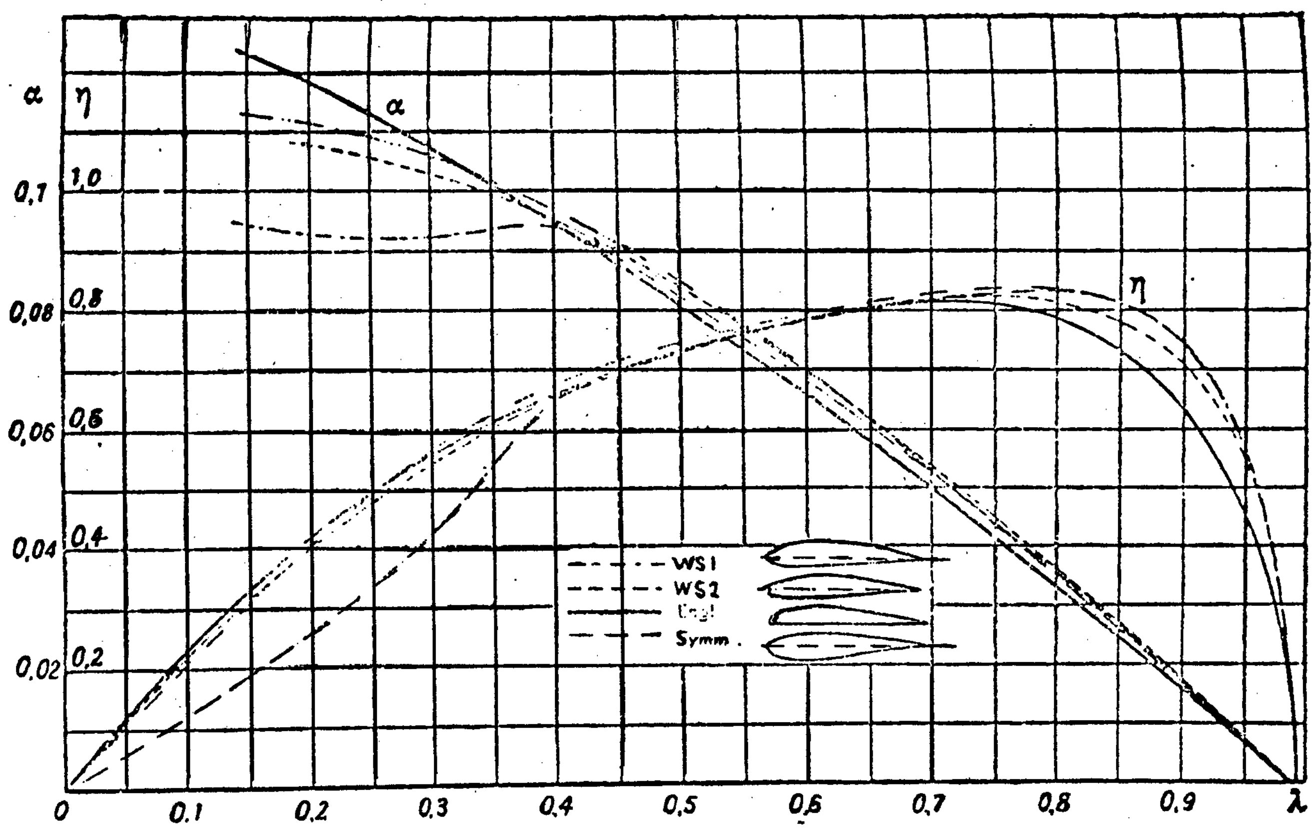


Figure 30. Section Profiles of the Propeller Blade

With regard to the value of λ and α see formulas (38).

The angle Θ (Fig. 32) of the tilt of the blade section to the propeller surface is important for the position of the propeller blade.

Figure 31. Diagram of the Coefficient α and η according to λ for Propellers with Various Profiles



In the conclusions concerning propellers in the literature, the following applications are found for the structure of the propeller surface:

The design surface of the propeller blade A_p. The surface is equal to the surface of the blade projection on the propeller surface vertical to the line of axis (Fig. 32).

The developed blade surface A_d.

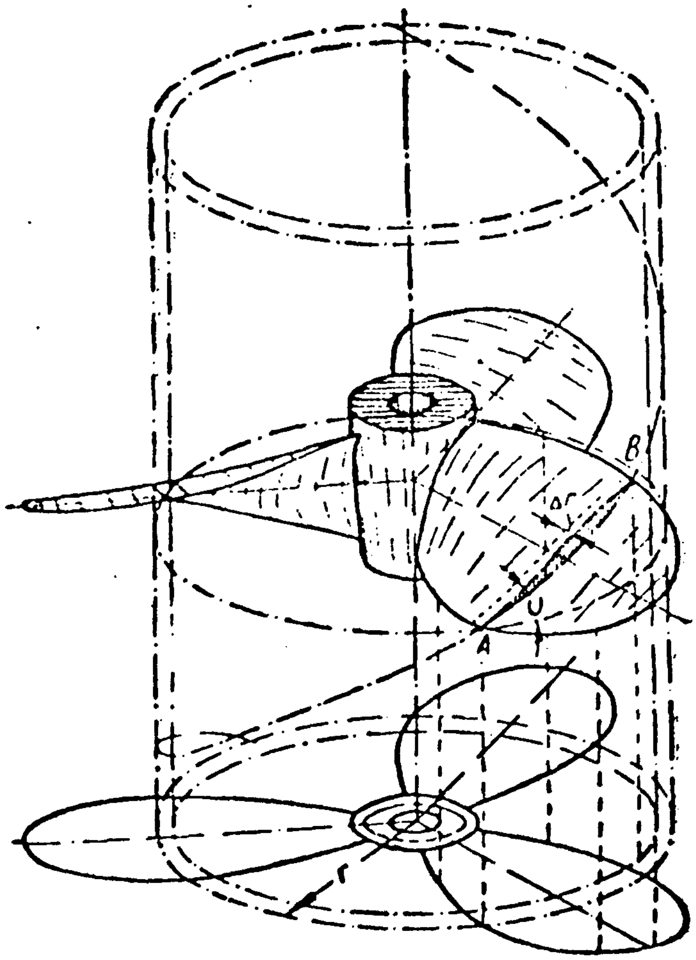


Figure 32. The Most Significant Propeller Values

This surface equals the sum of the surfaces which are developed to the arc surface AB which we obtain with the section of a blade having cylindrical surfaces, the line of axis of which coincides with the propeller line of axis while the radii exhibit successively the following values:

$$r, r - \Delta r, r - 2\Delta r.$$

The surface of each arc is B A r, where b is the propeller blade width developed to the surface at the given radius.

The surface of the propeller surface or working surface A (sic). This surface is as large as the circular surface, the diameter of which equals the diameter of the arc which describes the outermost points of the propeller.

a) Description of the Propellers from the Geometrical Standpoint of Fluid Dynamics.

The diameter D of a propeller is its most important characteristic. By propeller diameter we mean the diameter of a circle described by the outermost edges of the propeller blades.

The diameters of tank propellers range between 0.35-0.6 m.

The next characteristic of the propeller is pitch. The concept "pitch of the propeller" rests on the idea that it screws into the water.

The path which all points on the propeller traverses as it turns, without considering slip, is called (theoretical) slip.

To approximately determine the pitch of a finished propeller, the propeller must be placed vertically on a table surface and by lifting up the arc of the blade section, determine the distance of the extreme points of the propeller blade from the table as well as the projection width of the blade sections.

If the distances of the extreme section points from the table are h_1 and h_2 , and the width of the projection of the line of intersection is b, the tg of the pitch angle is determined according to the following equation:

$$\mathrm{tg}\,\Theta = \frac{h_2 - h_1}{b}$$

and the pitch of the propeller

$$H = 2\pi r_x > \mathrm{tg}\,\Theta.$$

where r is the radius of the arc section.

The path traversed by the tank during one revolution of the propeller is designated as "propeller path":

$$H_a = \frac{v}{n_s}.$$

where r is the tank speed in m/s and n_s is the propeller rotational speed per second.

As far as judging propeller efficiency is concerned, the difference between the propeller path and the vehicle path is important; this is called apparent reversal or slip.

The actual reversal or slip is represented by the amount of water acceleration on which propeller thrust depends. If this slip is zero, then propeller thrust and thus vehicle movement stops.

Theoretical speed of a propeller is designated as that speed at which the propeller moves forward in the water at its pitch:

$$v_t = H n_s \quad (30)$$

H is the theoretical propeller path and n is the rotational speed of the propeller per second.

By actual return or slip we mean the difference between the theoretical propeller path and water acceleration on which propeller thrust depends:

$$S = H - H_a.$$

The apparent return or slip of the propeller is the difference between the theoretical and actual propeller path and vehicle path:

$$s = \frac{S}{H} = \frac{H - H_a}{H} = 1 - \frac{v}{H n_s}. \quad (31)$$

In the case of sea-going ships, slip changes from 20 to 40%; in the case of tanks from 40-70%.

b) <u>Identification of Ship Propellers</u>

1. According to direction of rotation we distinguish between:

   a) left-handed and
   b) right-handed propellers.

   At the same rotational direction, propellers can be classified according to puller-type and pusher-type propellers.

2. According to pitch:

   a) propellers with constant, invariable pitch,
   b) propellers with variable pitch, i.e. that the pitch corresponds to each radius.

Usually the pitch is increased from the center point to the circumference. The propellers named here exhibit no advantages over the propellers with invariable pitch.

3. According to the number of blades:

a) two blades,
b) three-bladed and
c) four-bladed propellers.

The fewer blades the propeller has, the less the resistance against movement and the greater the return (slip) and the less steady its performance.

4. According to the constructional material:

	Permissible stresses in kg/cm^2	
	Compression Strength	Tensile Strength
a) Cast iron	550	180
b) Steel (cast steel)	1900	800
c) Bronze (manganese bronze)	900	800

5. Propellers whose blades can be adjusted during travel are used to control direction of travel without changing the rotational direction of the propeller.

c) Operational Method and Efficiency of the Propeller

The purpose of the propeller is to convert the rotational movements generated by the engine into thrust.

By turning a machine screw into a stationary nut and exerting a certain force the screw can turn a body in front of it (as a result of the reaction exerted by the thread of the nut, i.e. the nut is the stationary support of the screw).

There is no such support point for a ship's propeller. Ship's propellers can only be supported on the moving water.

For this reason, in order to achieve a reaction on the part of the liquid, it is necessary that a certain amount of water be centrifuged away at a certain speed in the opposite direction.

When we repel any surface of the water, we can generate no reaction of the liquid without giving it velocity because as a result of viscosity, forces of friction can only be effective with the movement of a liquid layer and the weight of the water can cause a reaction only with a non-uniform movement.

Consequently, in order to move a tank, it is necessary to centrifuge the water at certain speed in the direction opposite the movement which is accomplished by twisting the propeller blade from the hub to the blade tip.

While the leading edge of the blade moves in the water, the trailing edge projected on the water surface is at a greater distance from a certain point than the leading edge, so that the water layer which comes into contact with the blade surface is centrifuged backward when the propeller turns. The result is a reaction of the liquid which thrusts the tank forward.

When the tank moves, the following movement occurs in the water. At a great distance from the propeller, water velocity equals zero. In the immediate vicinity, 1-2 diameter lengths from the propeller, the water already exhibits a certain velocity which is increased more and more in the direction of the propeller.

The flow velocity directly behind the propeller (as a result of thrust) will be designated by us as actual return or slip.

Behind the propeller, in relation to slip, the water jet is constricted which generates a greater power consumption. In addition the mass of water in question is given a rotating motion which requires an additional power efficiency (Fig. 33).

The water flowing toward the propeller (suction) is due to the fact that the pressure in front of the frontal surface is negative, so that the water layer in front of the propeller flows toward the propeller at a certain velocity.

If the propeller is mounted too close to the stern, the water flow toward the propeller will cause a pressure decrease on the stern and thus the resistance against tank movement will be additionally increased.

At a certain (large) rotational speed, the water flow declines. The space in front of the propeller is filled with air bubbles which causes a considerable reduction of thrust and consequently of tank speed. This phenomenon is designated as "cavitation" of the propeller.

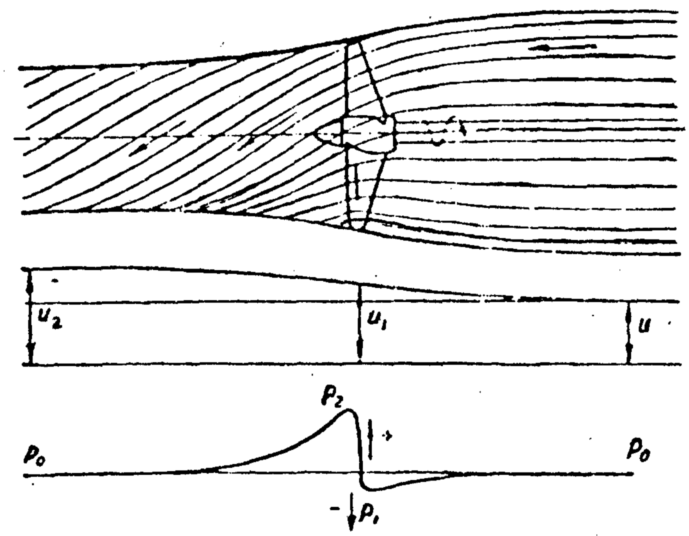


Figure 33. Behavior of the Water Filaments in the Propeller

In order to avoid this phenomenon, it is necessary to adhere to the following precondition for circumferential speed u and specific propeller load p:

$$u \leq 50\text{—}55 \text{ m/s},$$
$$p \leq 1 \text{ kg/cm}^2.$$

Consequently, the engine power at normal operation of the propeller is used for the following:

1. To overcome the resistance against forward movement of the tank N,
2. To constrict the water jet N_1,
3. To turn the water jet N_2, and
4. To overcome friction of the propeller blades N_3 on the water.

Consequently, we can write the following equation for the coefficient of progressive movement without considering the influence of the hull:

$$\eta_1 = \frac{N}{N + N_1 + N_2 + N_3}. \tag{32}$$

d) Behavior of Particles (Elements) of the Free Propeller

By a free propeller we mean a propeller which can be put into operation in water without the tank; i.e. it is installed in such a way that the water movements generated by the moving tank in the water do not touch the propeller.

By an "element" of the propeller, we mean the small arc of the propeller blade which was cut out with two cylindrical surfaces with the radii r and $r - \delta r$.

We wish to give closer consideration to the movement of the small arc and to the forces being exerted on it.

We select a small arc of a propeller blade which is located at an angle Θ to the propeller surface. By turning the propeller, the small arc of the propeller blade will traverse a distance of AB within one second, where u is the circumferential speed of the blade points with the radius of the section.

In the same time, this small arc together with the propeller hub will traverse a distance at speed v, the speed of the hub.

As a final result, the small arc will be shifted on the line AC at a speed equal to the section AC. Actually, when considering the water flow toward the propeller and a certain angular velocity of the water, the speed will equal the distance AD and be directed in direction AD.

Consequently, the section AD is the geometrical total of sections AC and DC.

Of course, the reaction of the water δR_y (the buoyancy force) which reacts on the small arc, will run parallel to the section CD, insofar as the movement of the liquid runs in the direction of action of force.

It is easy to see that if no friction forces occur, the section CD will be located vertically to AD, or, which amounts to the same thing, δR_y will be vertical to AD; for this reason we obtain the following equality:

$$\delta P v_1 = \delta Q u,$$

where v_1 is the axial velocity of the propeller in comparison to the turbulent water.

The friction force δR_x diverts δR_y into the position δR, and thus CD deviates from the vertical direction to AD.

If we break down δR into components, we obtain δP the resistance pressure and δQ the tangential force.

If the propeller would move in the water like a screw into a nut, then this section of the propeller blade would reach point E in one second. Section EF shows us the actual return or slip and section FC shows us the apparent return or slip due to suction.

The angle α which is formed by the chord of the propeller blade section and the direction line of relative speed AD is designated as the section angle of pitch.

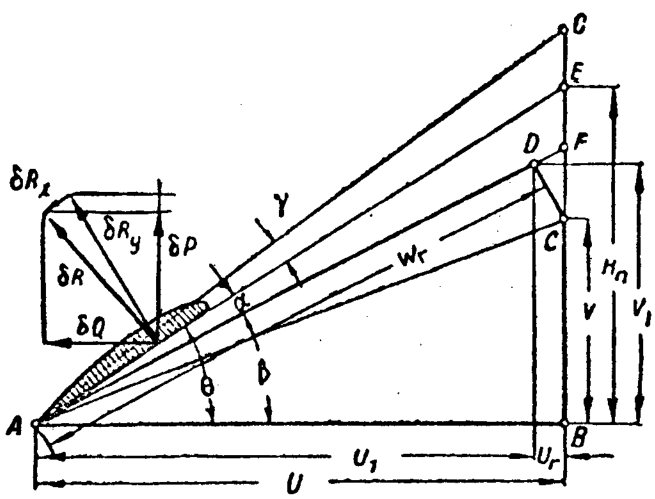


Figure 34. The Polygon of Velocities in the Propeller's Plane of Revolution

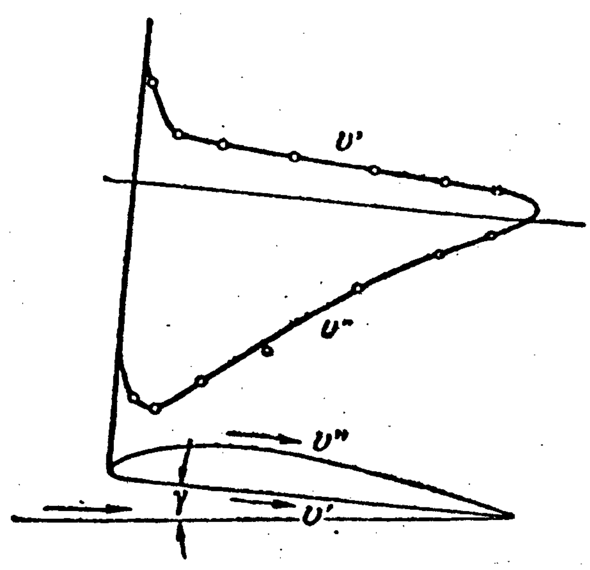


Figure 35. Pressure Distribution on the Blade Section

We are assuming that the section of the propeller blade has a symmetrical shape and when the propeller turns in the water it reaches point E within one second. In this case, the velocity CD of the water being churned up by the propeller and the angle of pitch α will equal zero and in accordance with these conditions the pitch BE = H_n is called the path of zero resistance.

Since in most cases the section of the propeller blade is asymmetrical, the speeds of the water currents which flow around the section on the front and rear operating surface are not equal (due to the inequality of the paths traversed by the currents) (Fig. 35).

As a final result the force of pressure being exerted on both surfaces (which are directed toward the opposite side) will also equal 0, and the small arc will receive a thrust from the side of the liquid which equals zero, if the pitch BG is greater than the theoretical propeller path BE (Fig. 34).

In this case we are dealing with an apparent slip GE and a negative angle of pitch. The dynamic pitch GE, which corresponds to zero resistance, will be designated as dynamic pitch.

Calculation of R_y R_x

The values R_y and R_x are calculated according to the experimental coefficients C_y and C_x, the magnitude of which were determined according to the law of similarity by measurements in the test channel and by recalculation of the forces determined from the test model and transferred to the actual equipment.

All such bodies, which are found in a similar position, undergo hydrodynamic forces on the side of the liquid which are calculated according to the following formulas:

$$\left.\begin{aligned} R_y &= C_y \varrho F v^2 \\ R_x &= C_x \varrho F v^2 \end{aligned}\right\} \qquad (33)$$

whereby

$$\varrho = \frac{\gamma \text{ kg/sec.}^2}{g \text{ m}^4}$$

F is the surface of the section in m^2 (projected surface),
v is the speed in m/sec.

According to the formulas cited above

$$C_y = \frac{R_y}{\varrho F v^2} \text{ and } C_x = \frac{R_x}{\varrho F v^2}.$$

According to flow measurements on the test channel, the result is usually represented in the form of curves, where one curve $C_y = f_1(\alpha)$ yields the other $C_y = f_2(\alpha)$.

c) Influence on the Propeller Efficiency when Mounted on the Rear of the Tank.

The operation of a free propeller, as is usually the case in tests, differs from the operation of propellers mounted on the stern.

Two additional preconditions must be considered:

1. The propeller suction as a result of a pressure decline between stern and propeller. For this reason, instead of thrust P which equals the resistance against movement of the tank R_o, the propeller must develop the following power:

$$P_0 = \frac{P}{1-t}$$

the coefficient

$$t = \frac{P_0 - P}{P_0} \qquad (34)$$

is designated as propeller suction coefficient.

2. The hull generates a concurrent flow, i.e. a flow with which a certain amount of water is drawn along by the tank at a certain speed in the direction of travel.

This flow (indraft) is caused by:

a) the friction between water and hull,
b) by the tendency of the ambient water to fill up the space from which water has been displaced by the underpart of the tank.

At a speed of this indraft of v_w the speed of the propeller stream is as follows:

$$v_w = v(1-w)$$

where w is the indraft coefficient.

The relation:

$$\frac{v}{v(1-w)} = \frac{1}{1-w} \qquad (35)$$

is designated as the coefficient of the influence of the indraft.

Consequently, this total coefficient of the influence of the hull is determined according to the equation

$$\eta_k = \frac{1-t}{1-w} \quad . \tag{36}$$

and the total coefficient (total efficiency) of forward movement of the propeller is determined according to the equation:

$$\eta = \eta_i \cdot \eta_k \quad . \tag{37}$$

Usually we assume that t = w, i.e. that the free propeller and the one behind the stern are in operation in the sense of the coefficient of forward motion under the same preconditions.

f) Special Remarks on the Influence of the Shape of the Tank Stern on the Efficiency of the Propeller.

Aside from the reasons already cited, the shape of the stern and the distance of the propeller from the stern surface has a very decisive influence on the efficiency of the propeller. We are assuming that we are dealing with the usual outline of the tank stern and with an anchored propeller.

At a large angle α and a small angle x, the water flowing under the bottom of the tank can flow further without coming in contact with the propeller, so that it will turn in "dead" water.

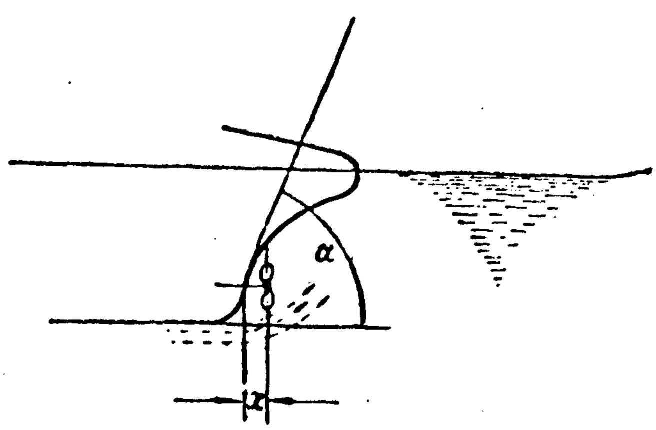


Figure 36. Propeller Behind the Stern

In order to avoid this, it seems desirable to have a small angle α and a sufficiently large value of x.

But the angle increases the space not utilized in the tank hull and at a higher value of x it can happen that the propeller breaks when the tank is surmounting obstacles.

These mutually opposite preconditions for the operation of the propeller can be fulfilled satisfactorily only by experience (tests).

The same must be said for the tunnel propeller which is used whenever the tank stern has a vertical surface according to design or has such an outline that special recesses must be provided to protect the propeller against break when traveling over terrain.

It can also happen here that the preconditions for the influx of water are not fulfilled satisfactorily and that the propeller does not generate the thrust necessary for forward motion.

As a result of these preconditions, the diameter of the propeller on the tank is not larger than 0.5 m which means that propeller efficiency is very small at large water resistance against tank forward movement and consequently there will be low thrust unless the manufacturer can make many changes.

g) Selection of the Propeller

The main purpose of studies devoted to ship propellers should offer the reader opportunity to select the best propeller with the greatest efficiency on the basis of general considerations in regard to design and method of operation.

The propeller is selected on the basis of tables drawn up by Taylor (USA) and Schaffran (Germany).

Drawings were made up on the basis of tests according to which it is possible to select the appropriate pitch and efficiency in relation to the size of the blade surface at given speed and performance.

The test results with screws of the respective (given) magnitudes at certain speeds can be extended to other propeller sizes and other preconditions (traveling speed, type of liquid); but care must be taken that the similarity of the propellers and the preconditions remain similar for their operation.

Similar propellers are those with which the mutual dimensional relationships have a constant value.

Similar operational conditions are those where an equality prevails between the following coefficients:

$$\left.\begin{aligned}\lambda &= \frac{v_w}{n_s D}\\ \beta &= \frac{N}{\varrho n_s^3 D^5}\\ \alpha &= \frac{P}{\varrho n_s^2 D^4}\end{aligned}\right\} \quad (38)$$

v_w is the amphibious speed of the tank in m/sec, i.e. the speed in the propeller plane with consideration of the indraft coefficient.

n_s = the rotational speed of the propeller per sec.,

D = the diameter of the propeller in m,

N = the power furnished by the engine in mkg/sec.,

ϱ = water viscosity; in fresh water is

$$\varrho = 0{,}102 \frac{kg/sec.^2}{m^4}.$$

The preconditions with regard to the similarity of propellers are adhered to if the values λ, β and α are equal.

It is easy to become convinced that the coefficient of progressive movement of the propeller can be expressed by the named coefficients in the following equation:

$$\eta = \frac{\alpha \lambda}{\beta} \quad (39)$$

Of all the reworked Schaffran diagrams, the simplest from the standpoint of applicability are those by Schmidt, which I introduce here as an example (Fig. 37).

Log β is divided on the axis of the ordinates and log λ on the axis of the abscissa. The numerical values of these magnitudes are given on the diagram.

To recognize how the change of the diameter or the rotational speed affects propeller efficiency at the given β and λ, it is sufficient to draw straight lines with the pitch 5/1 and 3/1 to the abscissa from the initial point of the coordinates.

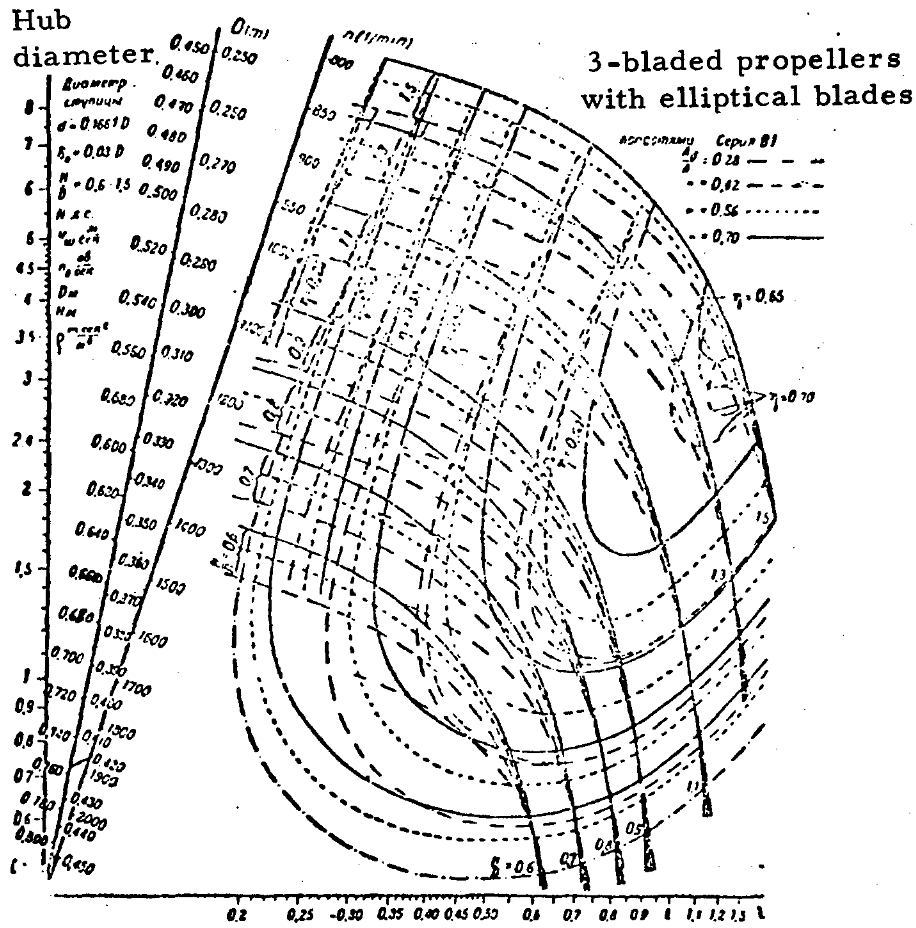


Figure 37. Diagram by Schaffran for the Selection of Propellers, reworked by Schmidt.

If we have a point on the diagram for the given magnitudes β and λ and we would like to achieve a change of the propeller diameter, then we will determine a change of the coordinates from the following equations:

$$\log\left(\frac{v}{n_s D_2}\right) - \log\left(\frac{v}{n_s D_1}\right) = -(\log D_2 - \log D_1)$$

$$\log\left(\frac{N}{\varrho n_s^3 D_2^5}\right) - \log\left(\frac{N}{\varrho n_s^3 D_1^5}\right) = -5(\log D_2 - \log D_1).$$

Consequently, the sections which correspond to the following values: $(\log D_2 - \log D_1)$ and $-5(\log D_2 - \log D_1)$ will come under consideration as the coordinates of the new point; i.e. all new points on the diagram which result from the change of the propeller diameter will lie on the straight line with the inclination 5:1.

With appropriate considerations we can also prove that all new points with a change of the rotational velocity rate will lie on a straight line with the angular coefficient 3.

I will now introduce an example for the choice of a propeller by using the aforementioned Schmidt diagrams.

I will assume that we are seeking a tank propeller whose travel resistance curve is known and whose desired velocity is planned as 8 km/h, so that the corresponding resistance will be 425 kg.

We will first take up the dimensional data for the propeller: according to design we will select a propeller with a 0.4 m diameter.

The propeller has three blades (with consideration of the specific load of the unit surface of the proposed surface AD).

The rotational speed is 1300.

We select the ratio between the proposed and the pass through surface:

$$\frac{A_d}{A} = 0{,}56.$$

We wish to select the theoretical speed of the tank at which the propellor thrust amounts to 425 kg.

For this purpose we turn to approximate formulas for thrust:

$$P = \varrho A_d v_T (v_T - v).$$

Here P is thrust; in our case P amounts to 425 kg and the water viscosity --- 1000 kg/m^3.

We will substitute the known values into the equation and obtain the following equation:

$$v_T^2 - 2.2\, v_T - 34{,}6 = 0$$

From this we obtain:

$$v_T = 1{,}11 \pm \sqrt{1{,}11^2 + 34{,}6} = +\ 7{,}2 \text{ m/sec.} = 26 \text{ km/h}.$$

In approximate magnitude (without considering the indraft coefficient) we determine the propeller efficiency according to the following equation:

$$\eta = \frac{v}{v_T} = 0{,}308.$$

Consequently, the necessary power efficiency for driving the amphibious tank at v = 8 km/h and P = 425 kg will be

$$N = \frac{425 \cdot 8}{270 \cdot 0{,}308} = 41\ HP$$

We are assuming that N = 40 HP.

Now we have the following initial basic values for selecting the propeller:

Propeller diameter D = 0.4 m,
Number of blades: 3,
Rotational rate of the propeller per minute 1300.

The power efficiency required from the engine N = 40 HP.

We are assuming that the indraft is w = 0.25 and the suction coefficient is t = 0.25.

The speed of the propeller plate is:

$$v_w = \frac{8}{3{,}6}(1 - 0{,}25) = 1{,}66\ \text{m/sec.}$$

$$\lambda = \frac{1{,}66}{21{,}7 \cdot 0{,}4} = 0{,}193$$

$$\beta = \frac{40}{0{,}102 \cdot 21{,}7^3 \cdot 0{,}4^5} = 3{,}74.$$

We select the diagram for the propeller with three blades, the one with the oval blades in row B 1.

If we divide the determined values λ and β we will find that h = H/D = 0.9 and η = approx. 0.23, because the point lies almost beyond the boundaries of the curves.

In order that the propeller does not "cavitate", the specific load must not be selected higher than 1.0-1.1 kg/cm^3 and the circumferential speed of the blades u (for ships) must not be selected higher than 50-60 m/sec.

In our case

$$p = \frac{P}{A_d} = 0{,}66\ \text{kg/cm}^2 \quad \text{and}$$

$$u = 27{,}0\ \text{m/sec.}$$

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13. ABSTRACT

This is a translation of the original Russian text and its contents include theoretical considerations of tractive effort, stability, turning ability and amphibiousness of tanks in addition to their various spring types.

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